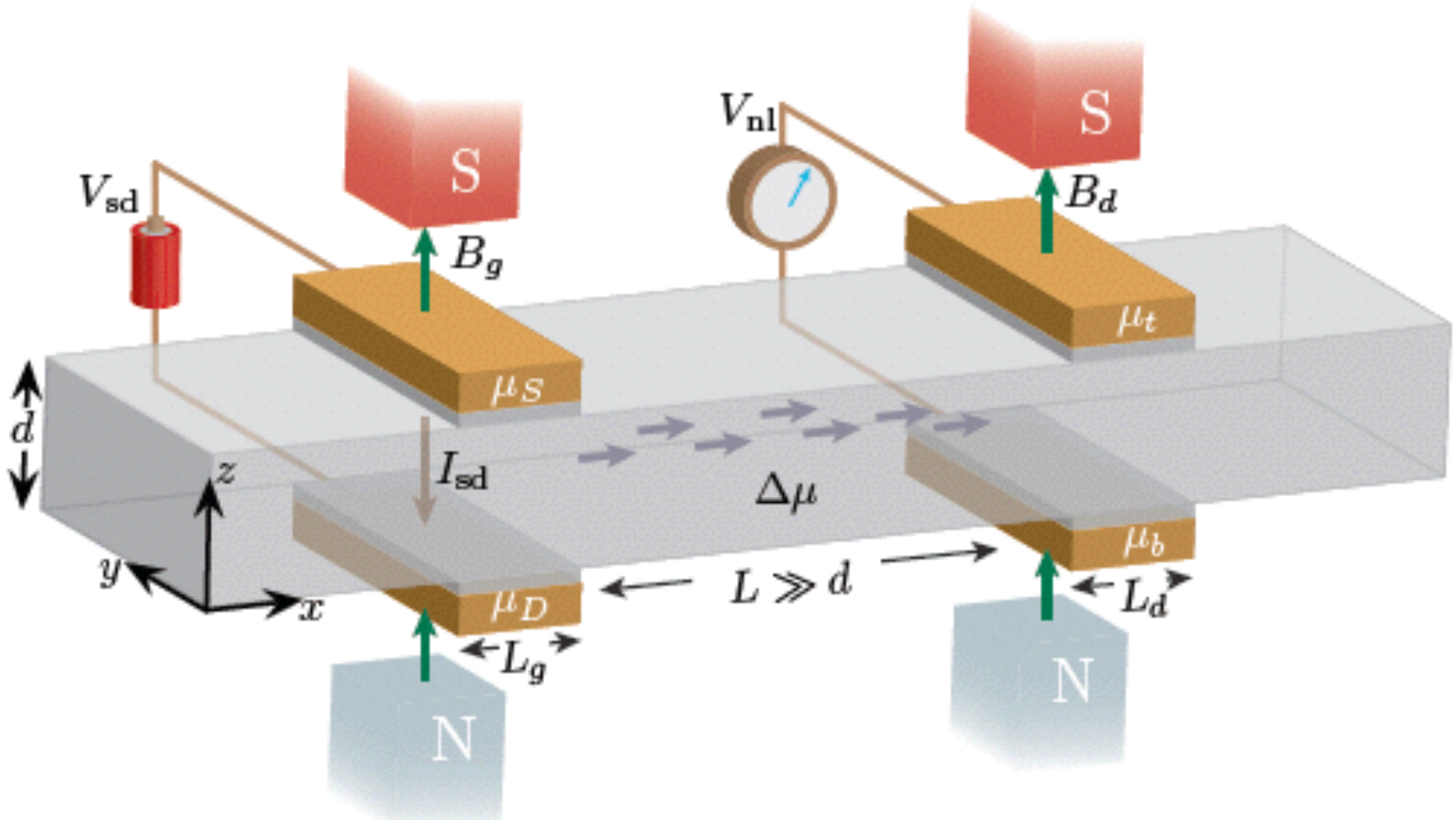
'Topological' Transport in Topological Semimetals



Siddharth Parameswaran

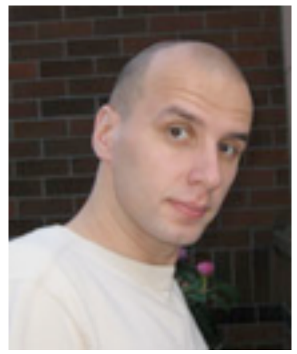
UC Irvine

Max Planck-UBC School, Vancouver, Oct 22-25, 2015

Collaborators

Ashvin Vishwanath

Pavan Hosur



Dmytro Pesin

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Yuval Baum

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Hosur, SP, Vishwanath, *Phys. Rev. Lett.* **108**, 046602 (2012)

SP, Grover, Pesin, Abanin, Vishwanath, *Phys. Rev. X* **4**, 031035 (2014)

Baum, Berg, SP, Stern, arXiv:1508.03047 (2015)

SIMONS FOUNDATION

Basic goal of condensed matter: classifying phases

One simple way

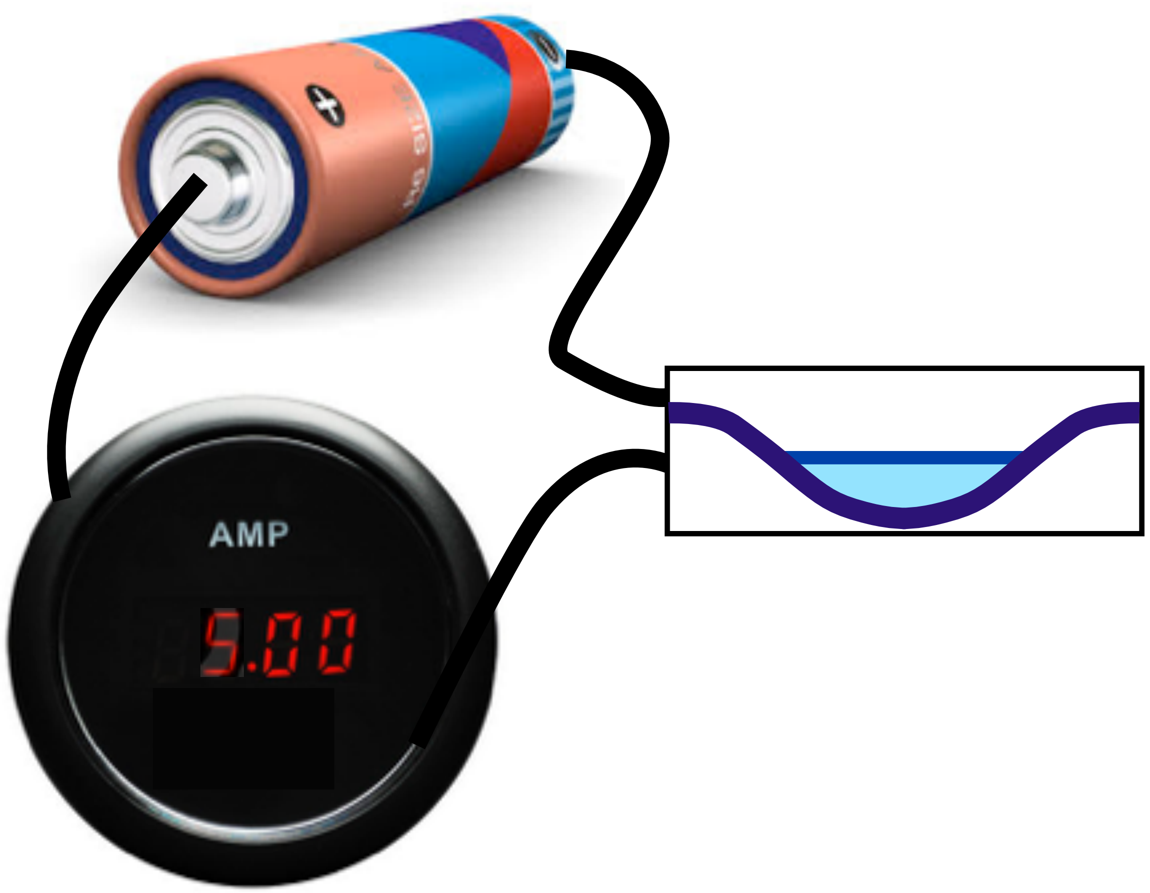
Metals

Insulators

Images: http://wikipedia.com, http://www.androidspin.com, http://www.nauticexpo.com

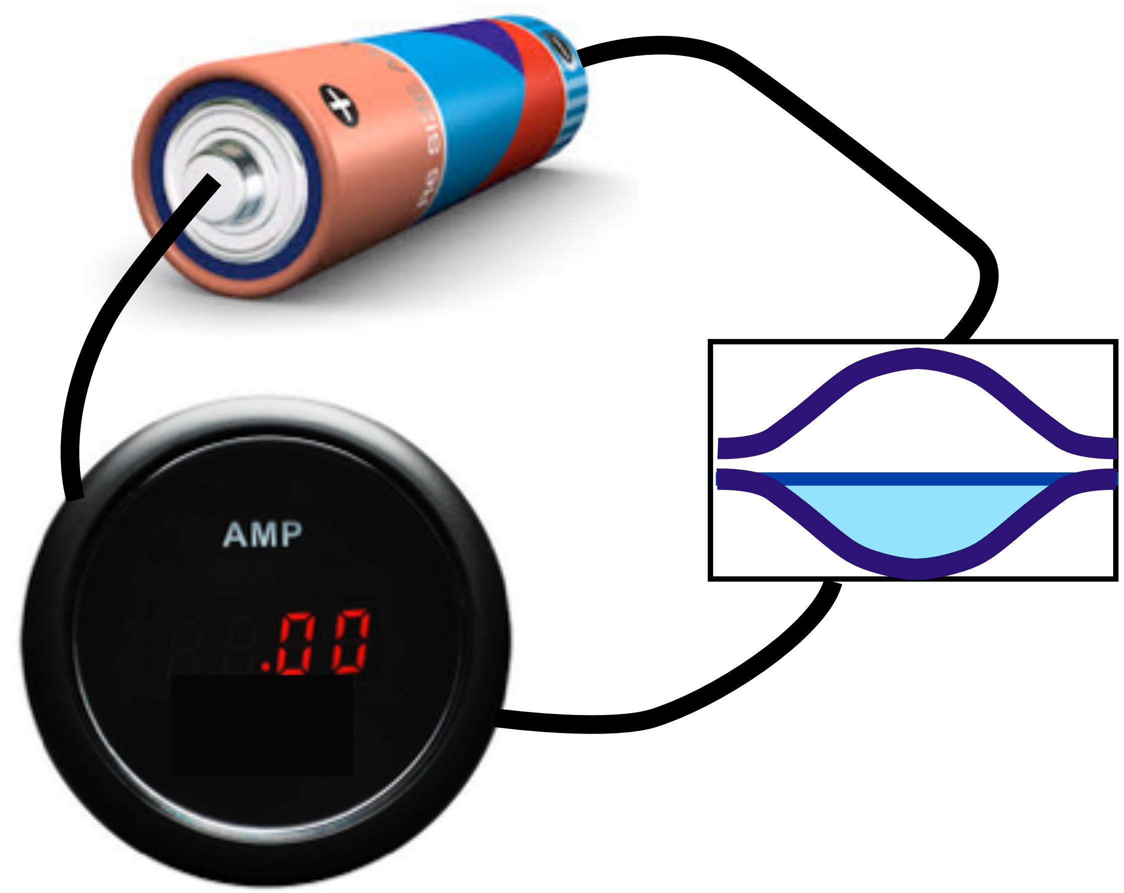
Band theory connects transport with symmetry

Bloch's theorem + Schrödinger equation ⇒ energy bands



Metals: partially filled/gapless

(semimetals~filled bands touch other bands)



Insulators: filled/gapped

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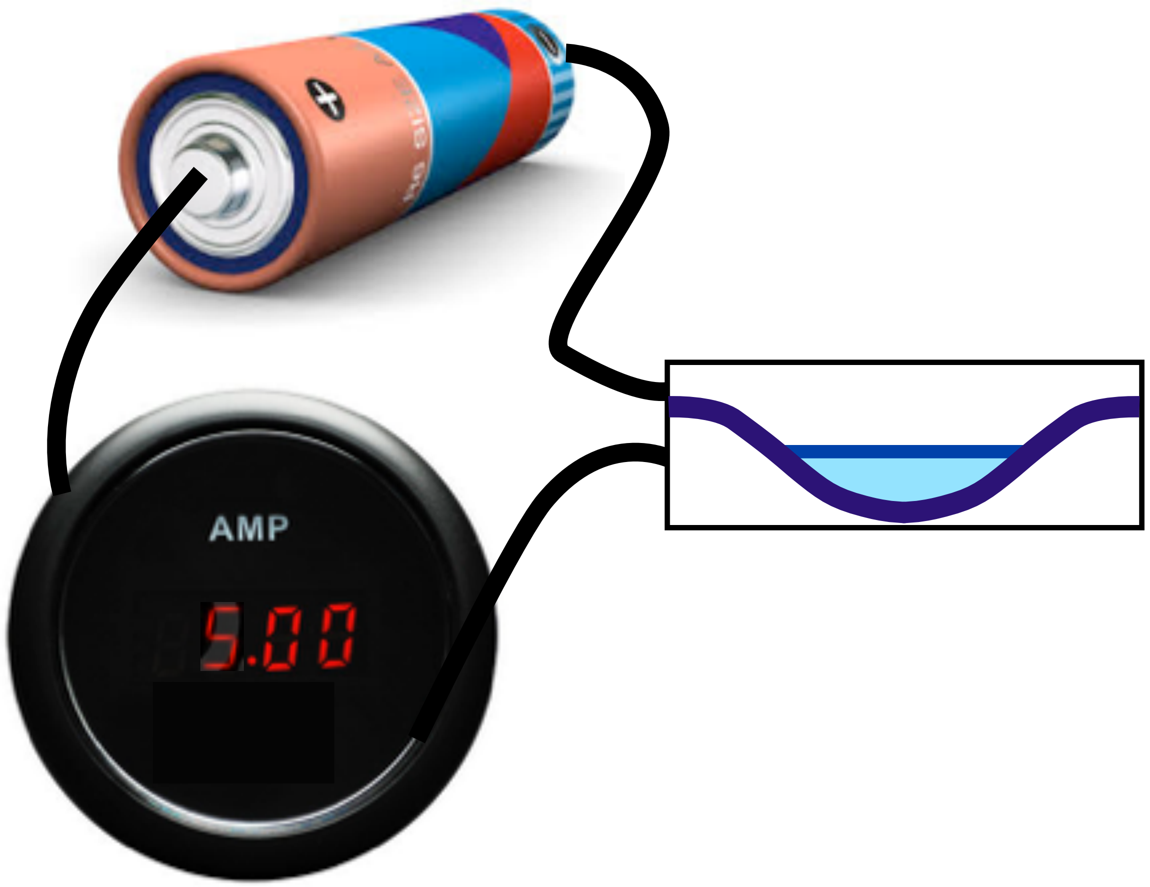

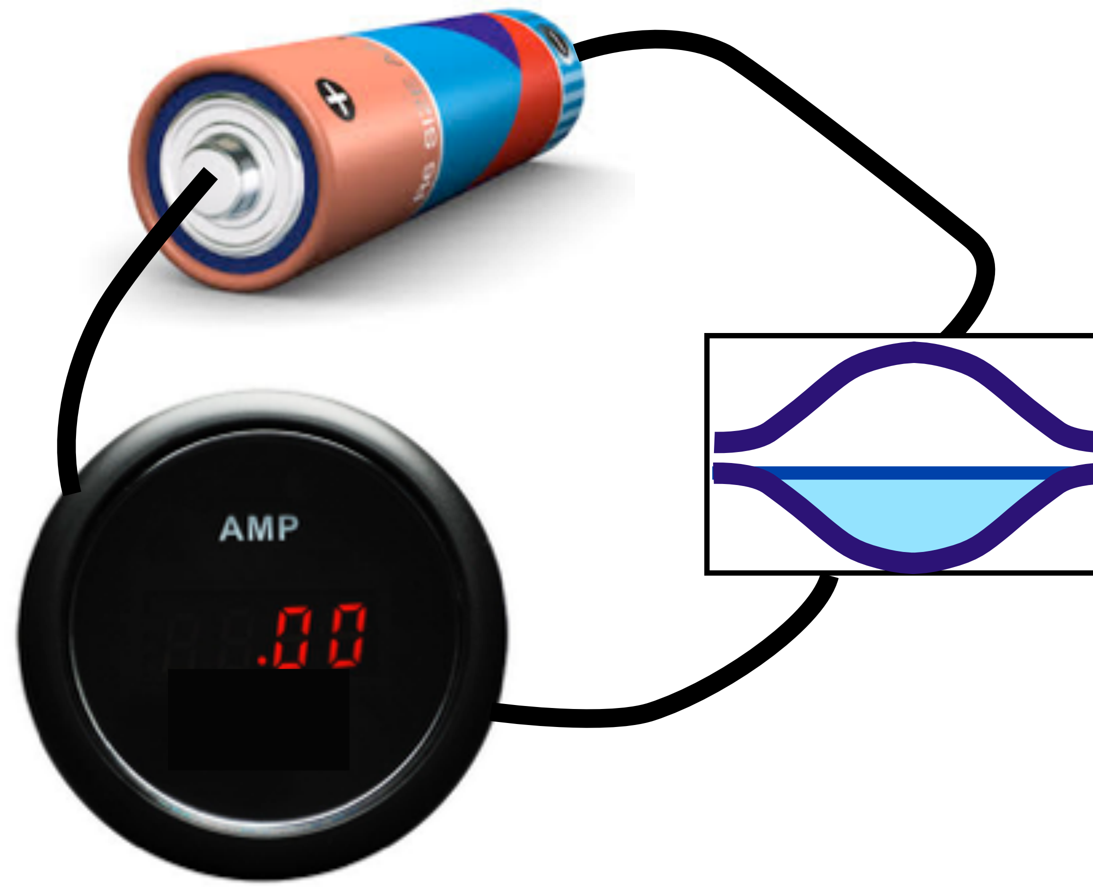

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How do we connect transport with *topology*?

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Semiclassical Transport in Solids

Electrons in a crystal + *E* and *B* fields

Solve Schrödinger equation with *E*, *B*=0, get Bloch bands

$$\hat{H}_0(\mathbf{k})|\psi_{0,n}(\mathbf{k})\rangle = \mathcal{E}_{0,n}(\mathbf{k})|\psi_{0,n}(\mathbf{k})\rangle$$

<u>Semiclassics</u>: wavepacket for Bloch electron, position *x* & average momentum *k*

How does this evolve when *E*, *B*≠0 ?

[Bloch & Peierls, '33; Jones & Zener '34; see Ashcroft & Mermin for intro.]

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velocity = 'band' (group) velocity

$$\dot{\mathbf{x}} = \frac{1}{\hbar}\frac{\partial \mathcal{E}_{0,n}(\mathbf{k})}{\partial \mathbf{k}}$$

force = Lorentz force

$$\hbar\dot{\mathbf{k}} = -e\,\mathbf{E} - e\,\dot{\mathbf{x}} \times \mathbf{B}$$

[Bloch & Peierls, '33; Jones & Zener '34; see Ashcroft & Mermin for intro.]

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Define 'electric' field in *k*-space $\tilde{\mathbf{E}}(\mathbf{k}) \equiv \frac{\partial \mathcal{E}_{0,n}(\mathbf{k})}{\partial \mathbf{k}}$

Then the semiclassical equations become

$(\hbar = e = 1)$

$$\dot{\mathbf{x}} = -\tilde{\mathbf{E}}(\mathbf{k})$$

$$\dot{\mathbf{k}} = -\mathbf{E}(\mathbf{x}) - \dot{\mathbf{x}} \times \mathbf{B}(\mathbf{x})$$

look similar, but
asymmetric

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missing piece: 'magnetic' field in *k*-space

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Key: <u>Berry's phase</u>

Berry's Phase: A Quick Review

Basic idea: adiabatically evolve *H* by changing its parameters R in time

system 'follows' *eigenstate* (adiabatic so no gap closing)

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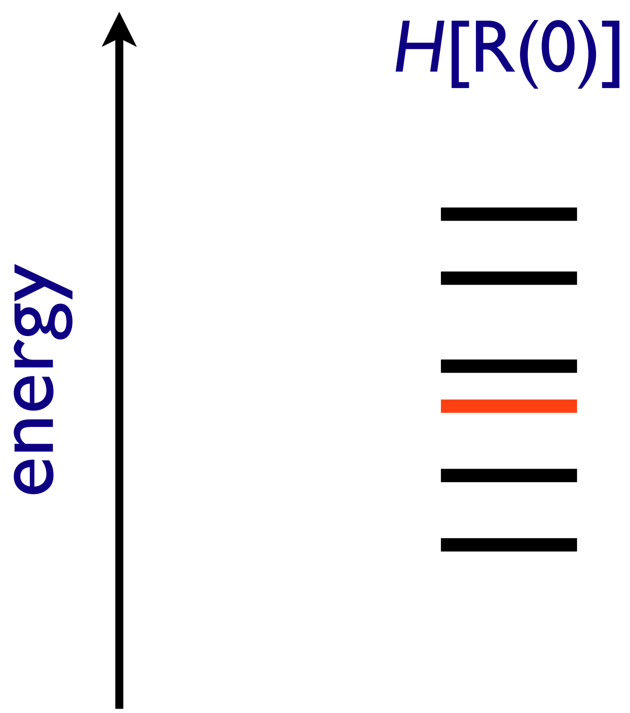

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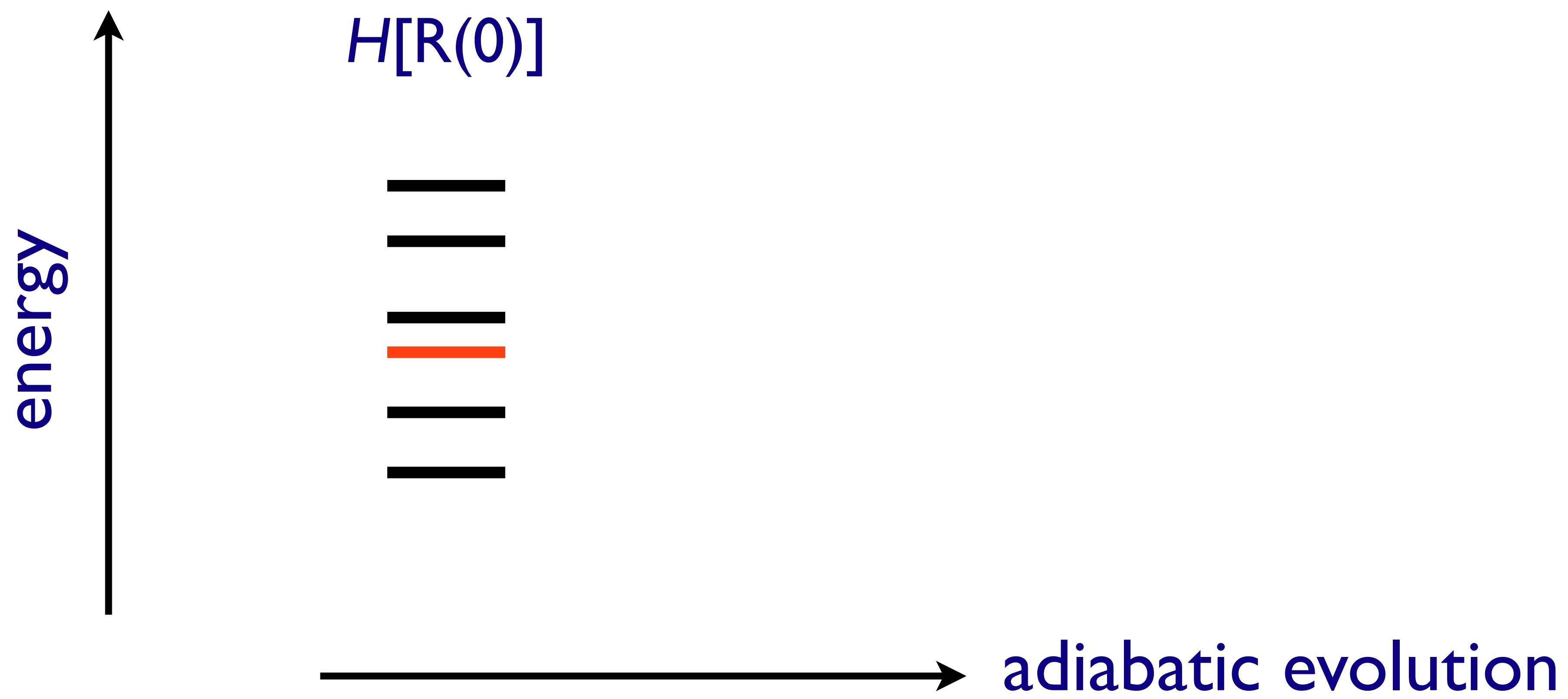

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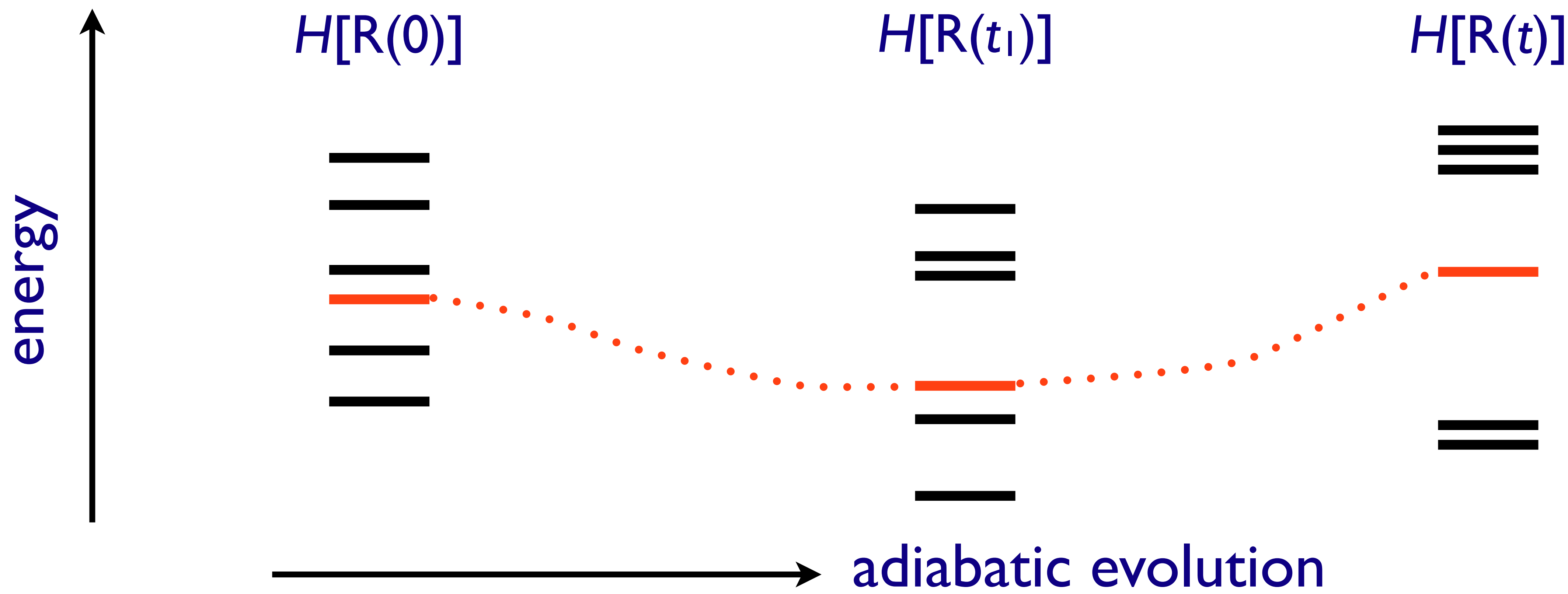

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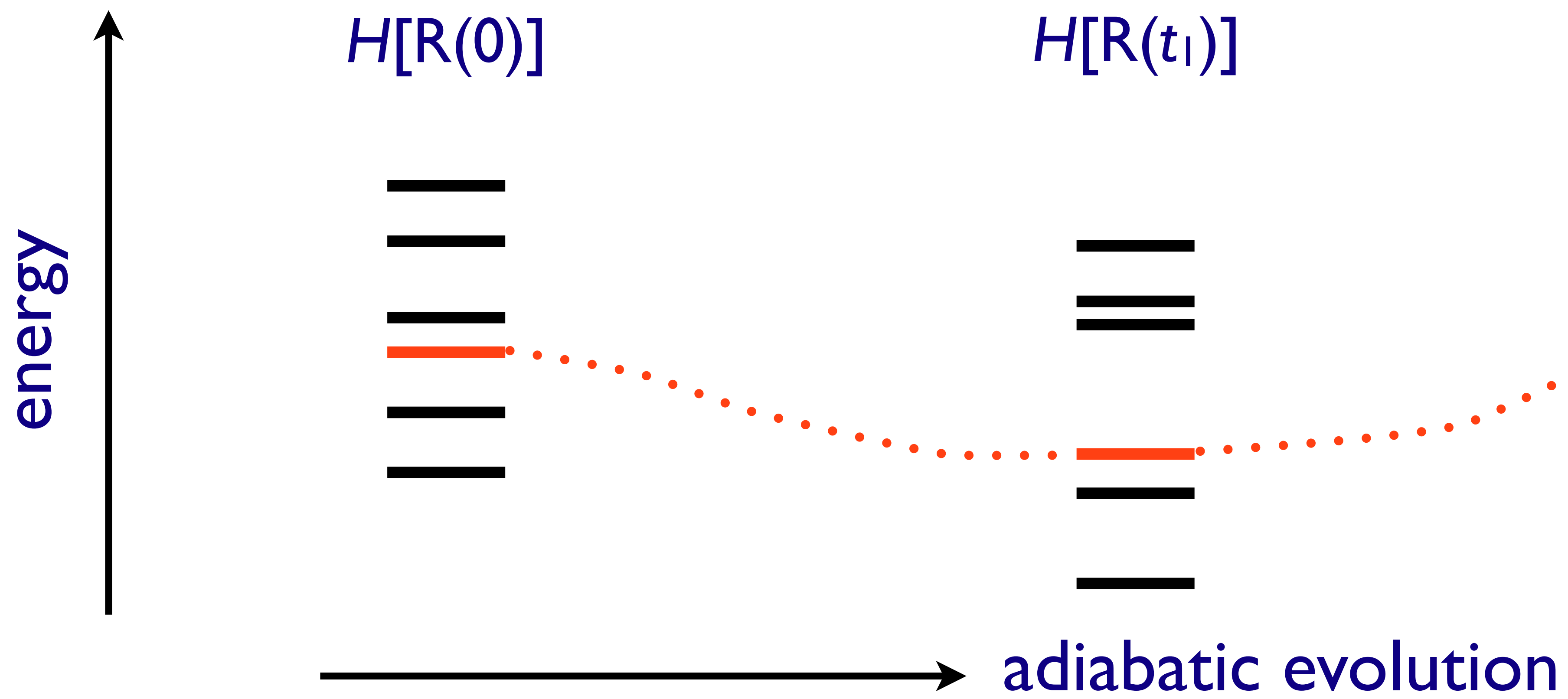

Natural guess for eigenstate evolution:

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$$|n(\mathbf{R}(0))\rangle \longrightarrow e^{-i\int_0^t E_n(t')dt'}|n(\mathbf{R}(t))\rangle$$

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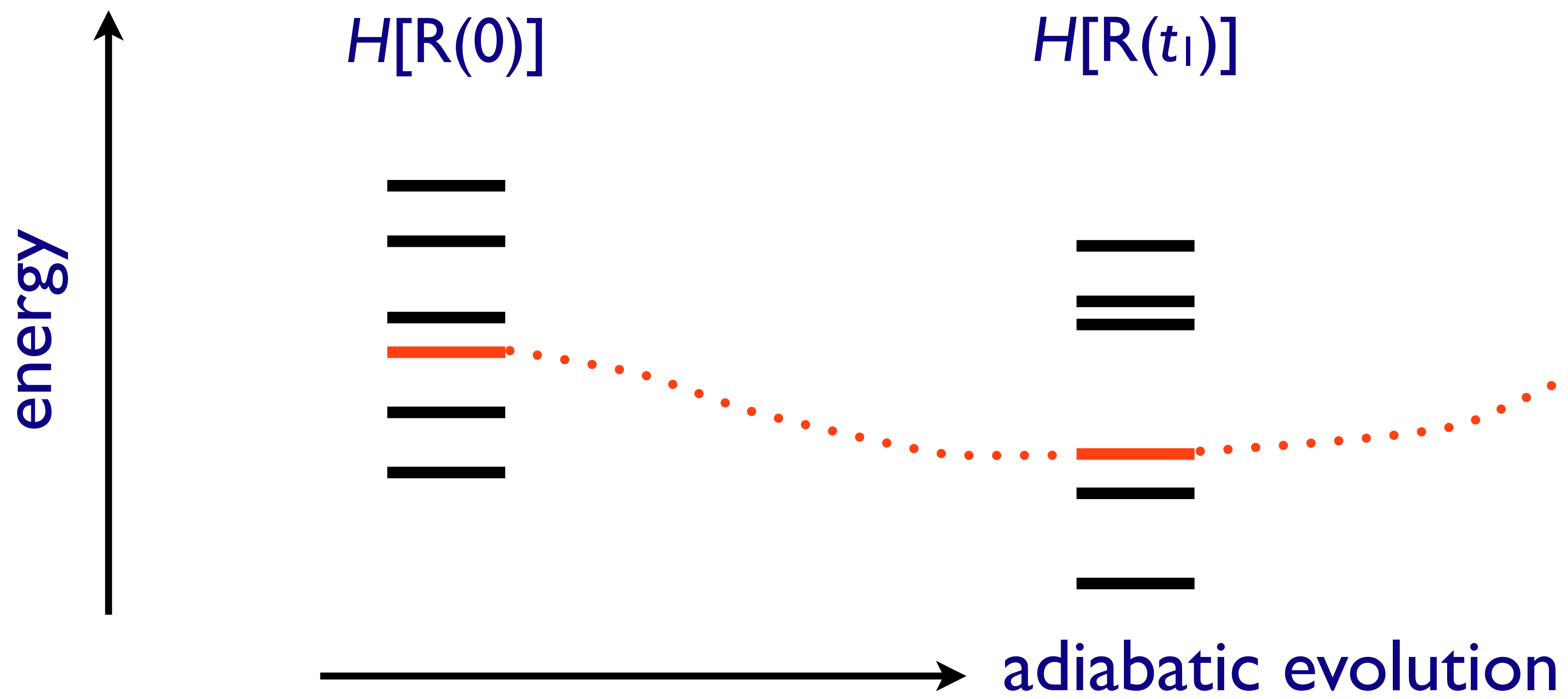

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$$|n(\mathbf{R}(0))\rangle \longrightarrow e^{i\gamma_n(t)} e^{-i\int_0^t E_n(t')dt'} |n(\mathbf{R}(t))\rangle$$

Missing piece: Berry's Phase

Berry's Phase

How did we get the 'dynamical phase' $e^{i\int_0^t E_n(t')dt'}$?

Between t, $t+\Delta t$, state n picks up phase $E_n\Delta t$

Add up that phase for each bit of time

But the *state* itself changes in that time!

$$|n(\mathbf{R}(t))\rangle \neq |n(\mathbf{R}(t+\Delta t))\rangle$$

Berry's phase is how we account for this change!

After some work, can show

$$\gamma_n(t) = i\int_{\mathbf{R}(0)}^{\mathbf{R}(t)} d\mathbf{R}\langle n(\mathbf{R})|\nabla_{\mathbf{R}}|n(\mathbf{R})\rangle$$

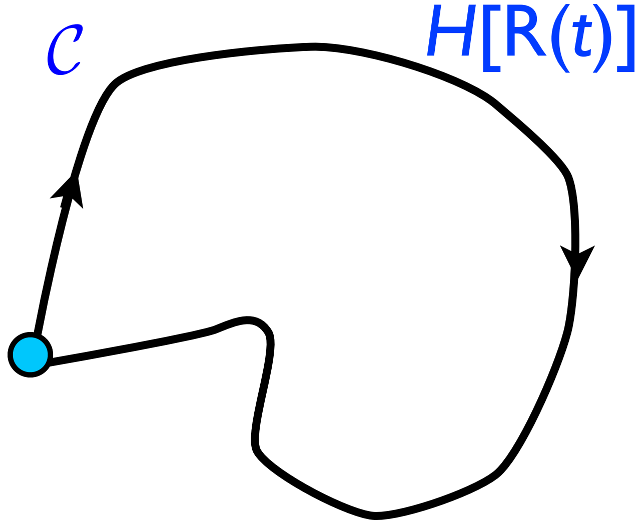
(depends only on path R(t), not on details of how R changes with t)

What's in a Phase?

Normally, overall phase unimportant

(can remove by suitable redefinition of kets at each point)

But in some cases, evolution is <u>periodic</u> *R*(t)=*R*(0)



Then, if Berry phase for a 'cycle' is nonzero

$$\gamma_{\mathcal{C}} = i \oint_{\mathcal{C}} d\mathbf{R} \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} | n(\mathbf{R}) \rangle$$

<u>no</u> choice of phase can make it zero

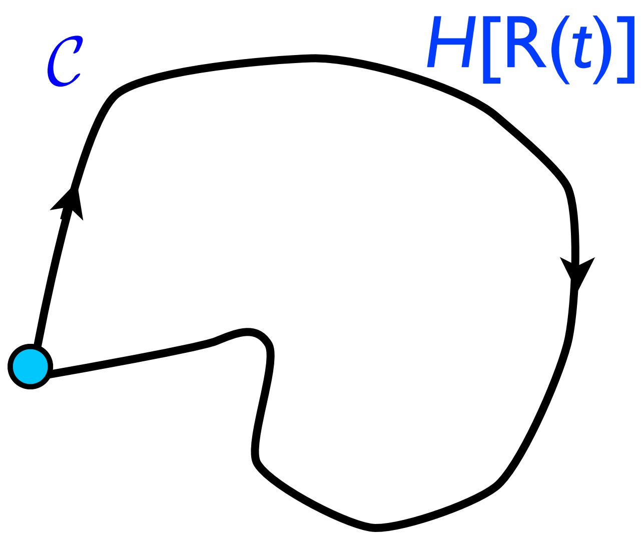
What's in a Phase?

One way to think about Berry phase is in terms of a 'vector potential'

$$\mathcal{A}_n(\mathbf{R}) = i\langle n(\mathbf{R})|\nabla_{\mathbf{R}}|n(\mathbf{R})\rangle \qquad \gamma_n = \int \mathcal{A}_n(\mathbf{R}) \cdot d\mathbf{R}$$

Most vector potentials are unimportant! ('gauge away')

'Magnetic Field' $\tilde{B} = \nabla \times \mathcal{A}$ is what matters



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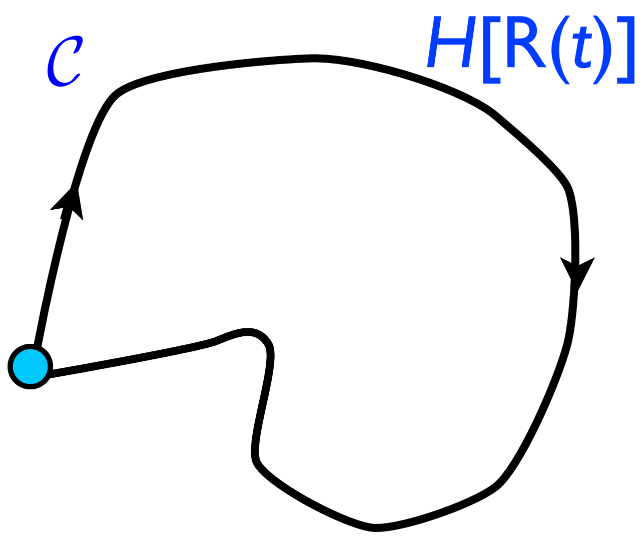

Stoke's Theorem

$$\gamma_{\mathcal{C}} = \oint_{\mathcal{C}} \mathcal{A}_n(\mathbf{R}) \cdot d\mathbf{R}$$
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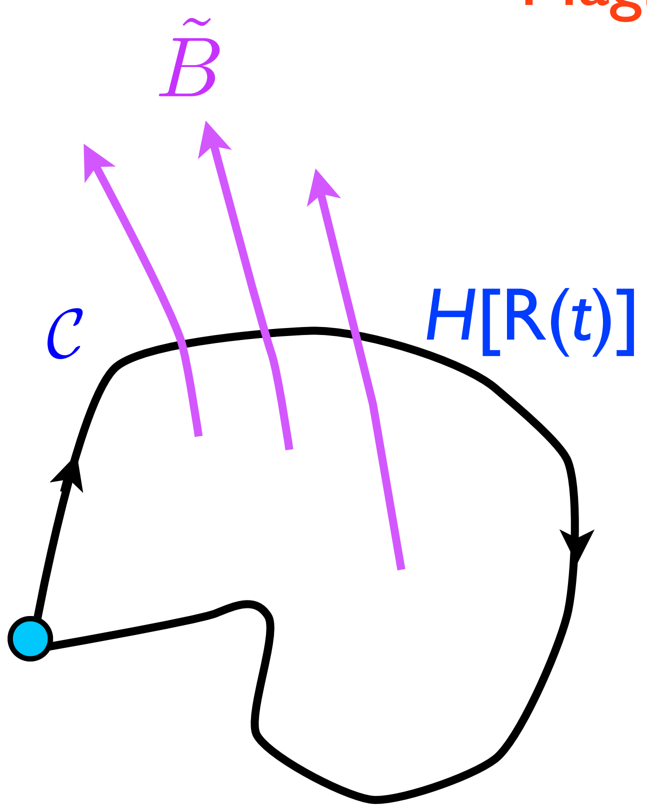

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Berry phase for loop = 'Berry flux' through it

Semiclassical Transport in Solids

Bloch states live on 'torus' in *k*-space (**k** ≡ **k** + 2π)

Their evolution is always periodic!

$$\hat{H}_0(\mathbf{k})|\psi_{0,n}(\mathbf{k})\rangle = \mathcal{E}_{0,n}(\mathbf{k})|\psi_{0,n}(\mathbf{k})\rangle$$

k evolves as electron wavepacket moves, plays role of *R*(t)

Berry phases matter!

[Karplus & Luttinger 1954; Niu & Sundaram 2000; Haldane 2004]

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Berry flux ~ momentum-space magnetic field

$$\tilde{B}(\mathbf{k}) = \nabla_{\mathbf{k}} \times [i\langle\psi_{0,n}(\mathbf{k})|\nabla_{\mathbf{k}}|\psi_{0,n}(\mathbf{k})\rangle]$$

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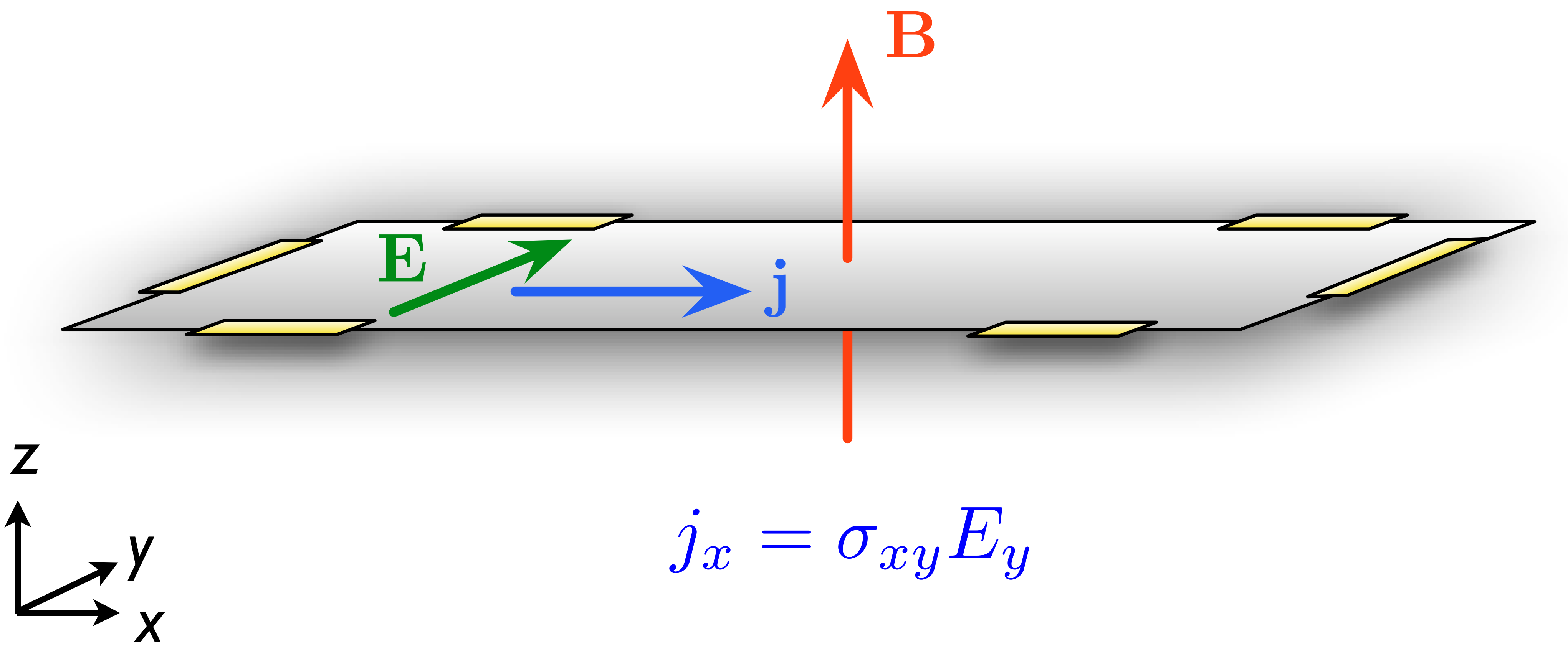
additional contribution often called 'anomalous velocity'

[Karplus & Luttinger 1954; Niu & Sundaram 2000; Haldane 2004]

An example: 'Intrinsic' Anomalous Hall Effect

Conventional Hall effect:

- charge carriers deflected due to external magnetic field
- additional voltage <u>transverse</u> to current



$$j_x = \sigma_{xy} E_y$$

- important probe: gives sign of charge carriers, etc....

An example: 'Intrinsic' Anomalous Hall Effect

no external magnetic field (B=0)

microscopic time-reversal symmetry-breaking
e.g. due to magnetism

'extrinsic': due to asymmetric scattering with TRSB

'intrinsic': due to Berry phase of energy bands

$$\sigma_{xy}^{\text{AH-int}} = -\frac{e^2}{\hbar}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^d}\hat{\mathbf{z}}\cdot\tilde{B}_n(\mathbf{k})\, f[\mathcal{E}_n(\mathbf{k})]$$

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Berry flux
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[Karplus & Luttinger 1954; Niu & Sundaram 2000; Haldane 2004; review: Nagaosa *et al* 2009]

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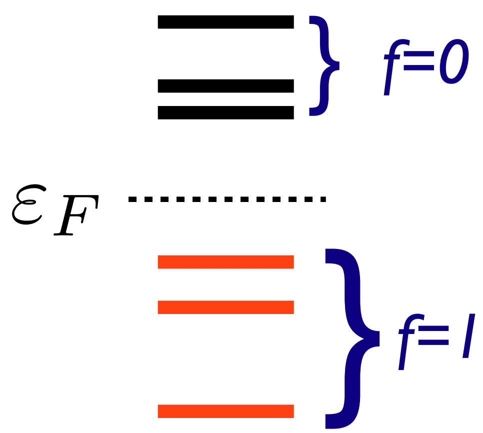
Berry flux of band *n*

occupation of band *n*

[Karplus & Luttinger 1954; Niu & Sundaram 2000; Haldane 2004; review: Nagaosa *et al* 2009]

Quantized Anomalous Hall Effect

$$\sigma_{xy}^{\text{AH-int}} = -\frac{e^2}{\hbar}\sum_n \int \frac{d\mathbf{k}}{(2\pi)^d}\hat{\mathbf{z}}\cdot\tilde{B}_n(\mathbf{k})\, f[\mathcal{E}_n(\mathbf{k})]$$



Fermi energy in gap ~ insulator

pick up total
Berry flux from each filled band

total Berry flux of a band must be 2π × integer

$$\sigma_{xy}^{\text{QAH}} = \frac{e^2}{h}\times \text{integer}$$

fixed as long as Fermi energy in gap!

[expt. in $(Bi, Sb)_2Te_3$: C-Z Chang *et al.*, *Science* **340**, 167 (2013)]

'Strong' TIs : no TRSB so $\sigma_{xy}=0$, but nonzero 'Z_2 invariant' ⇒ helical edge states

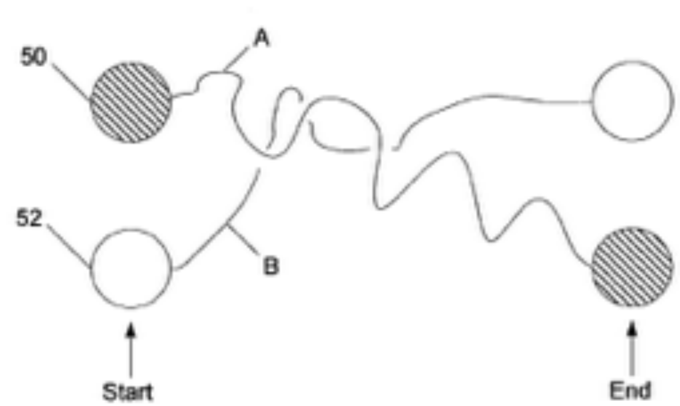
Why do we care?

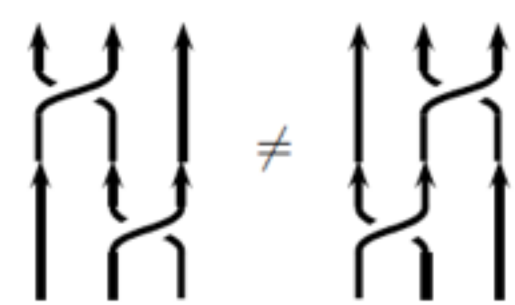
Topological insulators and related systems* represent fundamentally new phases of matter

Topological properties are very universal: robust to disorder, sample imperfections, etc...

May be possible to engineer 'topologically protected' quantum computers

Microsoft® Research
Station Q





Exotic materials, with potential for unforeseen applications

*also quantum Hall states, topological superconductors, quantum spin liquids...

<u>This Talk</u>: Topological Semimetals

topological <u>insulators</u> ⇒ gapped in the bulk

only surface is 'interesting': Dirac cones, transport, WAL etc...

The Invisible Man, HG Wells, Signet Ed.; Son of Man, Rene Magritte

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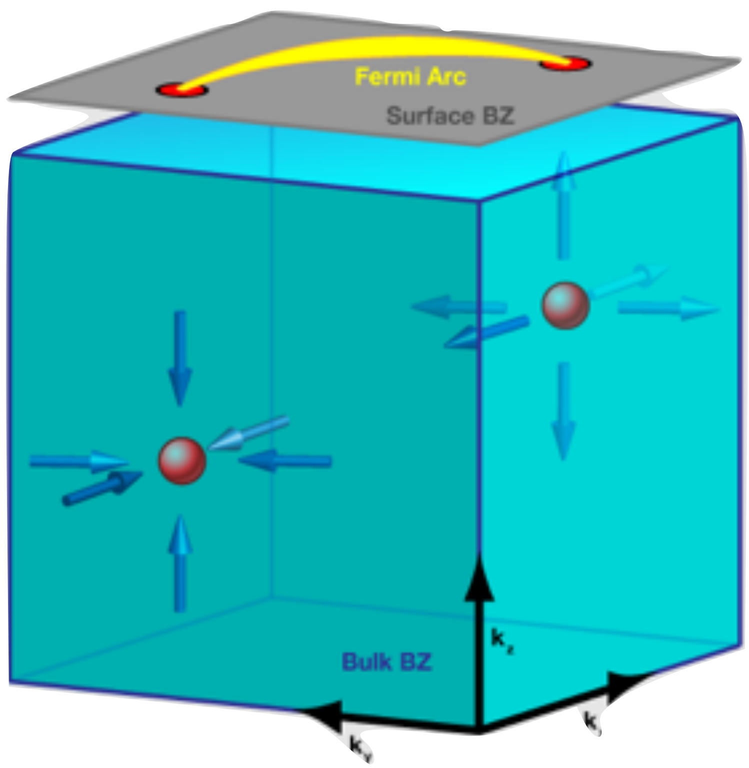
Can band topology play a role even when bulk is gapless?

The Invisible Man, HG Wells, Signet Ed.; Son of Man, Rene Magritte

‘Weyl semimetals’ ~ 3D graphene

Two bands touch at a point node w/o fine-tuning in d=3

(needs broken inversion/time reversal)



[Herring, ‘37; Abrikosov, Beneslavskii ’71; X. Wan *et al.* ’11]

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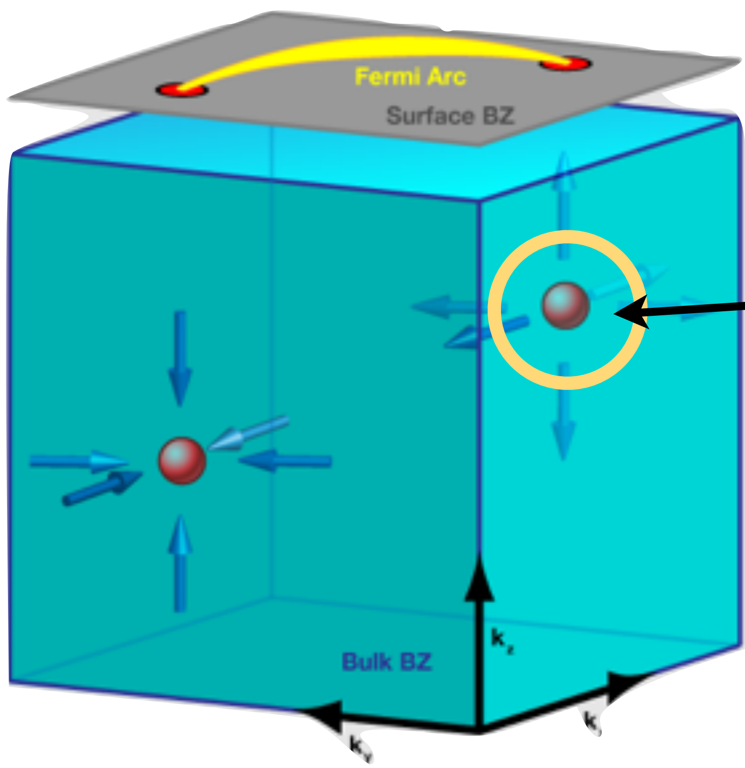

To describe energy levels near node: 2x2 matrix equation

$$H \approx \mu \mathbf{1} + k_x\sigma_x + k_y\sigma_y + k_z\sigma_z$$

~ 'Weyl equation'

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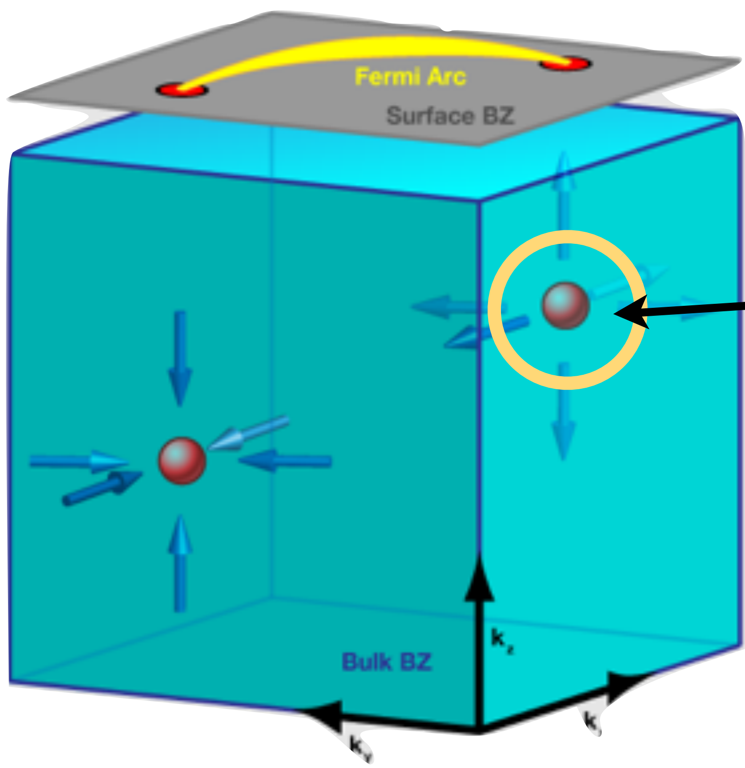

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all 3 Pauli matrices used ⇒ cannot gap single node

Pairs of nodes separated in BZ ⇒ robust w/ transl. symmetry

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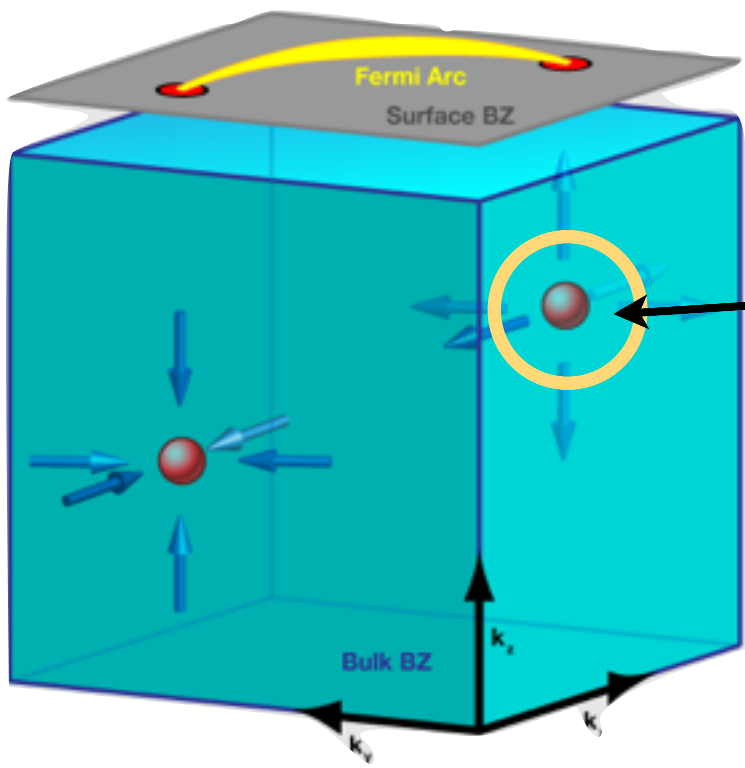

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For 'topological' reasons, nodes come in pairs. Why?

[Herring, '37; Abrikosov, Beneslavskii '71; X. Wan *et al.* '11]

Node ~ 'monopole' source or sink of Berry Flux

BZ = torus, 'Berry-Gauss law' ⇒ zero <u>total</u> charge

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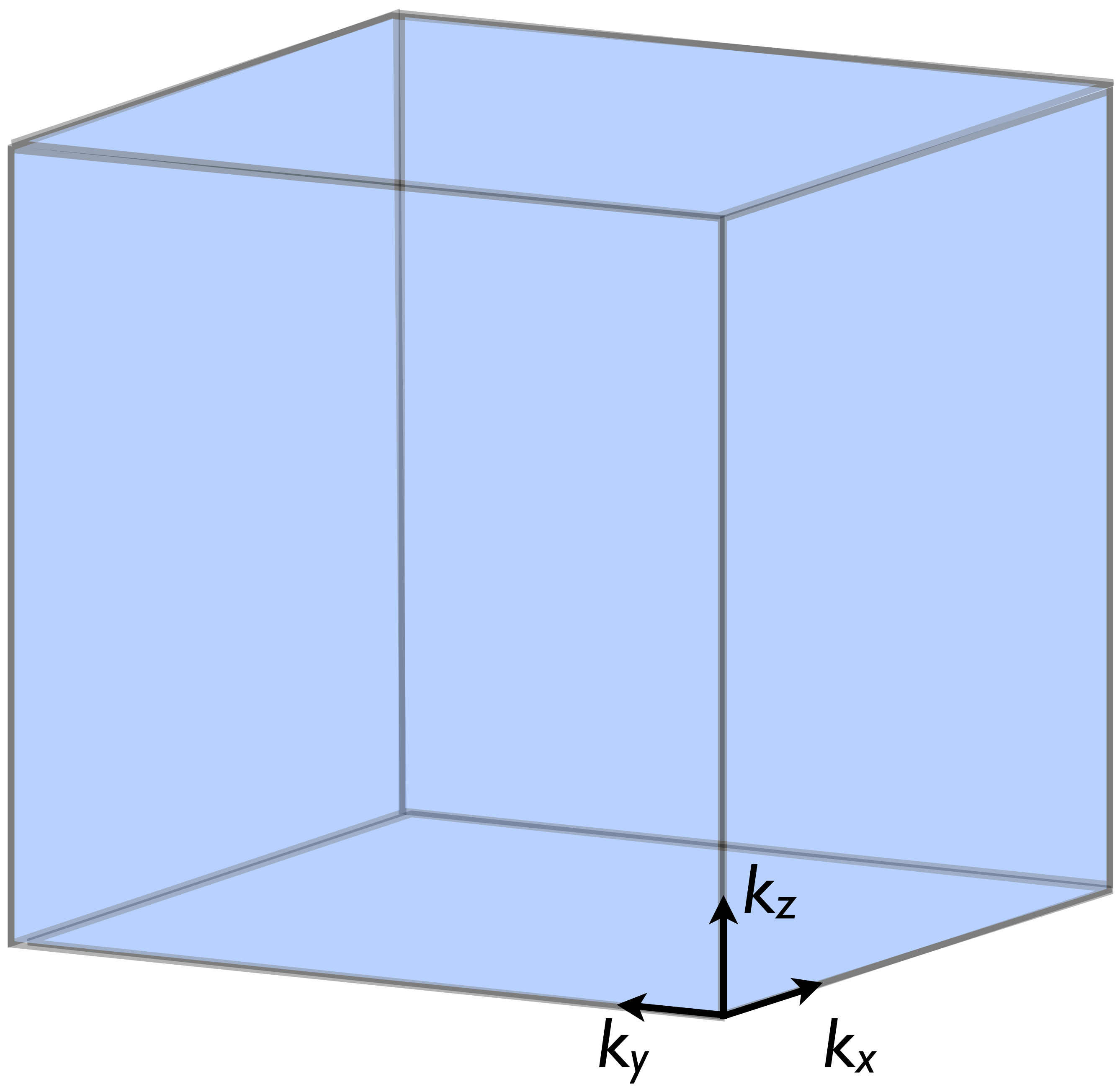

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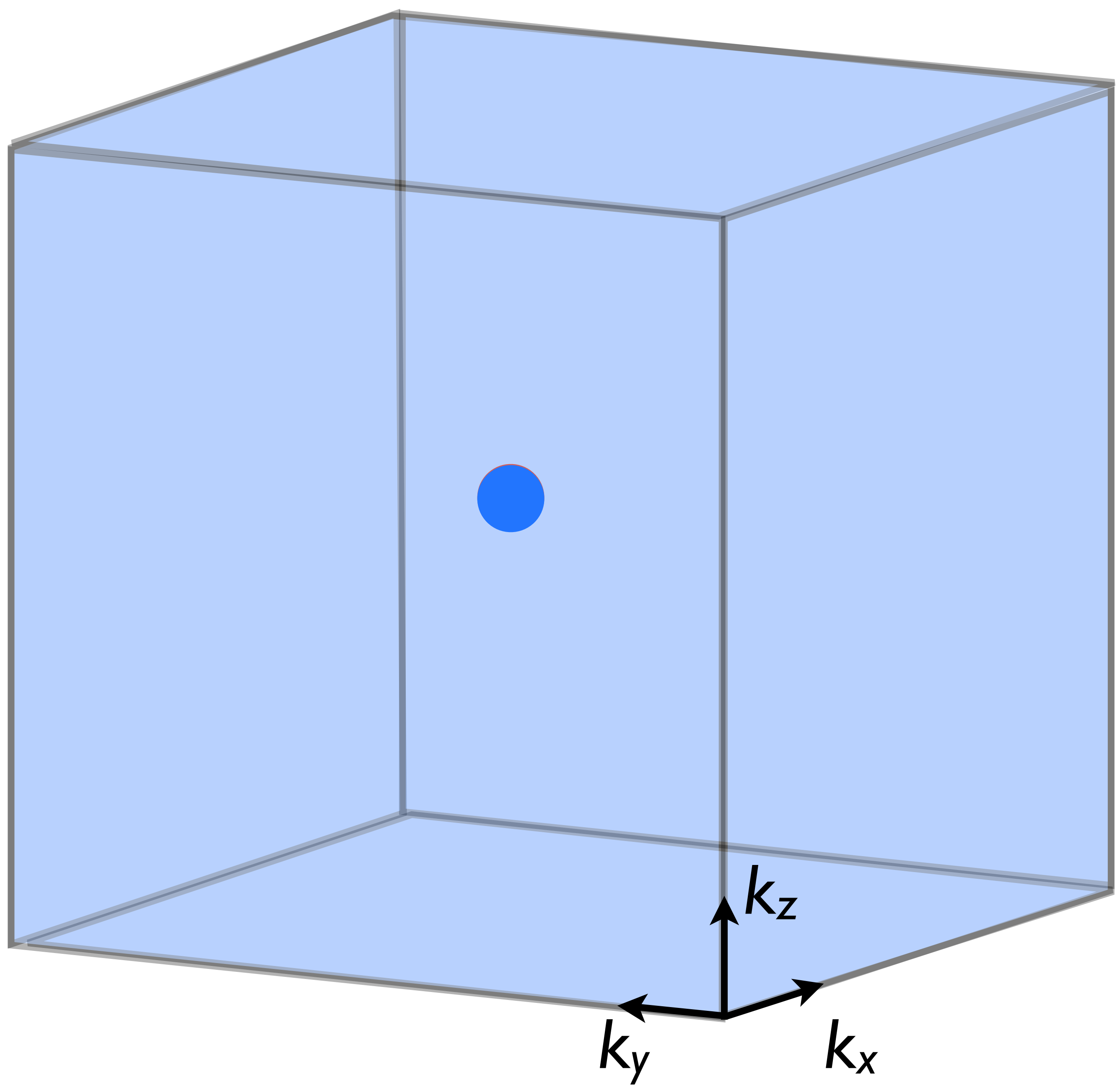

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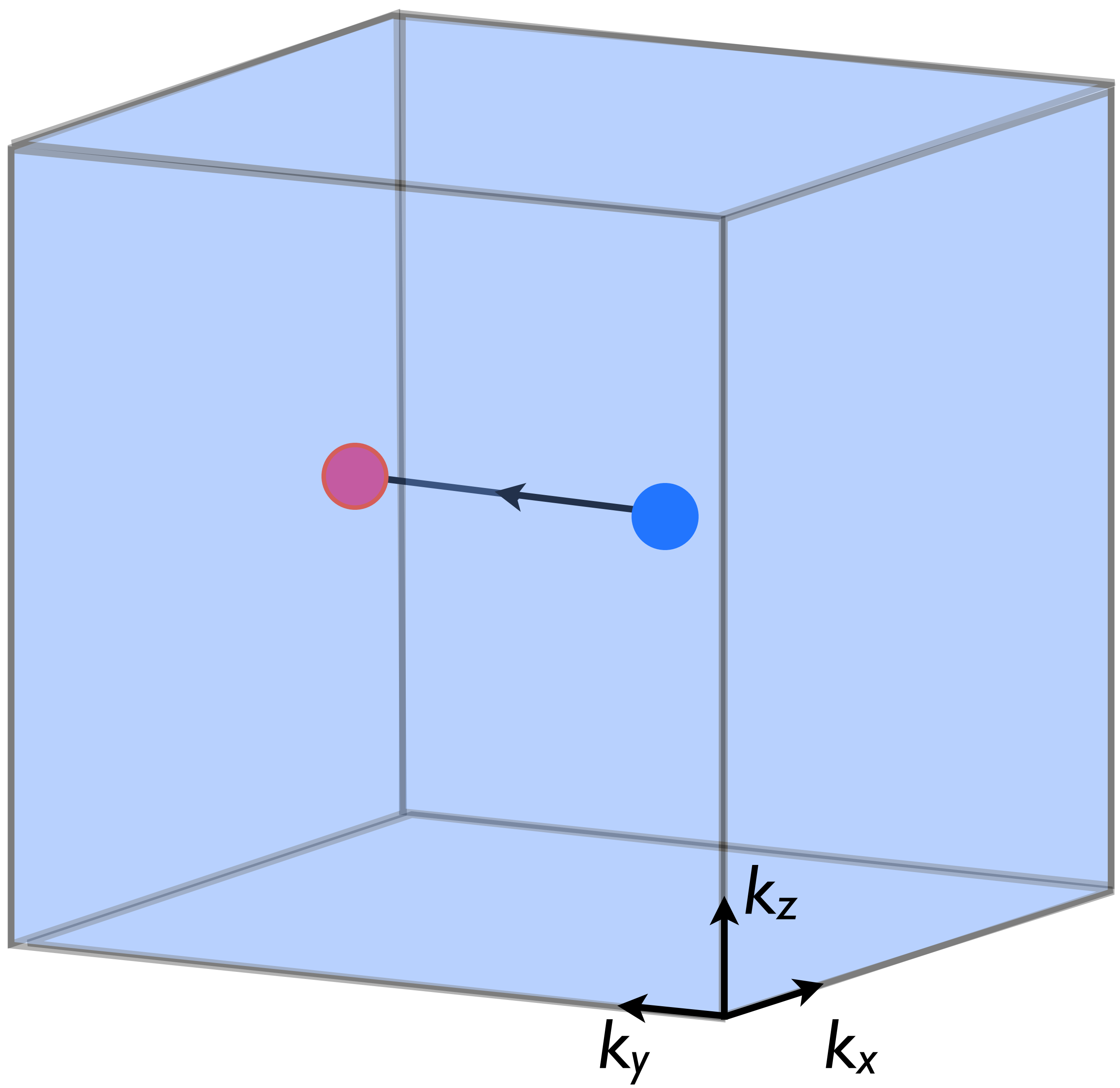

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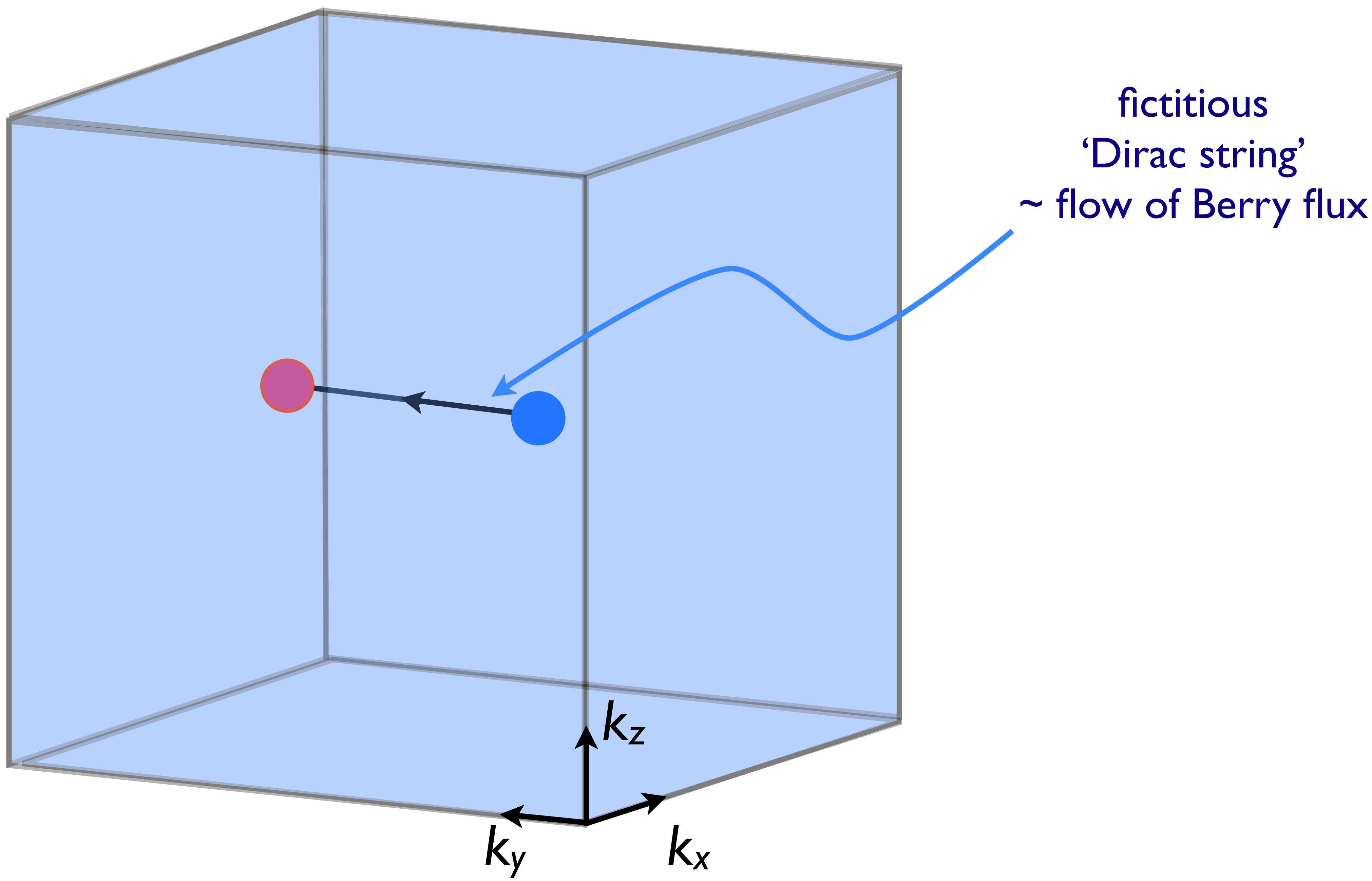

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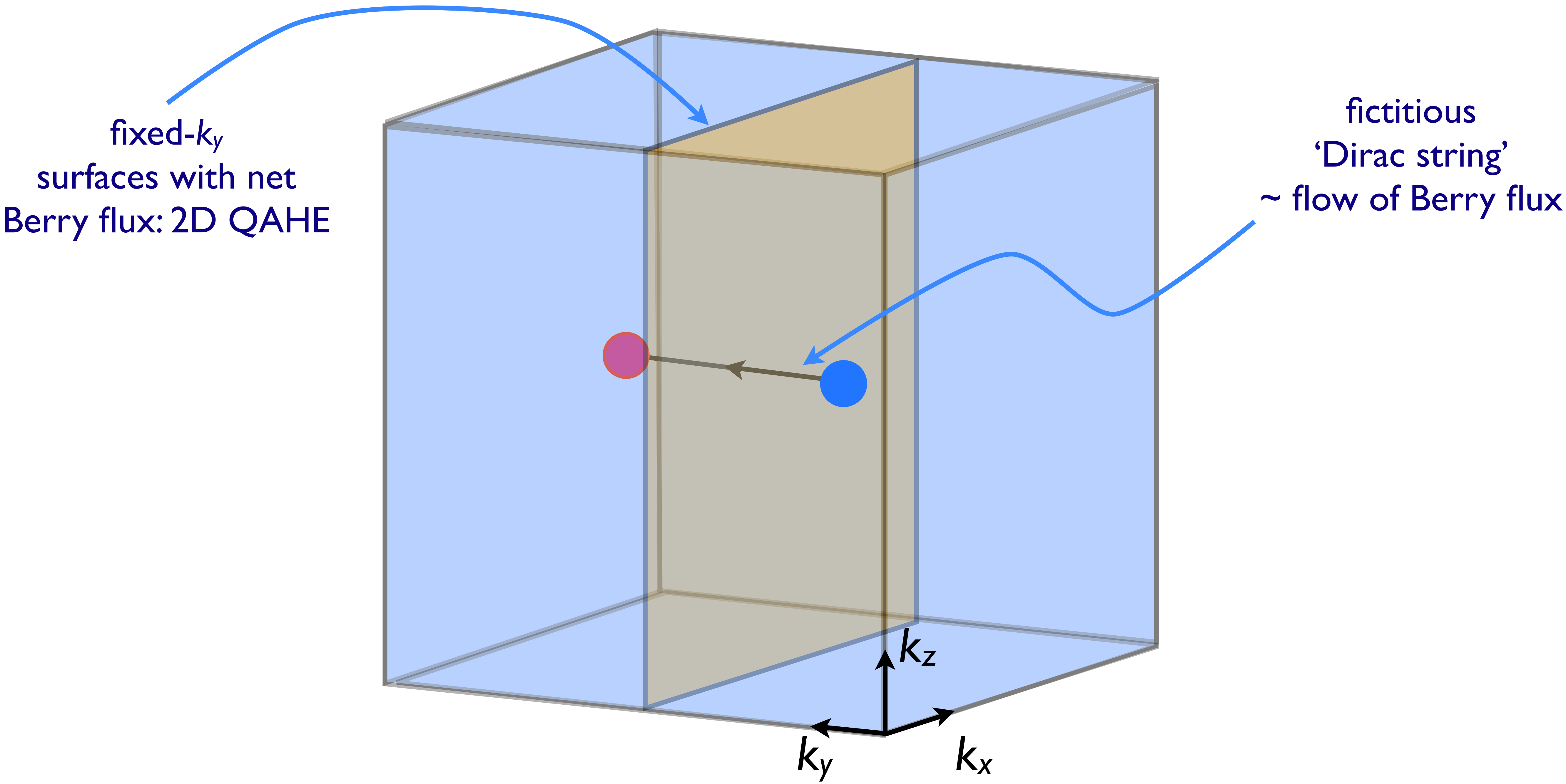

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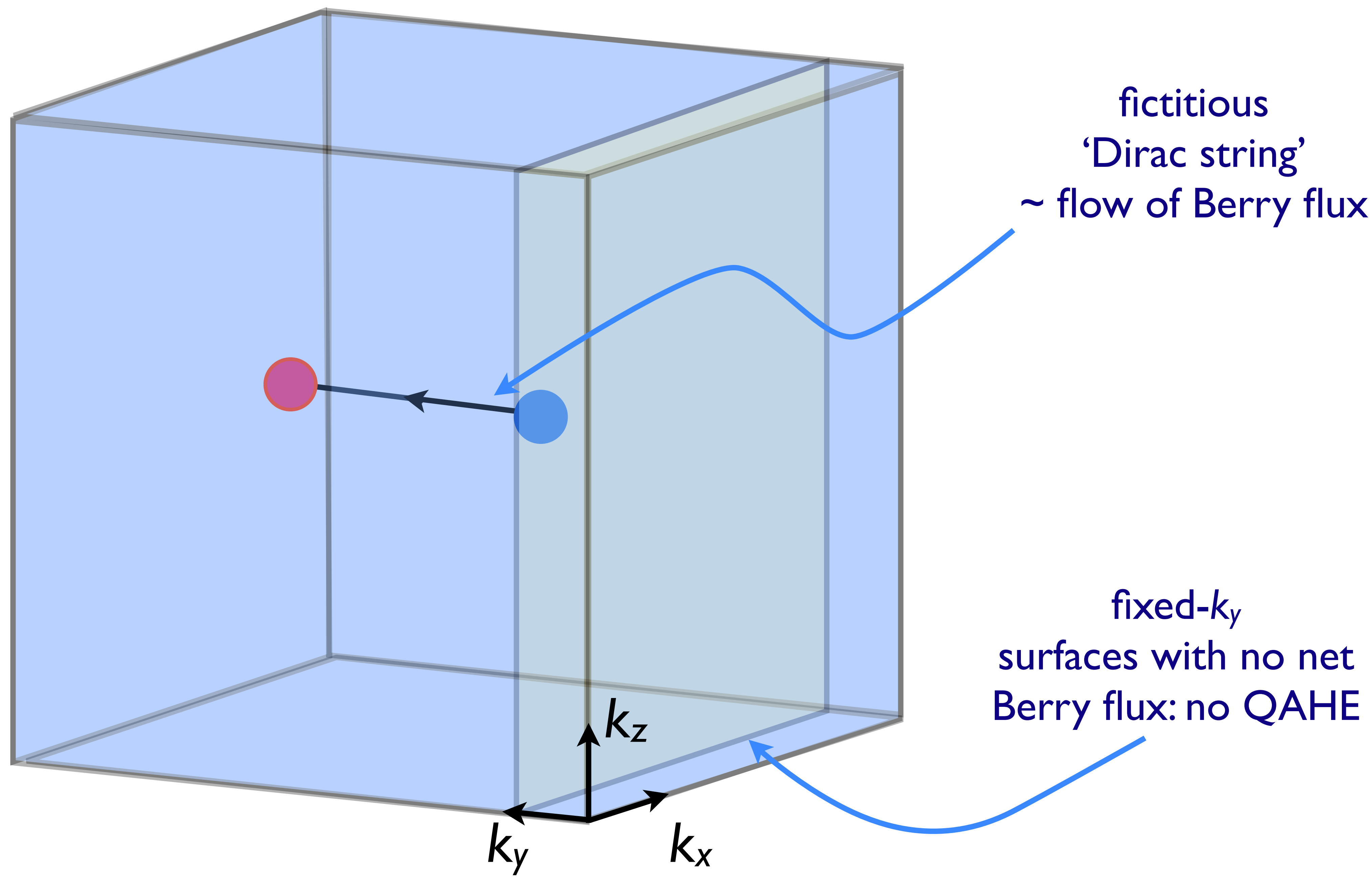
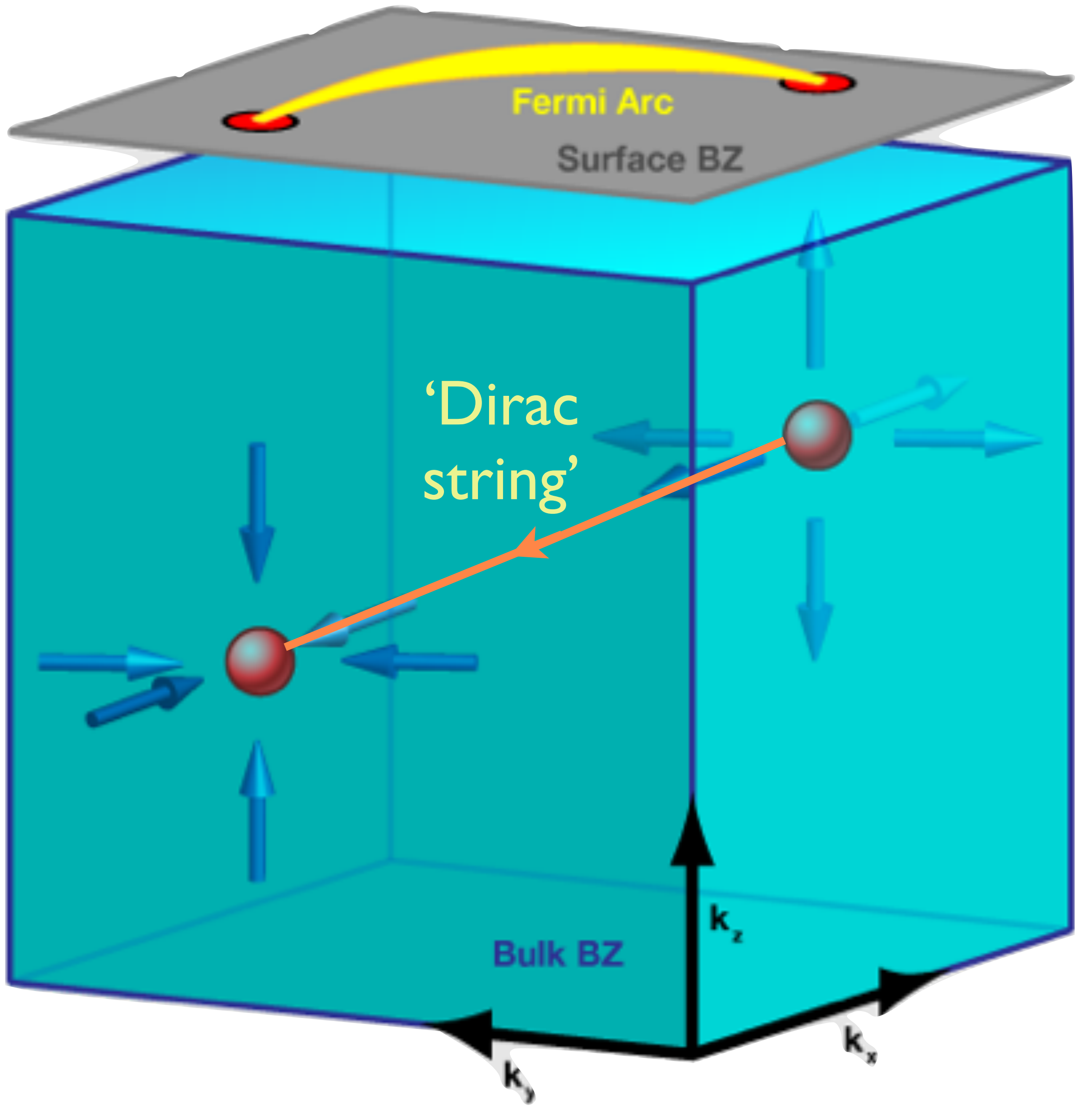


‘Fermi arcs’



For every k_y value w/ QAHE, surface BZ normal to k_z has *chiral* surface state

[X. Wan, A. Turner, A Vishwanath, S. Savrasov, PRB **83**, 205101 (2011)]

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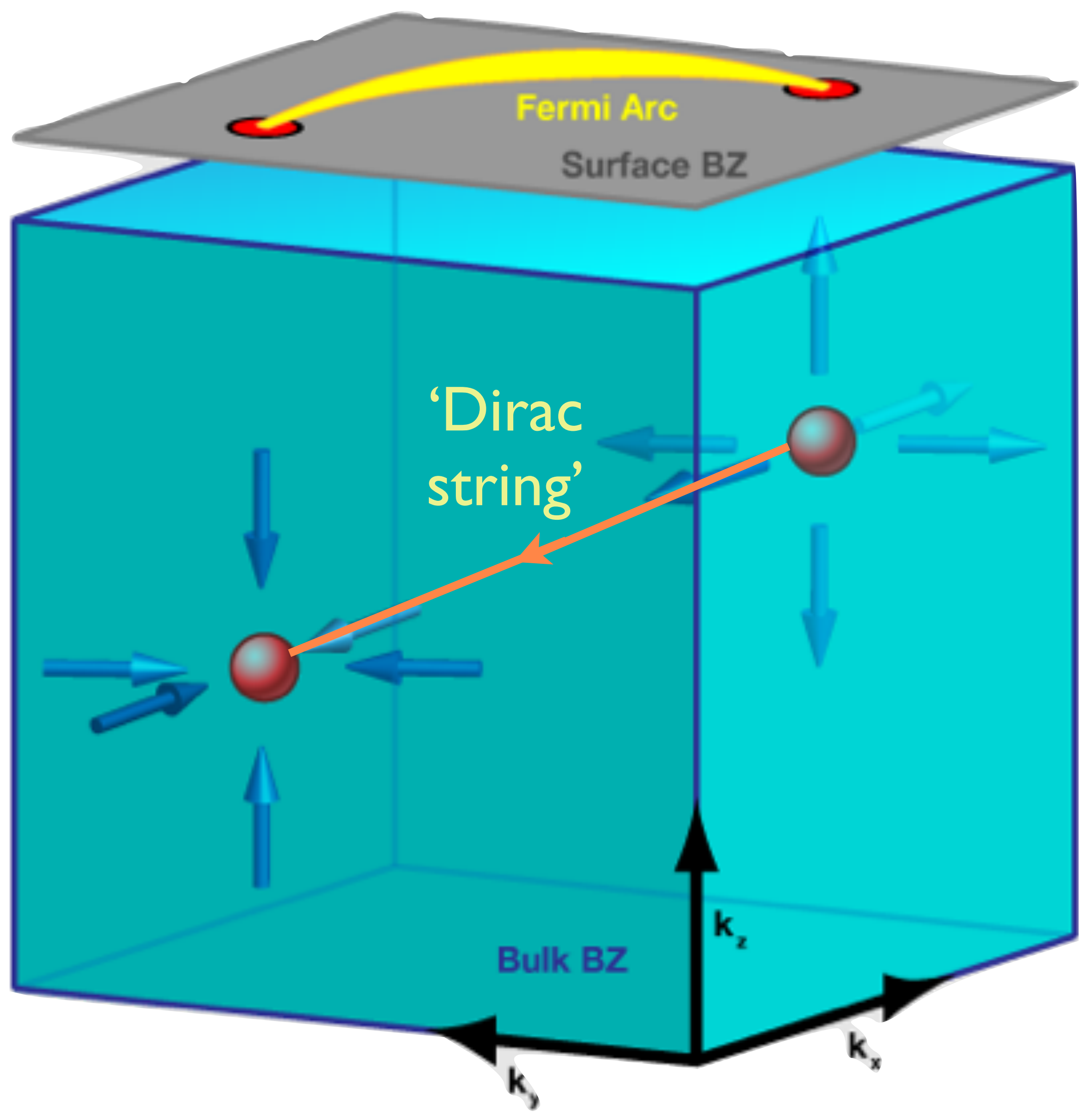

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'half' of a 2D Fermi surface on top
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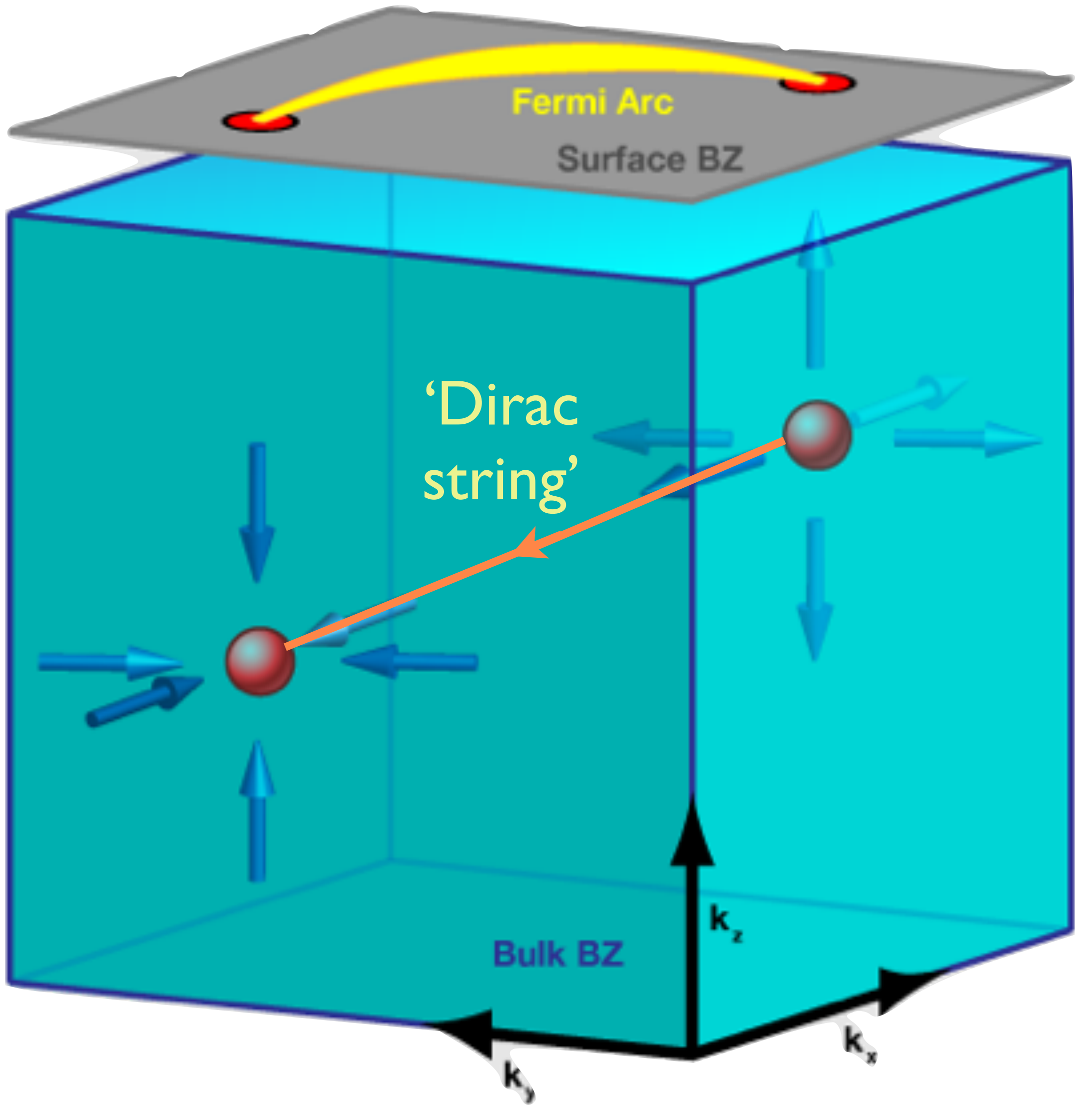

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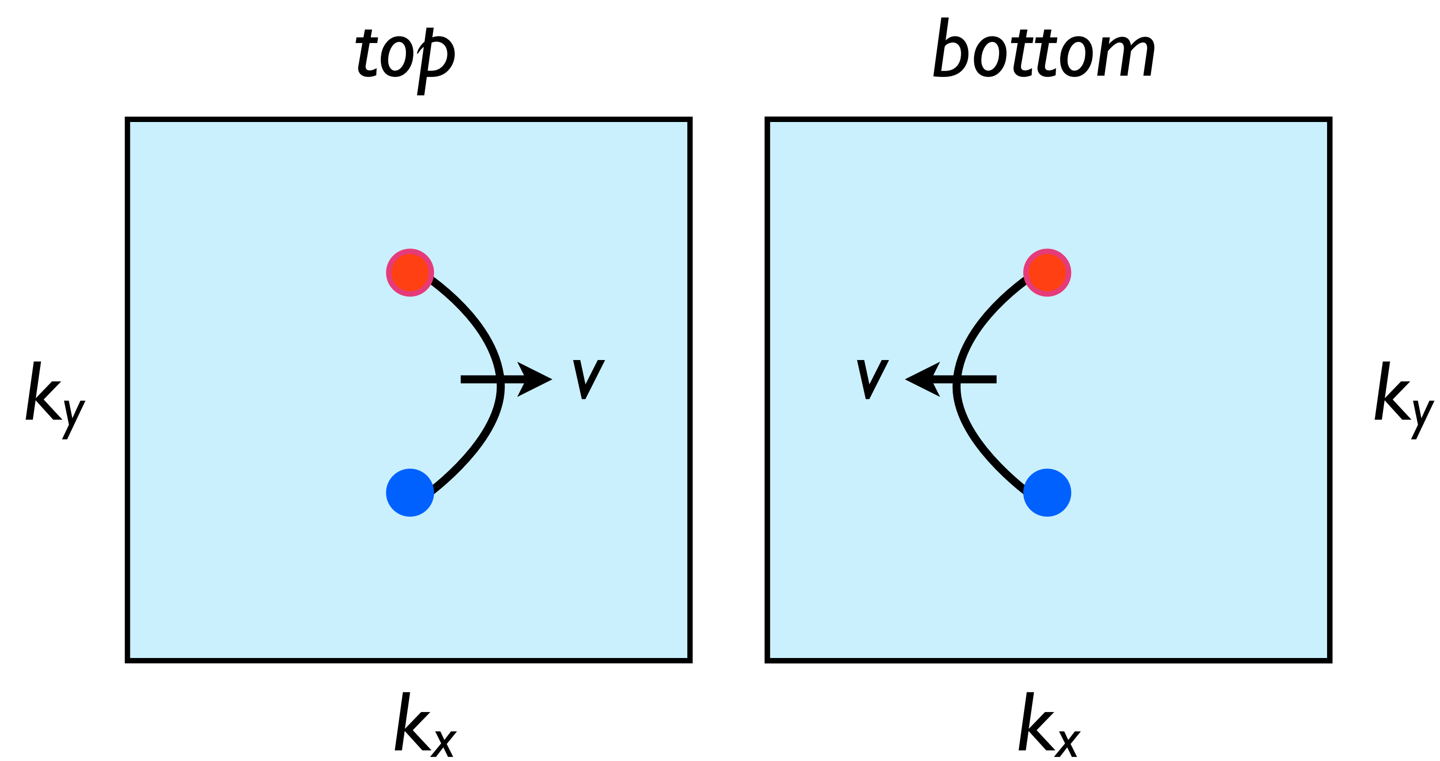

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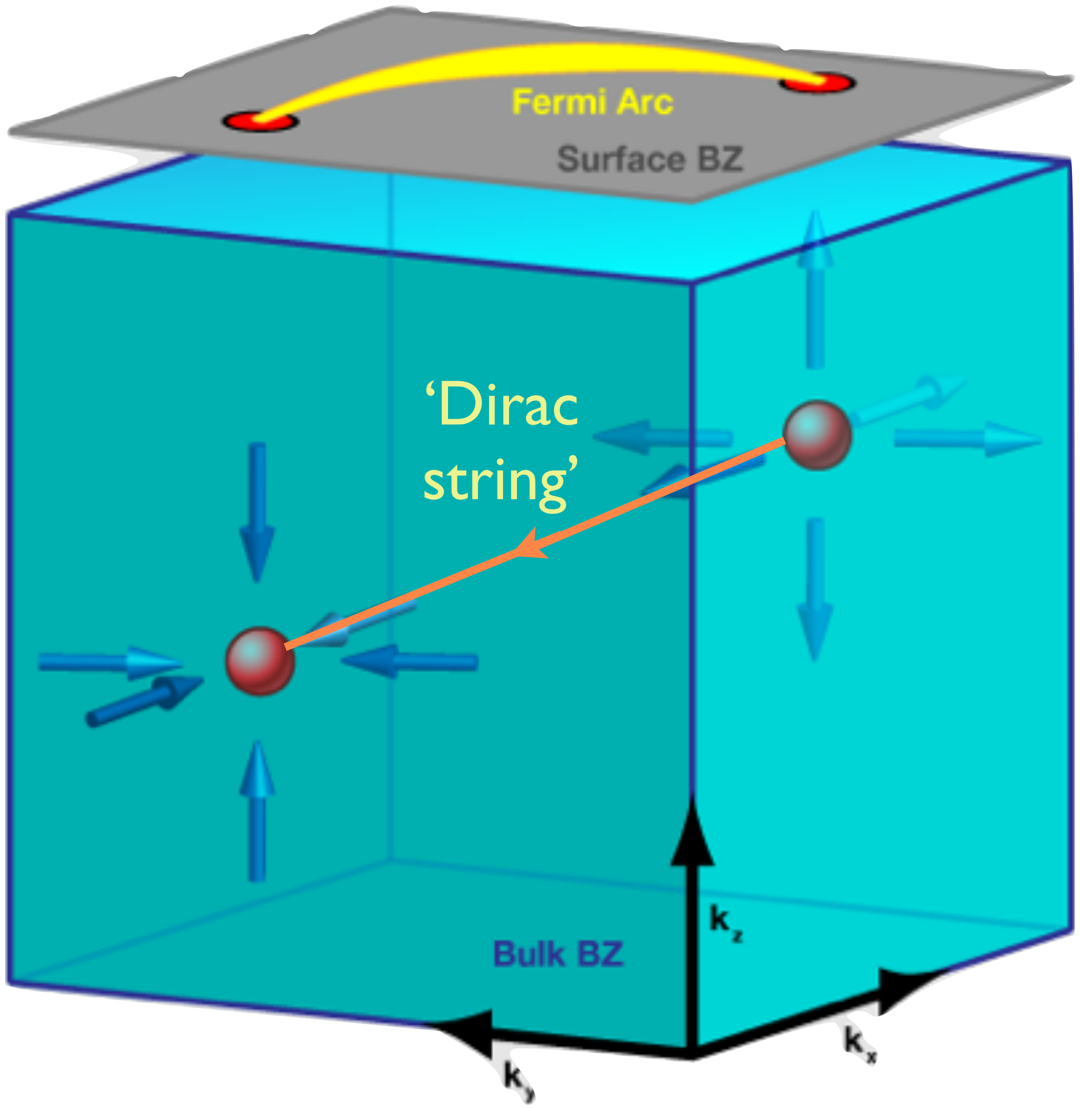

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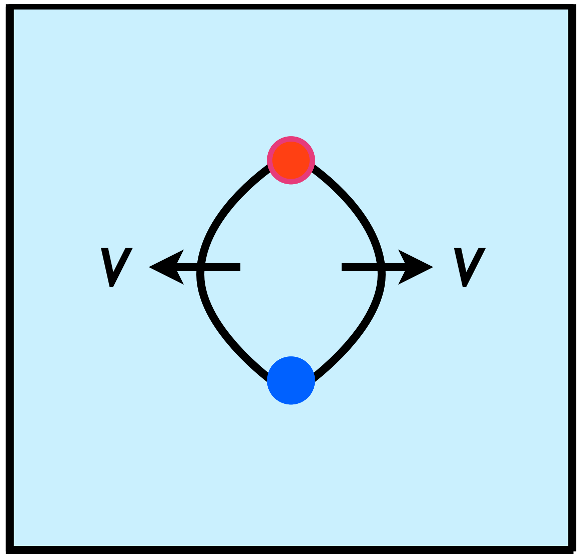
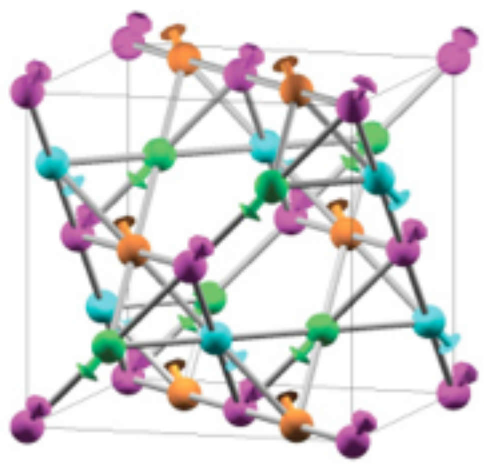


top+bottom = 'legal' 2D Fermi surface

[X. Wan, A. Turner, A Vishwanath, S. Savrasov, PRB **83**, 205101 (2011)]

Where to look?

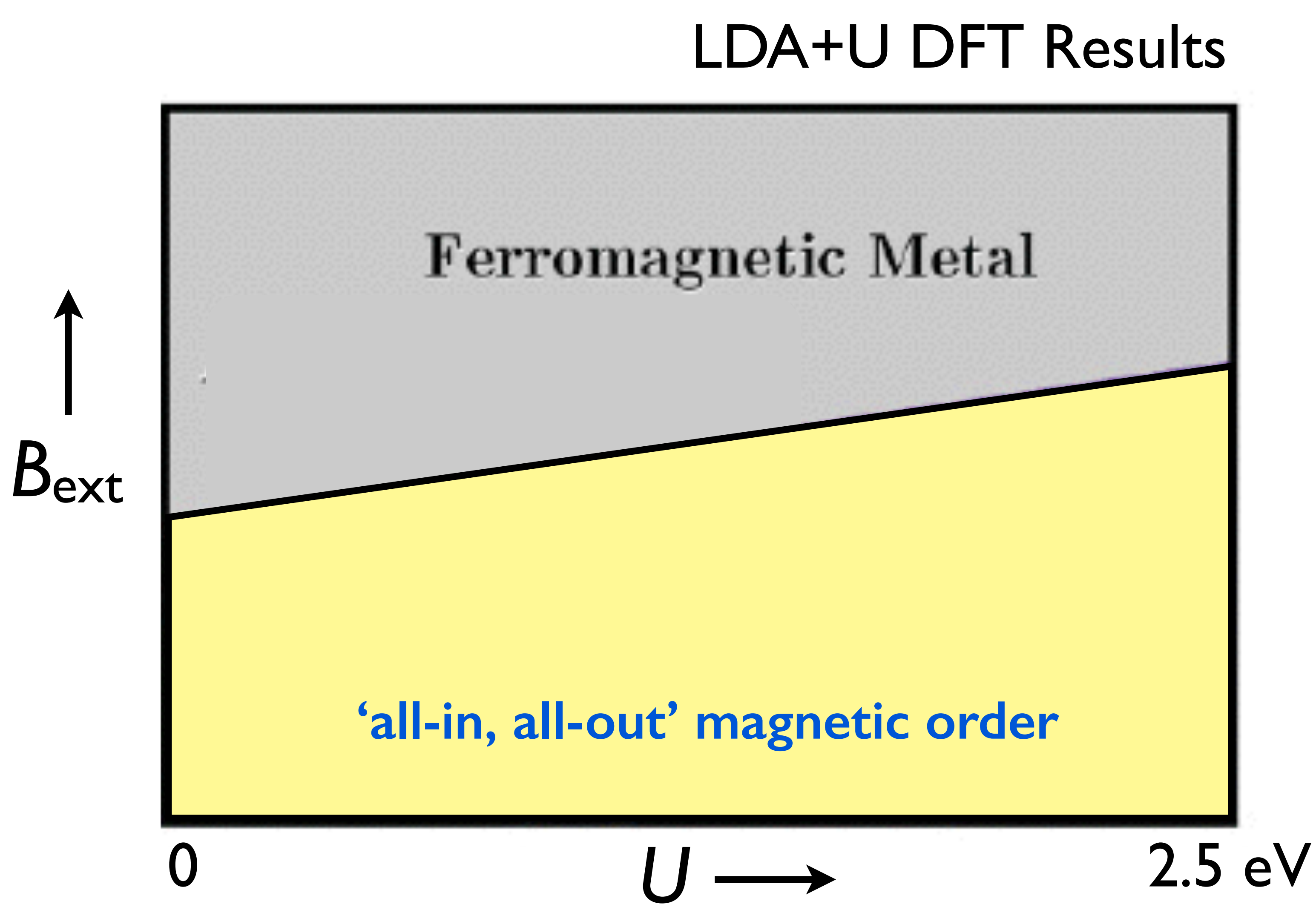
Original Candidate: Pyrochlore Iridates



$A_2Ir_2O_7$ (A = Y, Eu)

Ir: strong spin-orbit (Z~77)

varying A: tunes interaction ('U') between Ir 5*d*



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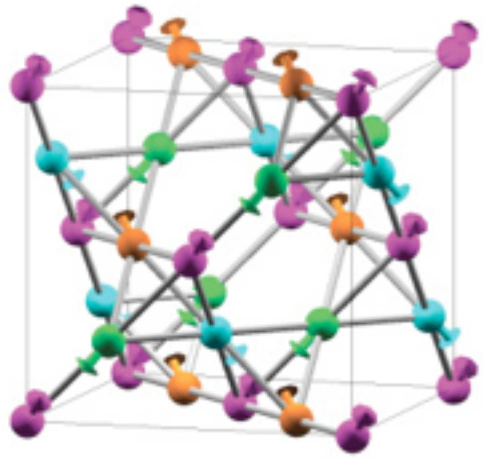

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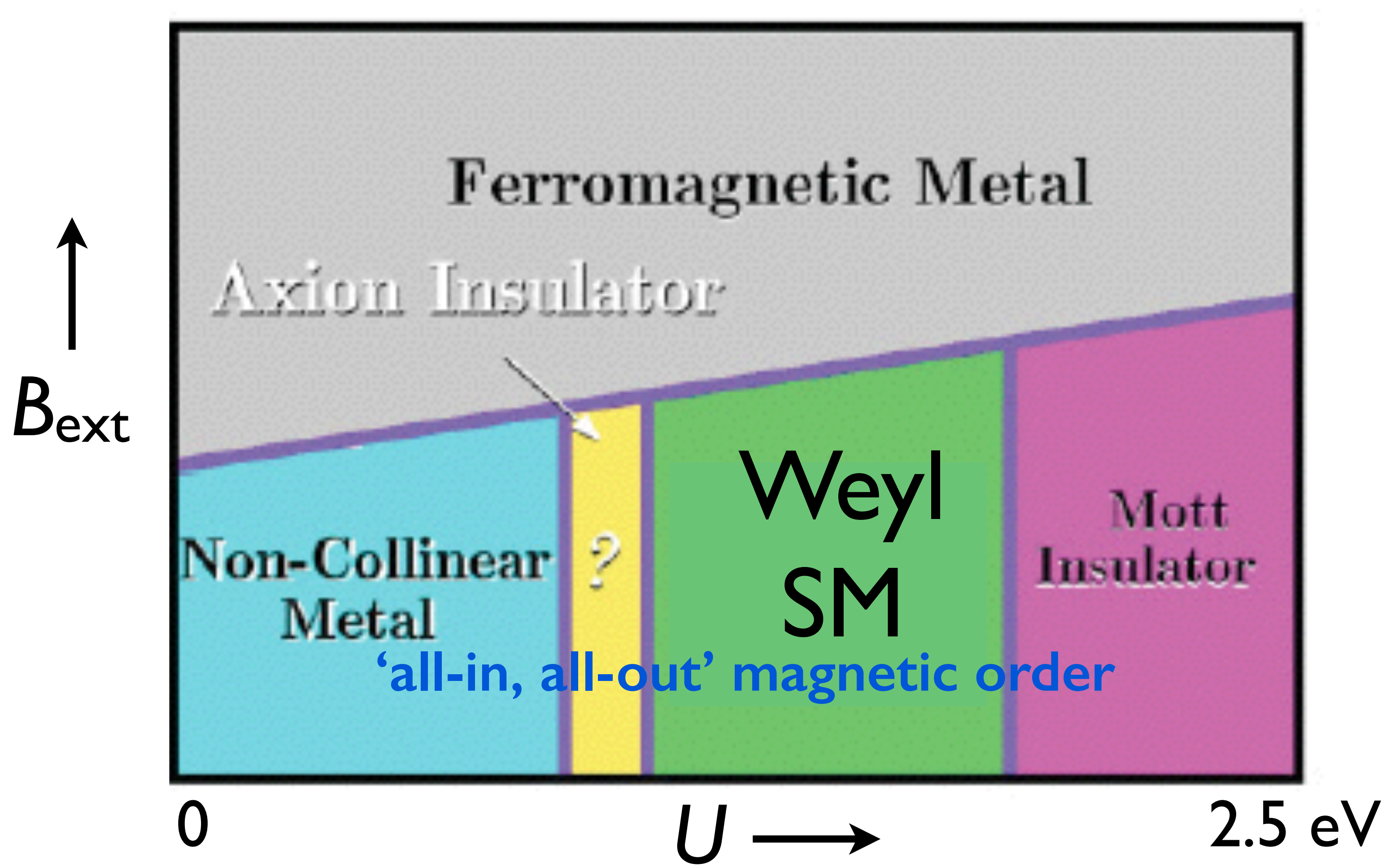

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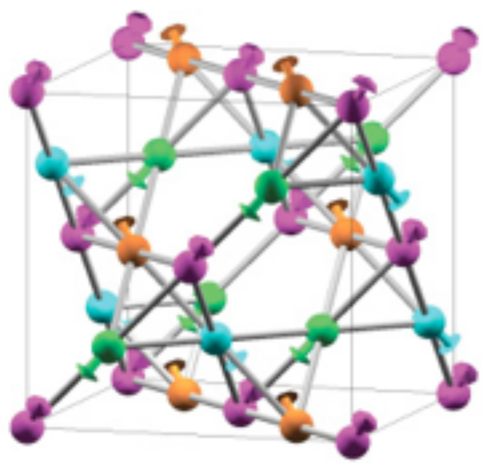

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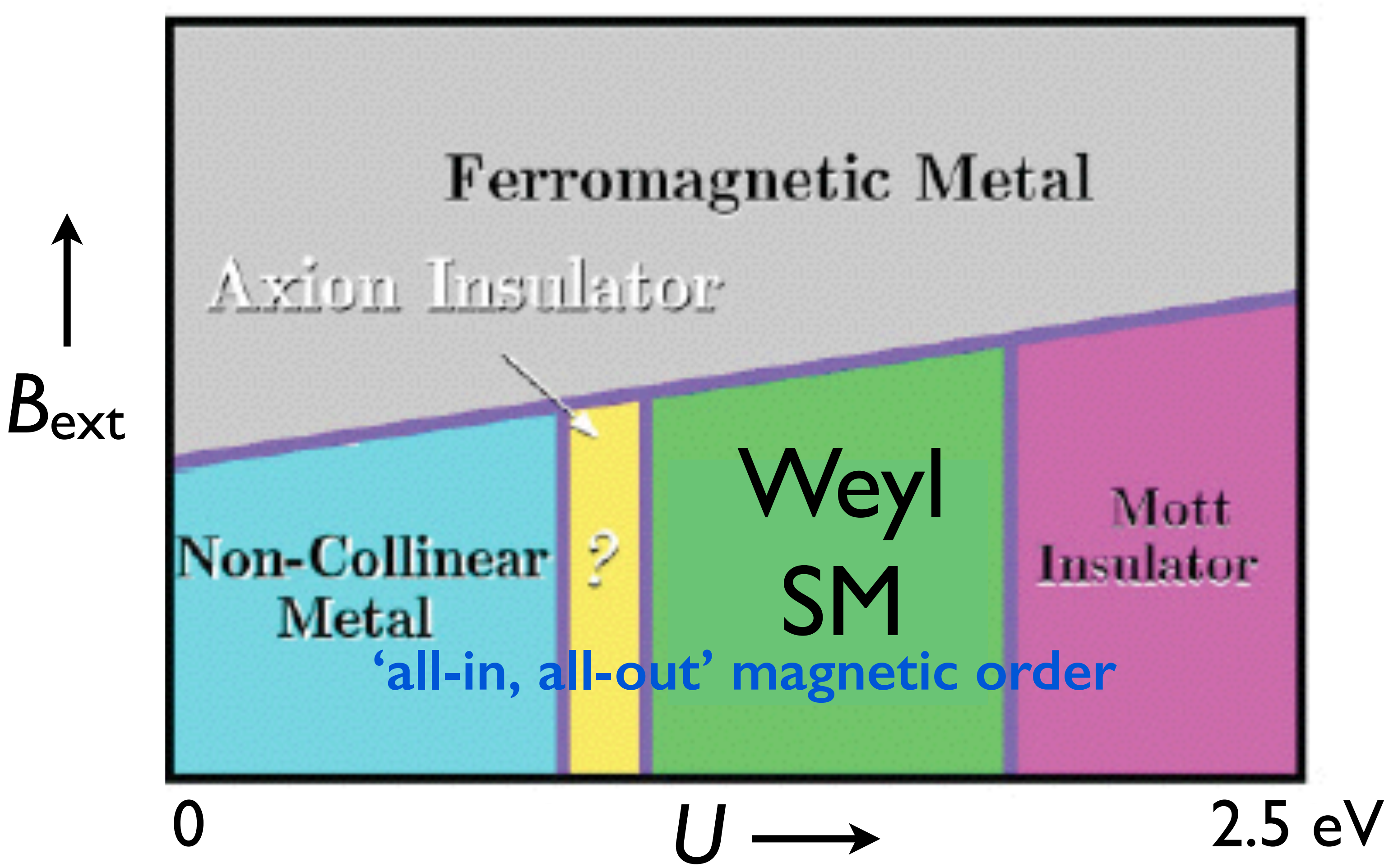
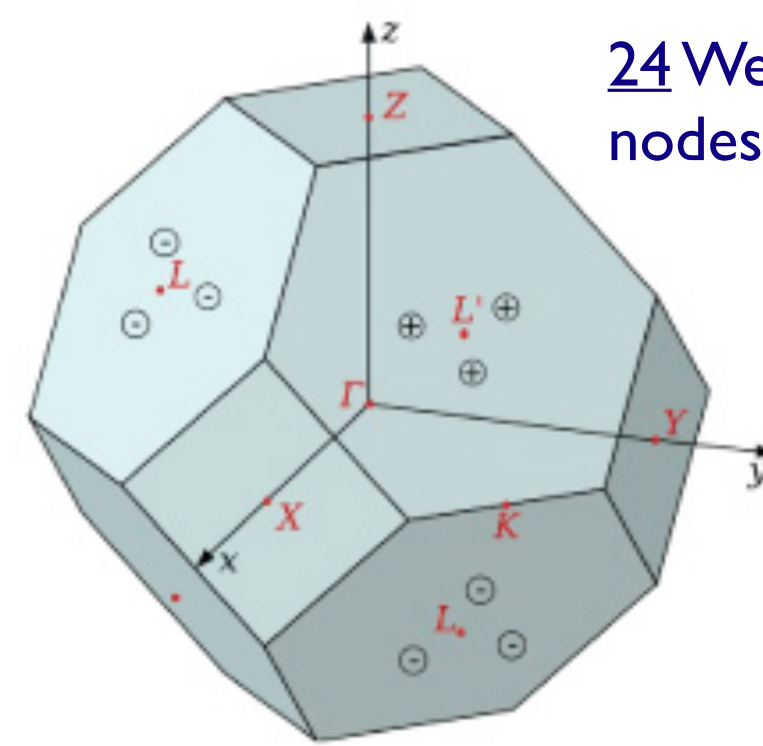
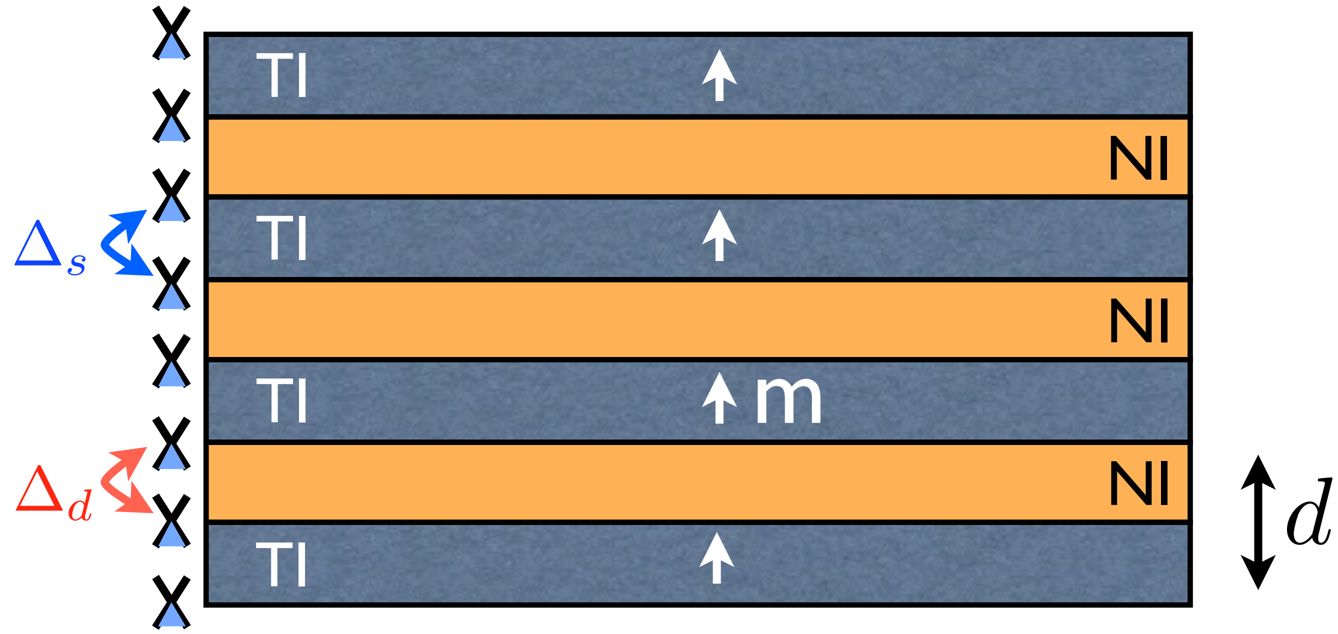




24 Weyl nodes

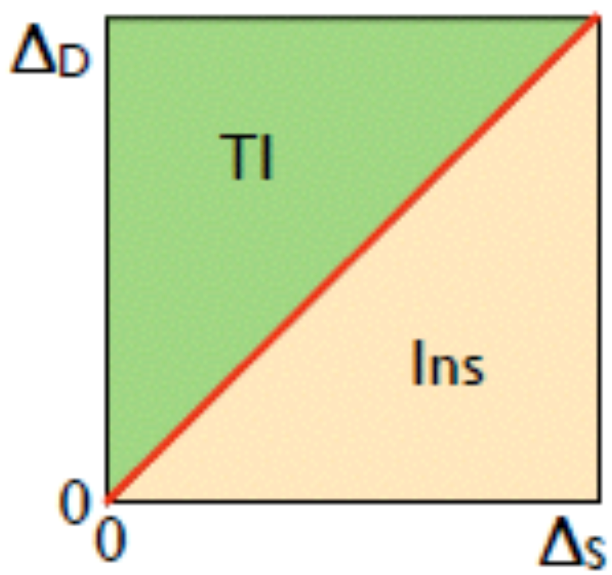
[X. Wan, A. Turner, A Vishwanath, S. Savrasov, PRB **83**, 205101 (2011)]

Toy Model: TI/NI Heterostructures w/ Magnetism



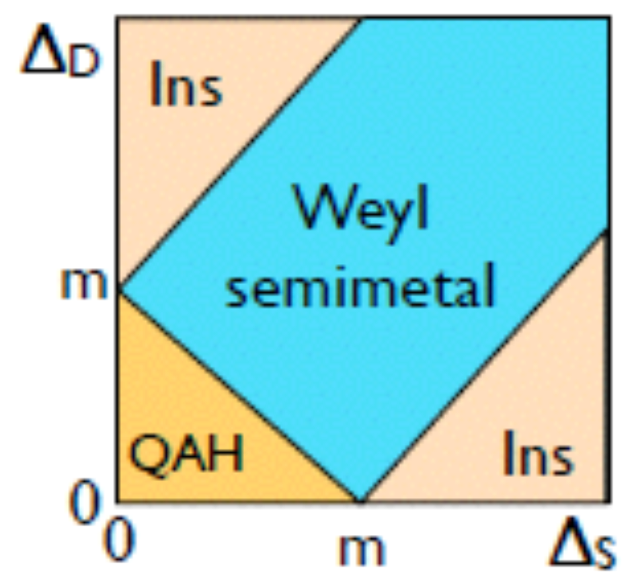
TI-NI transition when $\Delta_d = \Delta_s$; split by magnetism (induced/spontaneous)

with TRS



(a) m=0

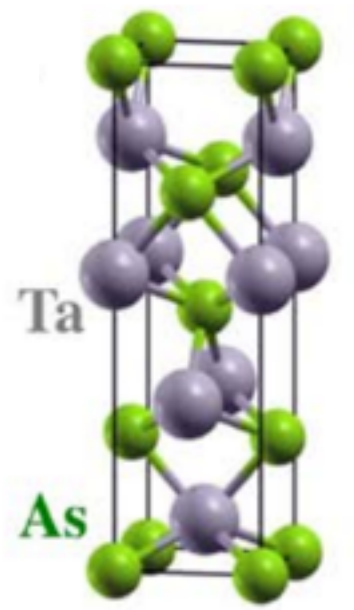
broken TRS



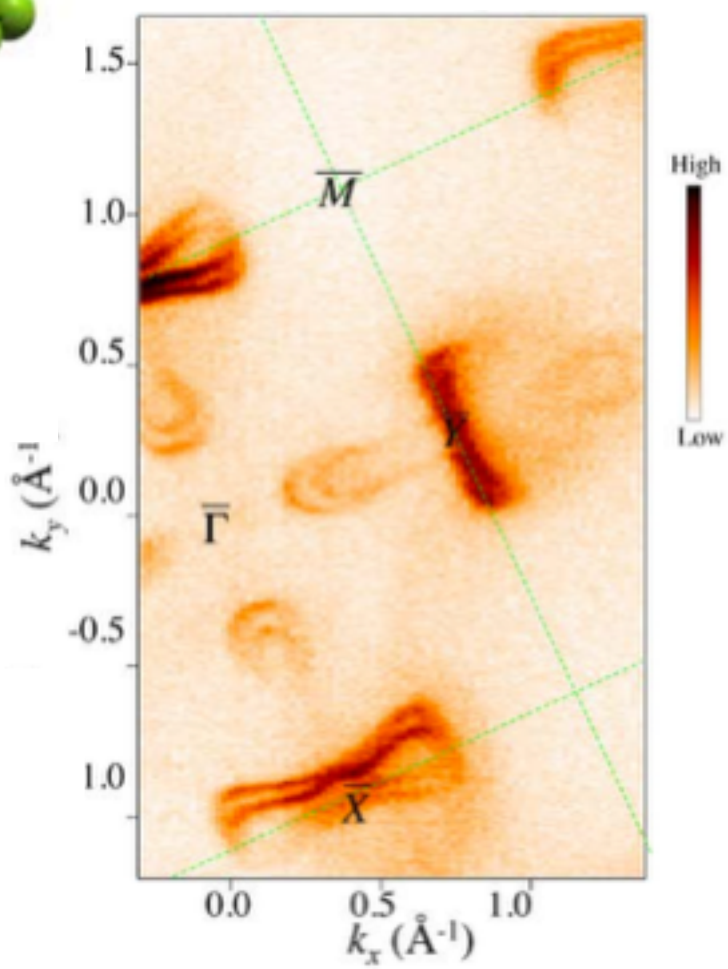
(b) m≠0

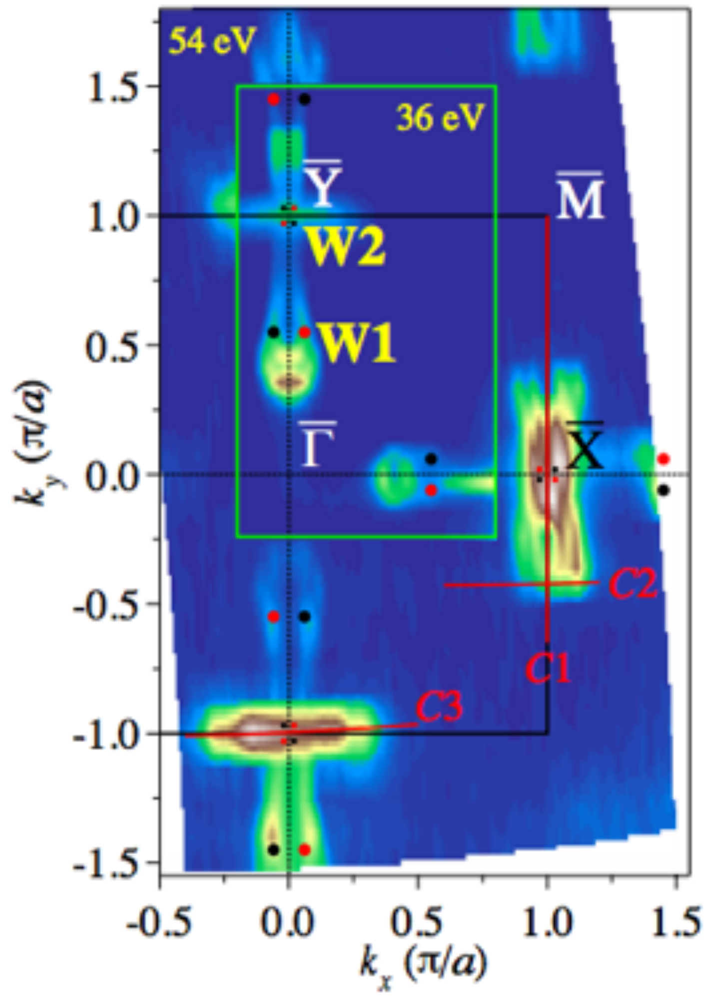
[A. Burkov, L. Balents, PRL **107**, 127205 (2011)]

Experimental Observation of WSM



ARPES: Fermi arcs on (001) surface in TaAs





[S.-Y. Xu *et al.*, *Science*, July'15]

[B.Q. Lv *et al.*, *PRX* '15]

Other examples: NbAs,… (many more!)

Topological 'Dirac' Semimetals

Typically, pair of Weyl nodes at same point 'gap out'

(2x2 mass matrix impossible in 3D, 4x4 mass matrix allowed)

Can remain gapless if crystal symmetry forbids mass term

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Na_3Bi	[Z. Wang, *et al.*, *Phys. Rev. B* **85**, 195320 (2012)]	stable, volatile
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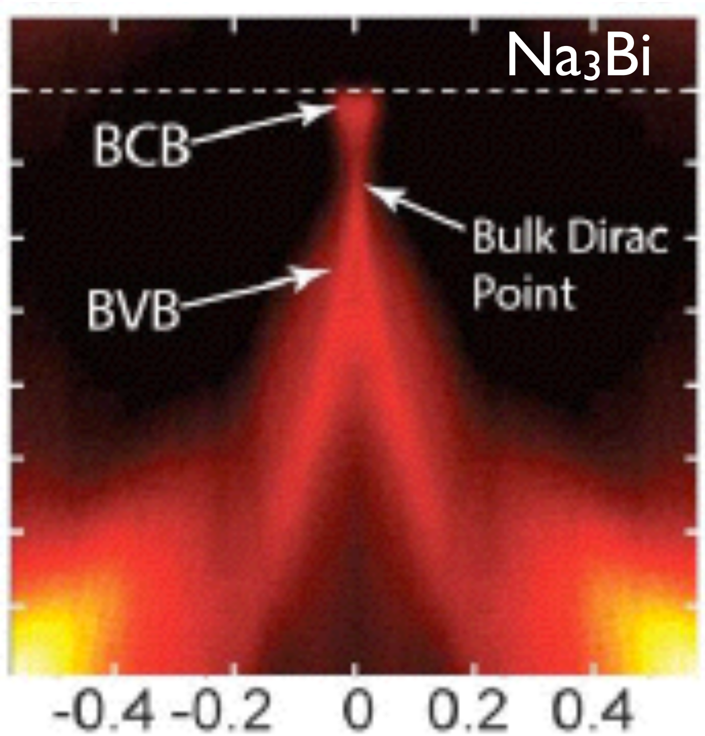
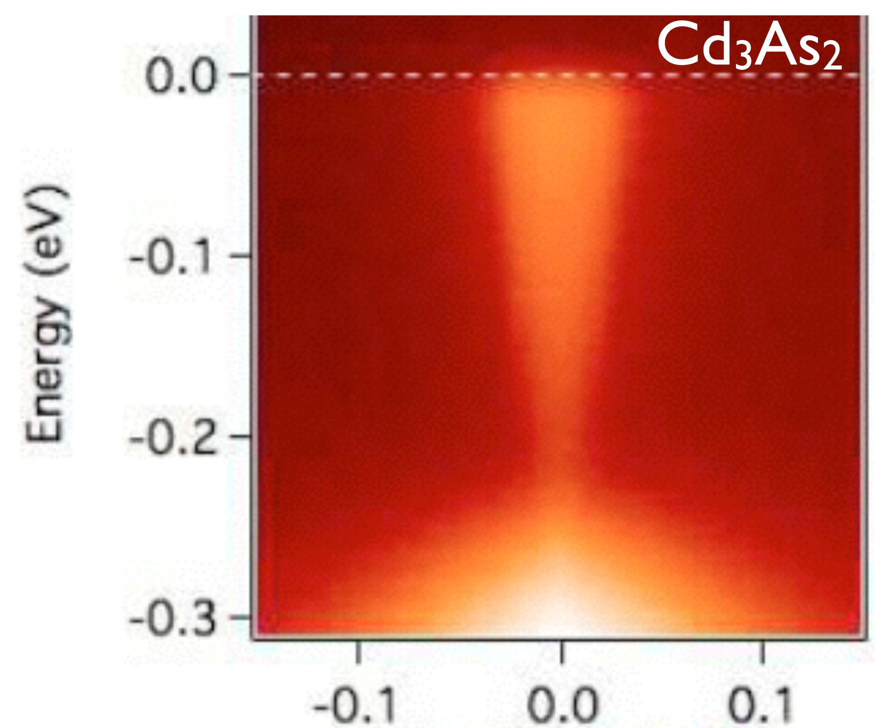


[Z. Liu *et al.* *Science* '14;
S.-Y. Xu *et al.* arXiv:1312.7624]



[M. Neupane *et al.* *Nat. Comm.* '14]

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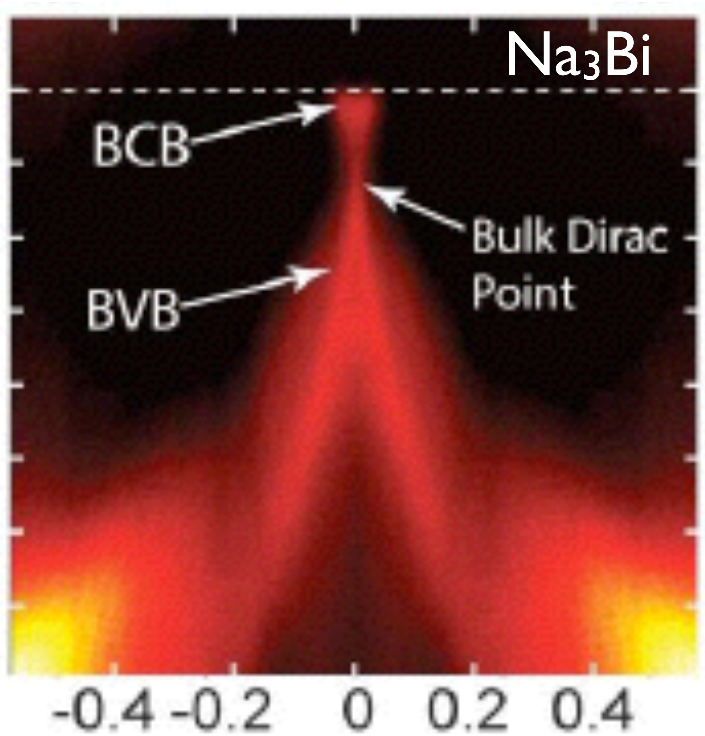

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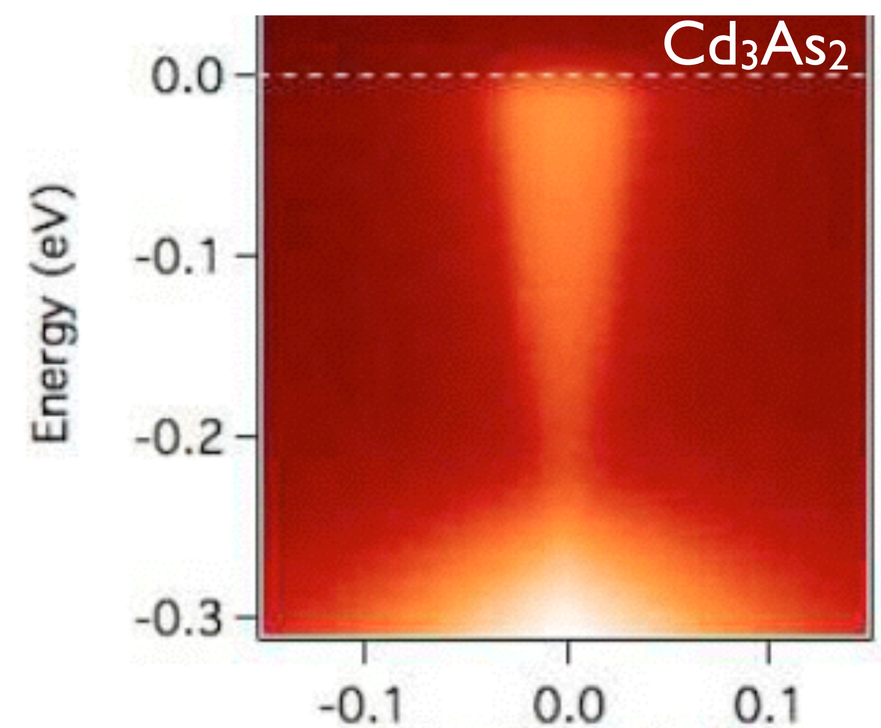
~ "2 x WSM"



[M. Neupane *et al.* *Nat. Comm.* '14]

Detecting Topological Semimetals

Conductivity Scaling

'quantum critical' resistivity $\rho \sim T^{-1}$

(assumes chemical potential at nodes)

[Th.: Hosur, SP, Vishwanath, PRL '11)]

[Expt.: Yanagashima & Maeno, JPSJ '01)]

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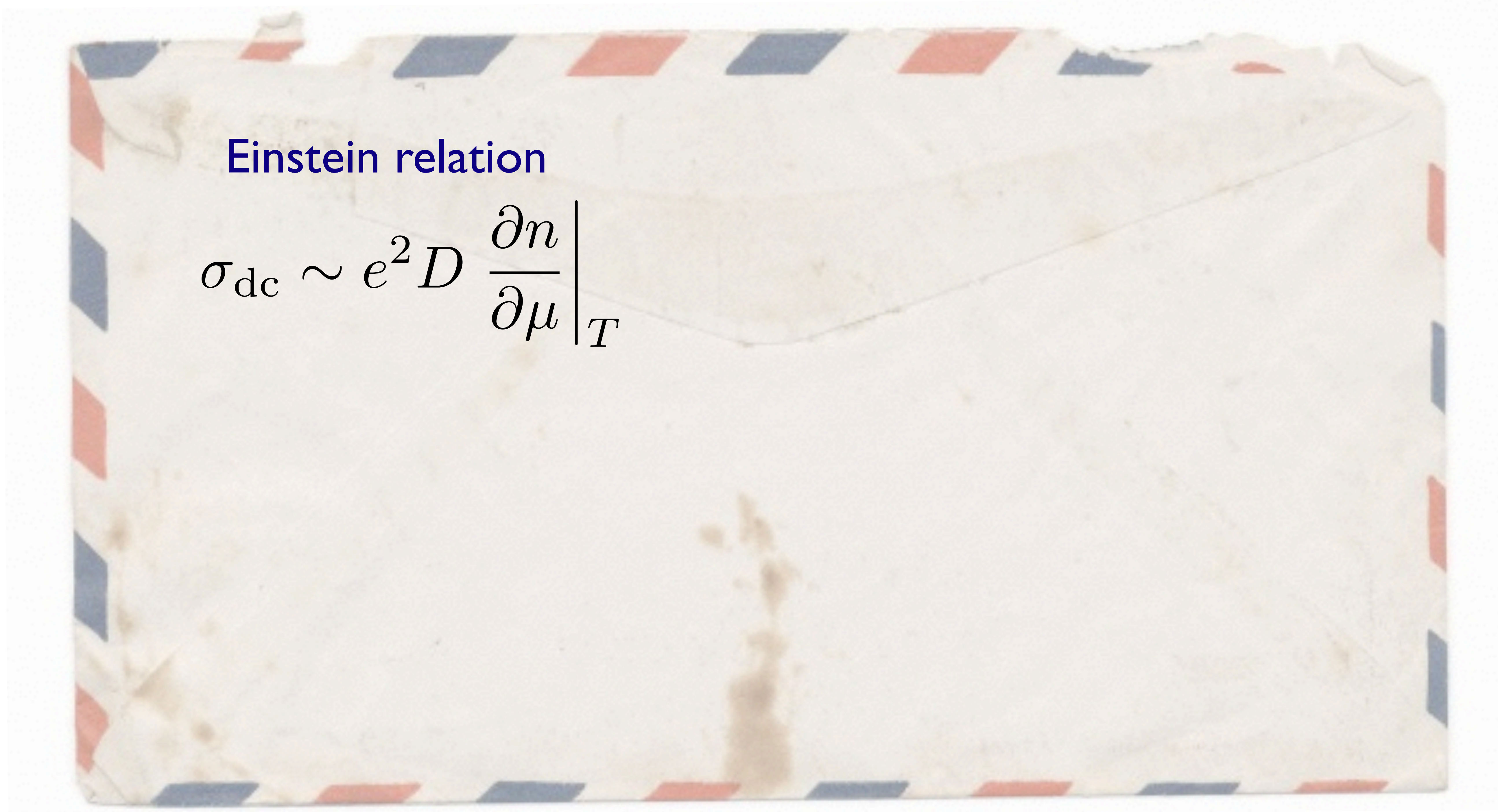

Einstein relation

$$\sigma_{\mathrm{dc}} \sim e^2 D \left.\frac{\partial n}{\partial \mu}\right|_T$$

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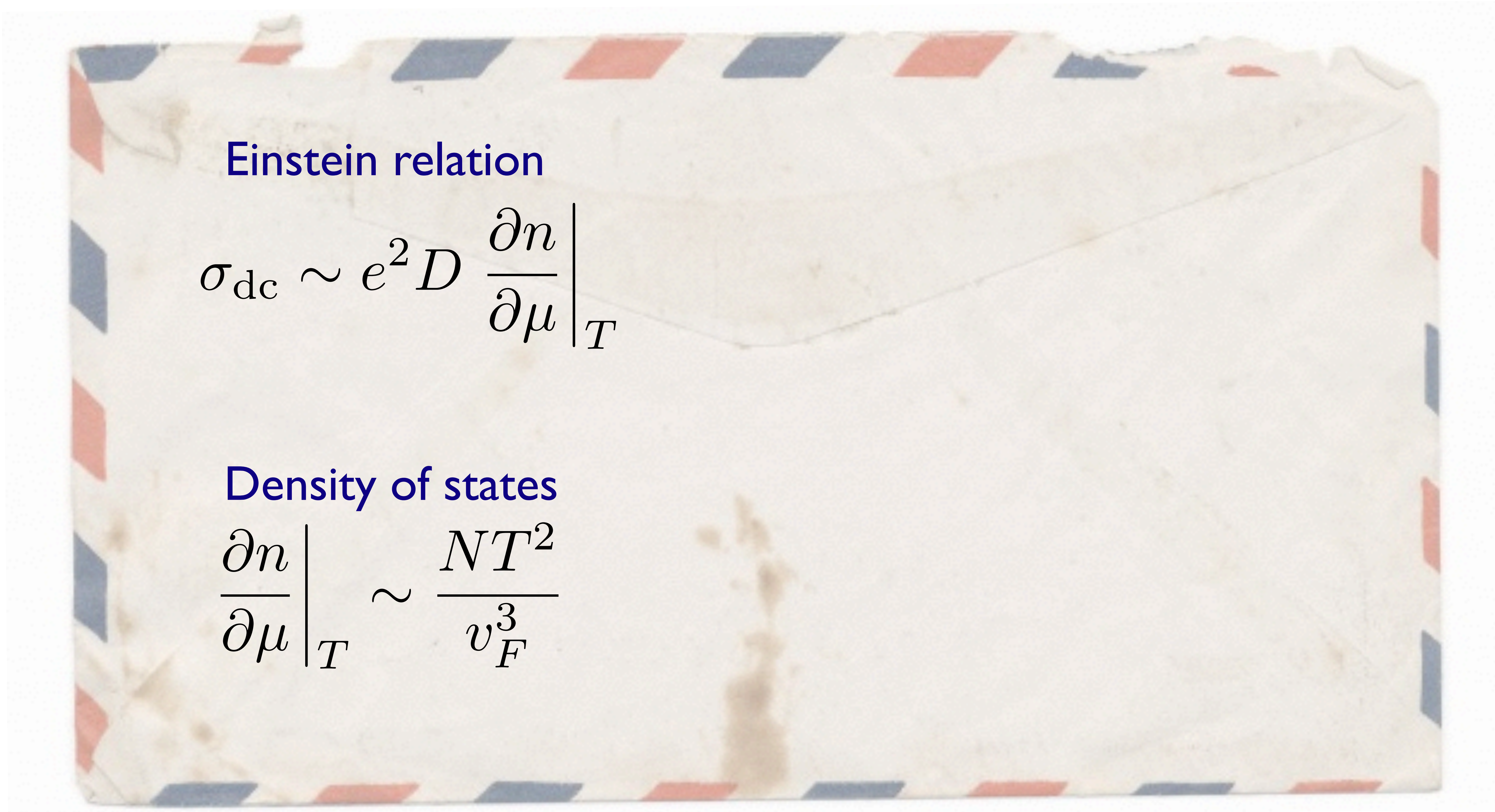

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Scattering rate

$$\tau^{-1} \sim N\alpha^2 T$$

(assumes chemical potential at nodes)

[Th.: Hosur, SP, Vishwanath, PRL '11)]

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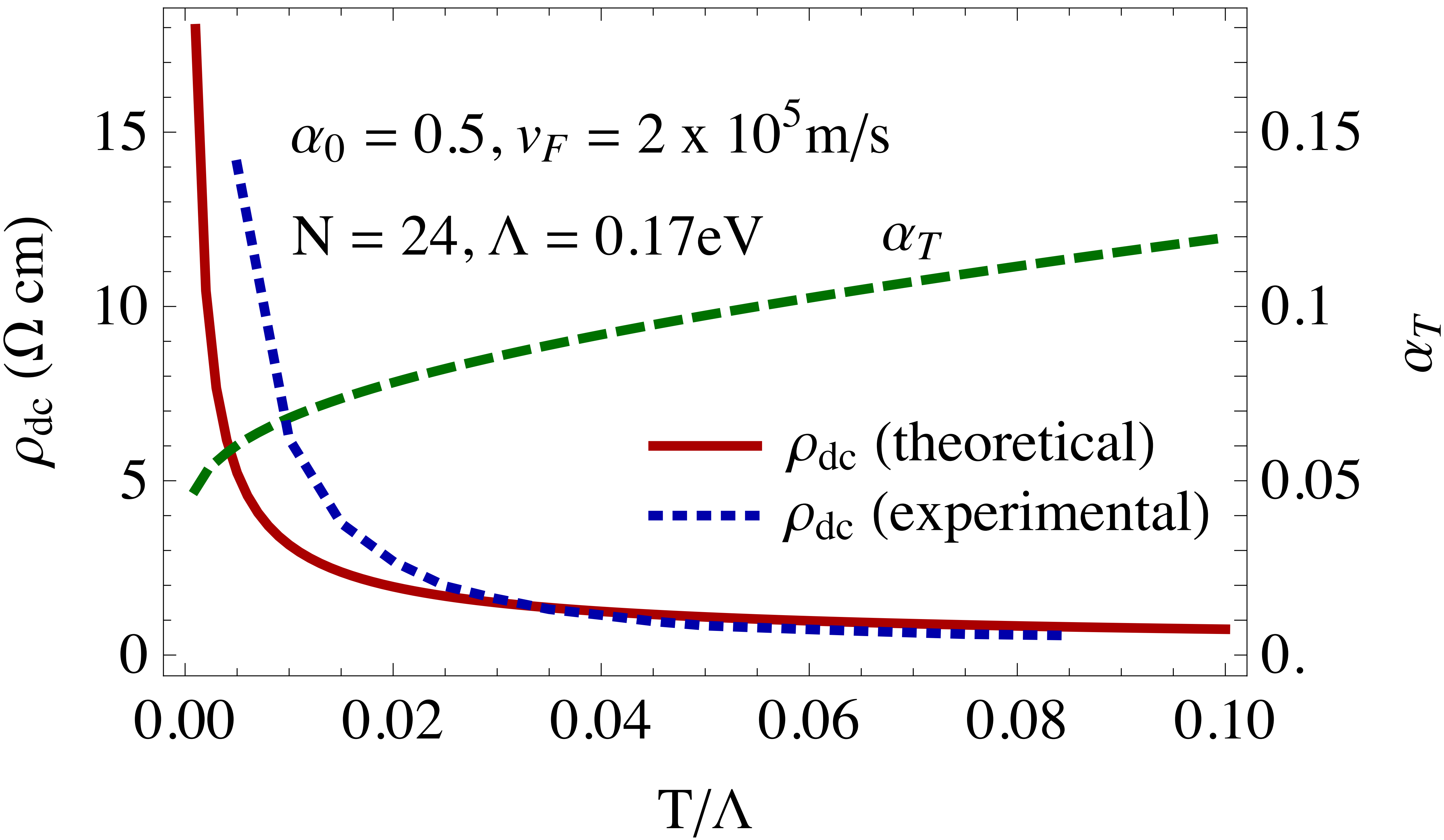
Experimental Signatures

'quantum critical' resistivity $\rho \sim T^{-1}$



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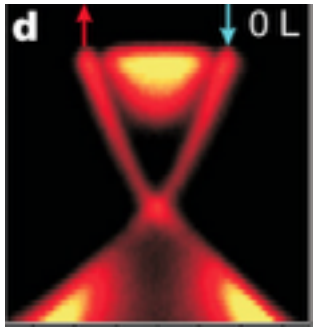
Topological signatures?

	Topological Insulators	Topological Semimetals
Boundary	protected 2D Dirac node  ARPES, STM, surf. transport	
Bulk	magnetoelectric effect $\mathcal{S} = \mathcal{S}_{em} + \frac{\alpha}{4\pi} \int \boldsymbol{E} \cdot \boldsymbol{B}$	

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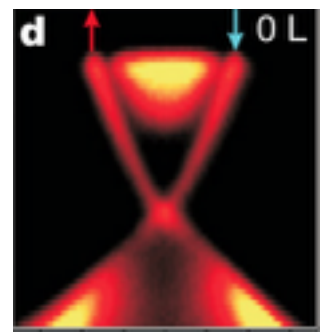
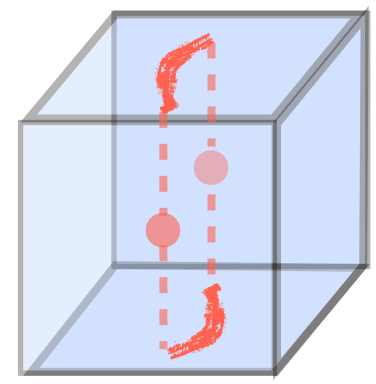
Boundary	protected 2D Dirac node ARPES, STM, surf. transport 	'Fermi arcs'  ARPES Quantum Oscillations
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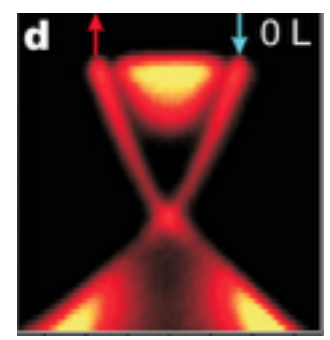
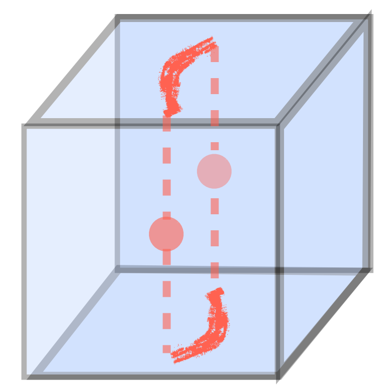
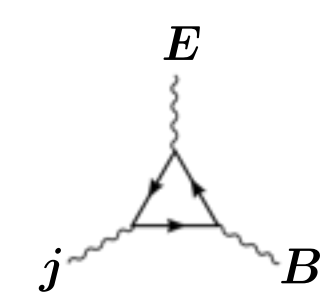
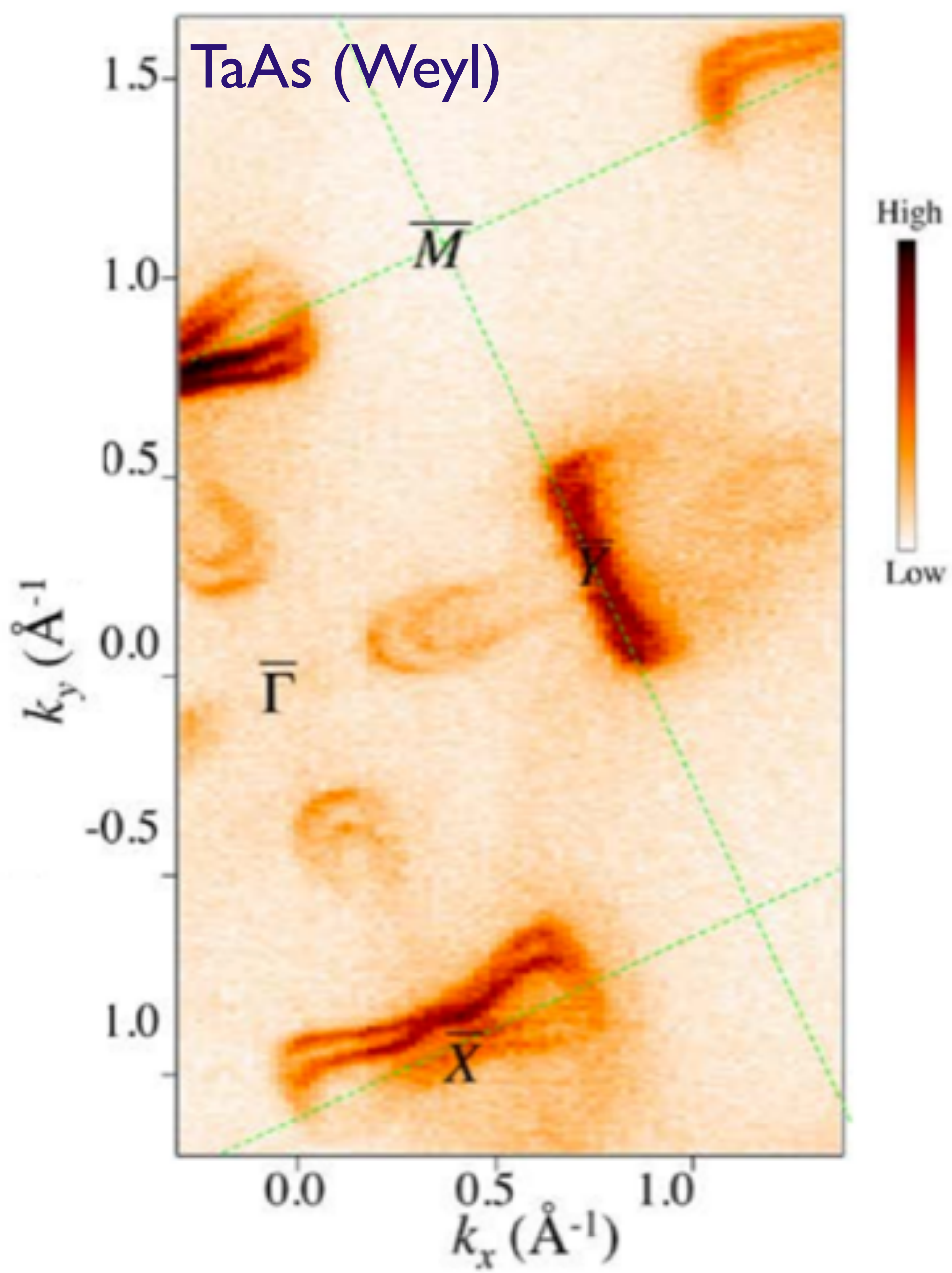
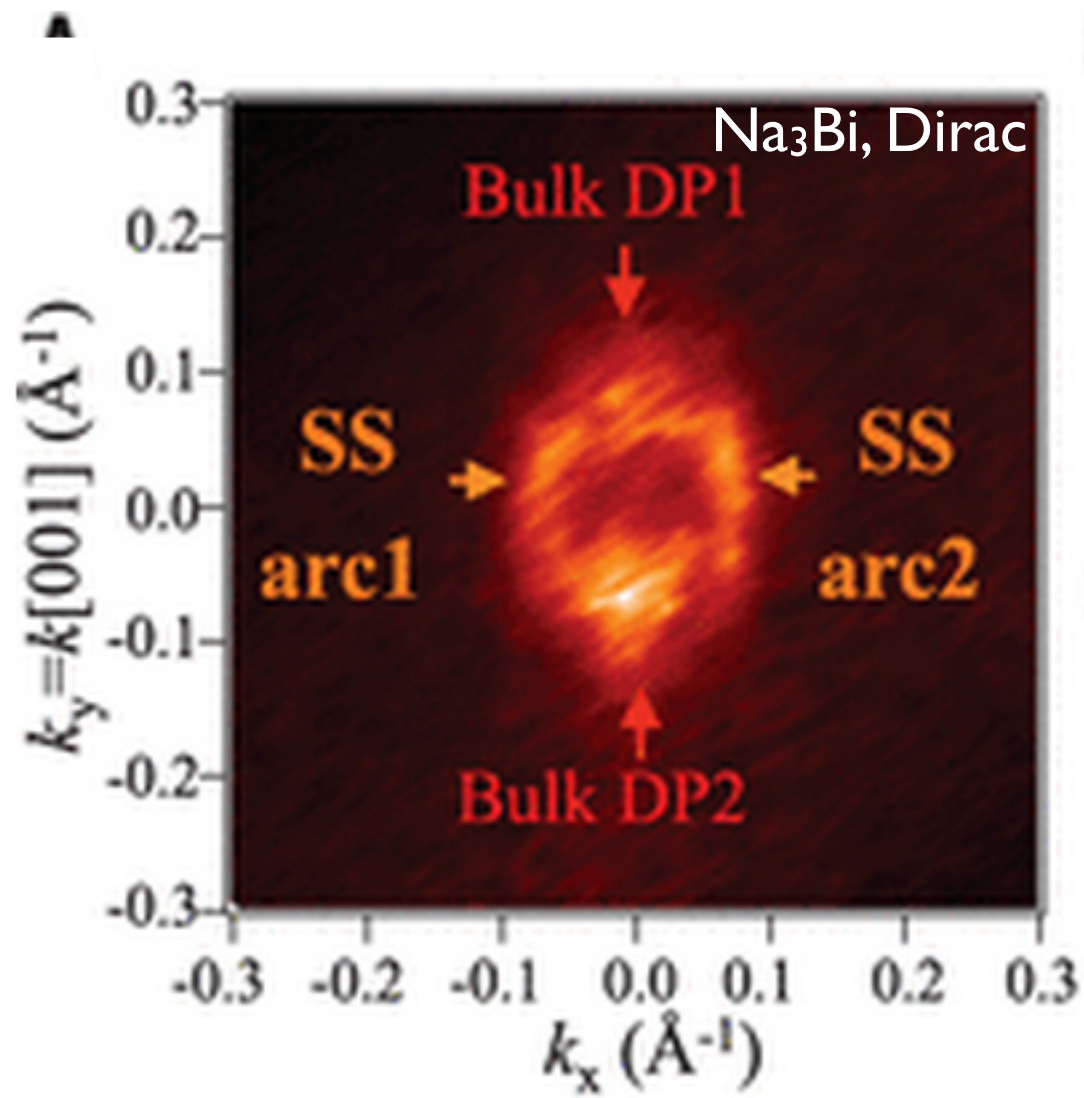
Boundary	protected 2D Dirac node ARPES, STM, surf. transport 	'Fermi arcs'  ARPES Quantum Oscillations
Bulk	magnetoelectric effect $\mathcal{S} = \mathcal{S}_{em} + \frac{\alpha}{4\pi}\int \boldsymbol{E}\cdot\boldsymbol{B}$	Quantum Anomalous Hall Eff. Adler-Bell-Jackiw Anomaly 

Boundary Effects

1. "Seeing" Arcs in ARPES



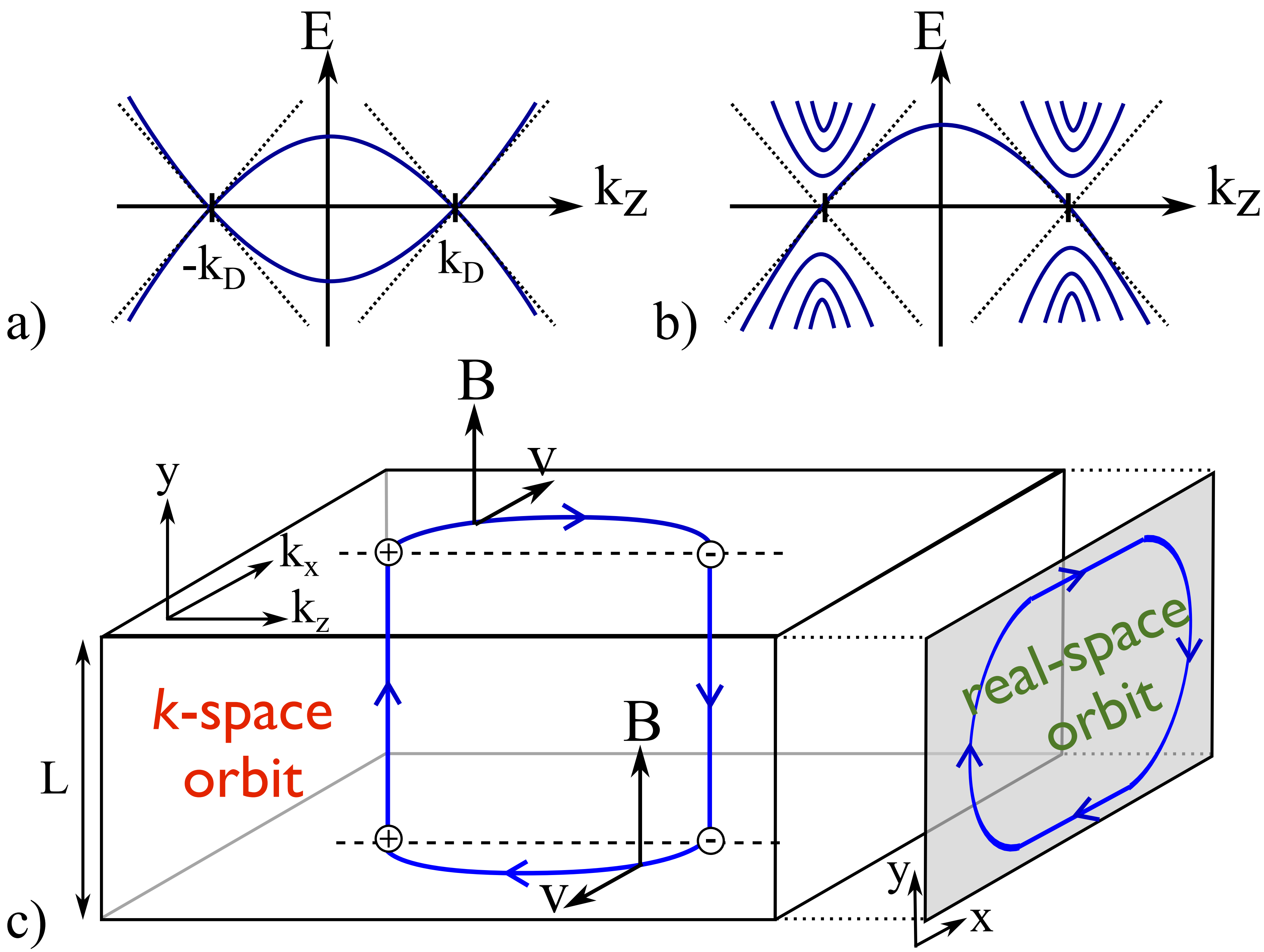


[S.-Y. Xu *et al.*, *Science*, July '15]

[S.-Y. Xu *et al.*, *Science*, Jan '15]

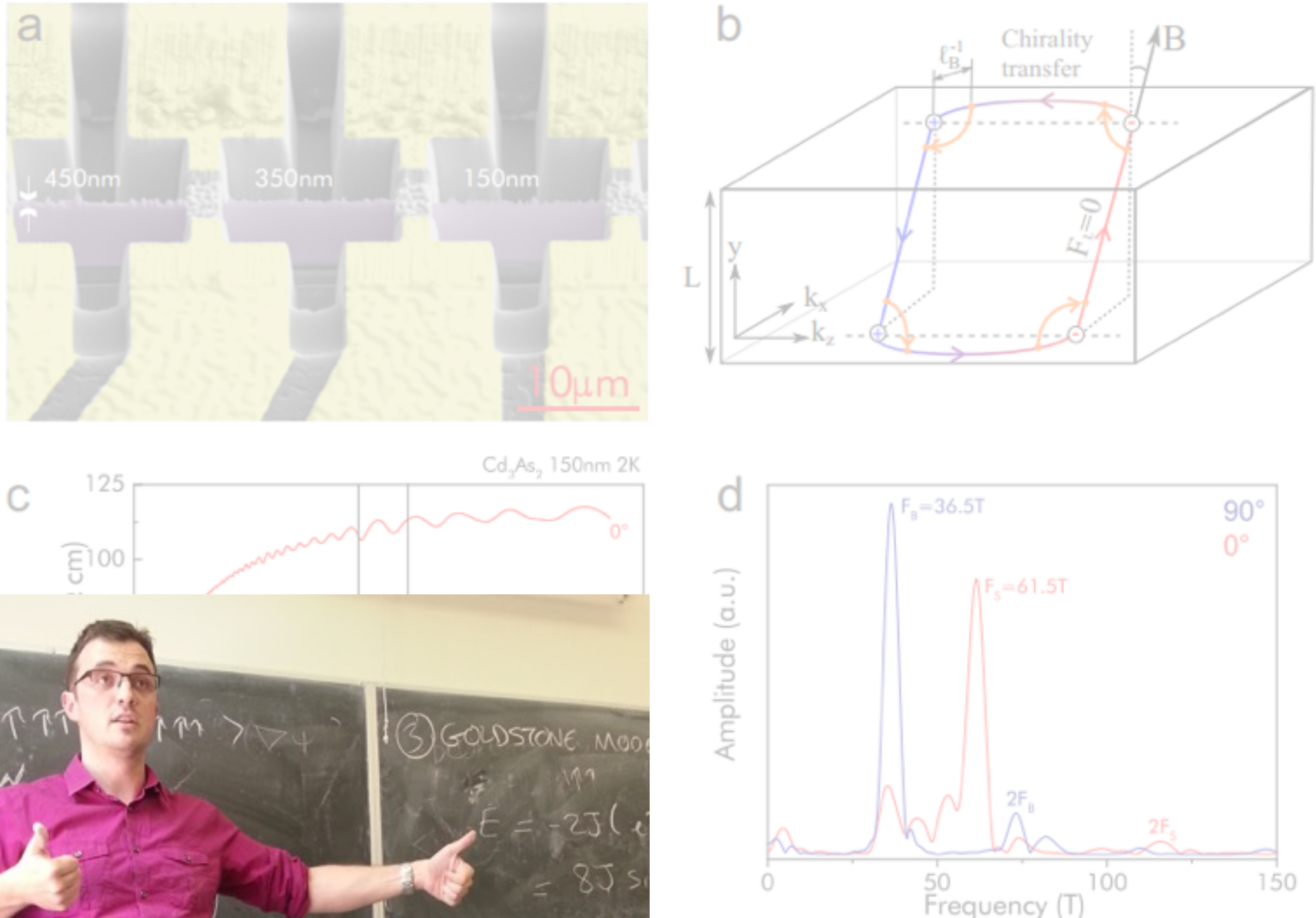
2. Quantum Oscillations

Semiclassical orbits on nonlocal Fermi surface ⇒ quantum oscillations



[Theory: A.C. Potter, I. Kimchi, A. Vishwanath, *Nat. Comm.* '14]

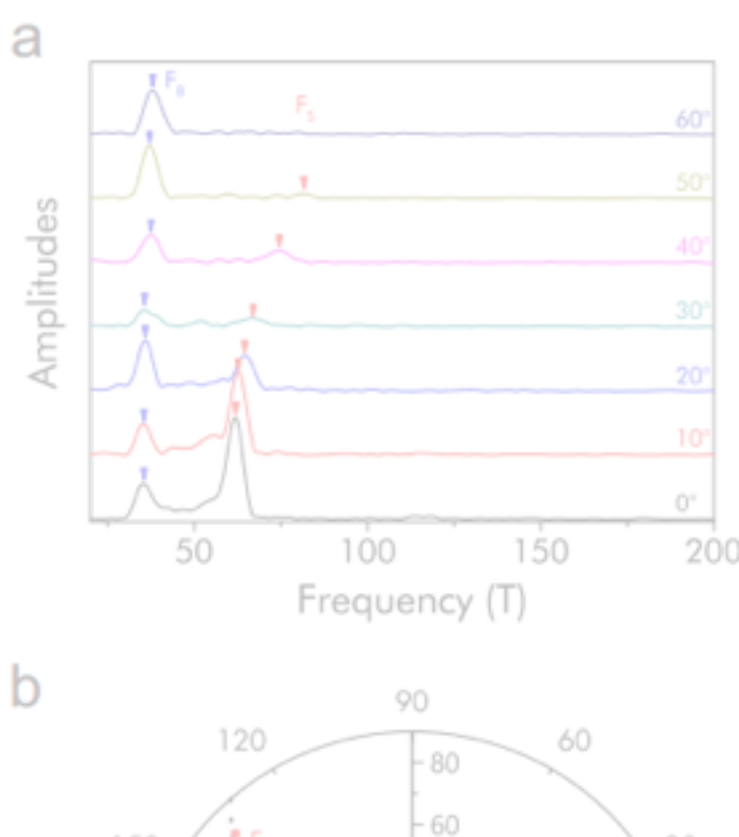
Quantum Oscillations: Experiments



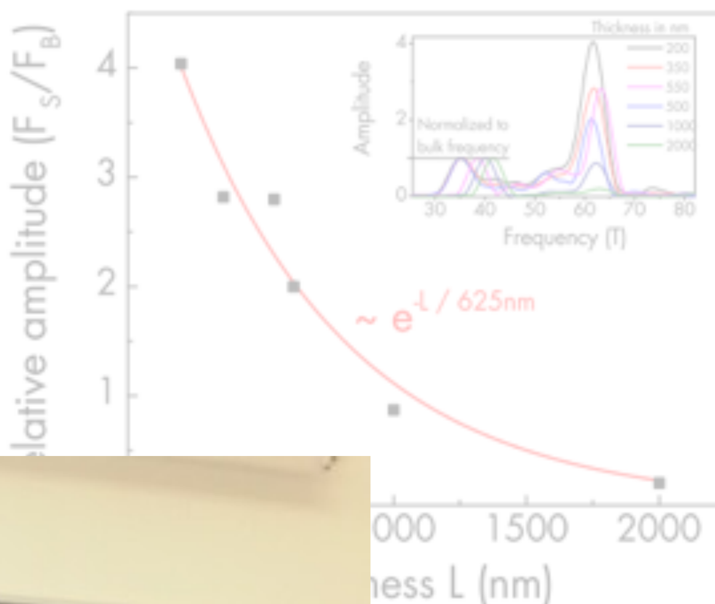
[J.W. Moll *et. al.* arXiv:1505.02817]

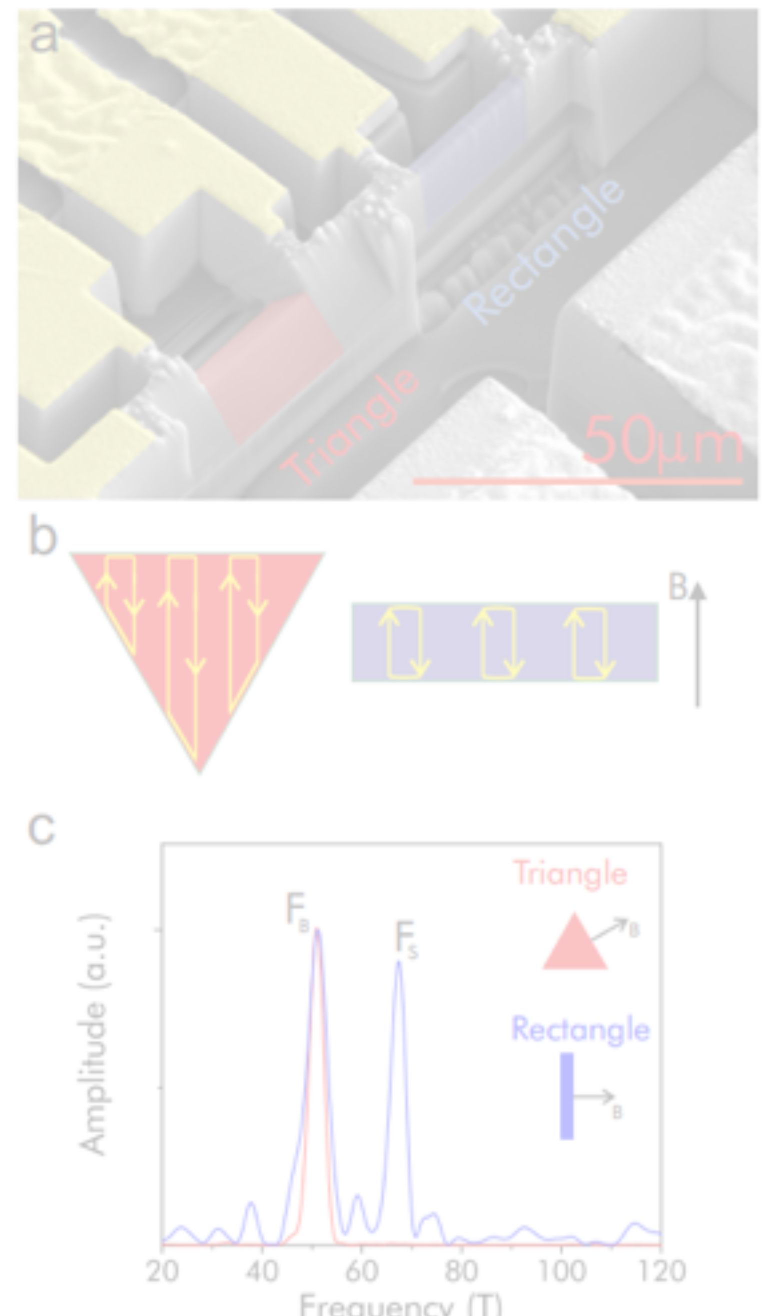
Quantum Oscillations: Experiments

Angle-dependence



Thickness-dependence + Interference



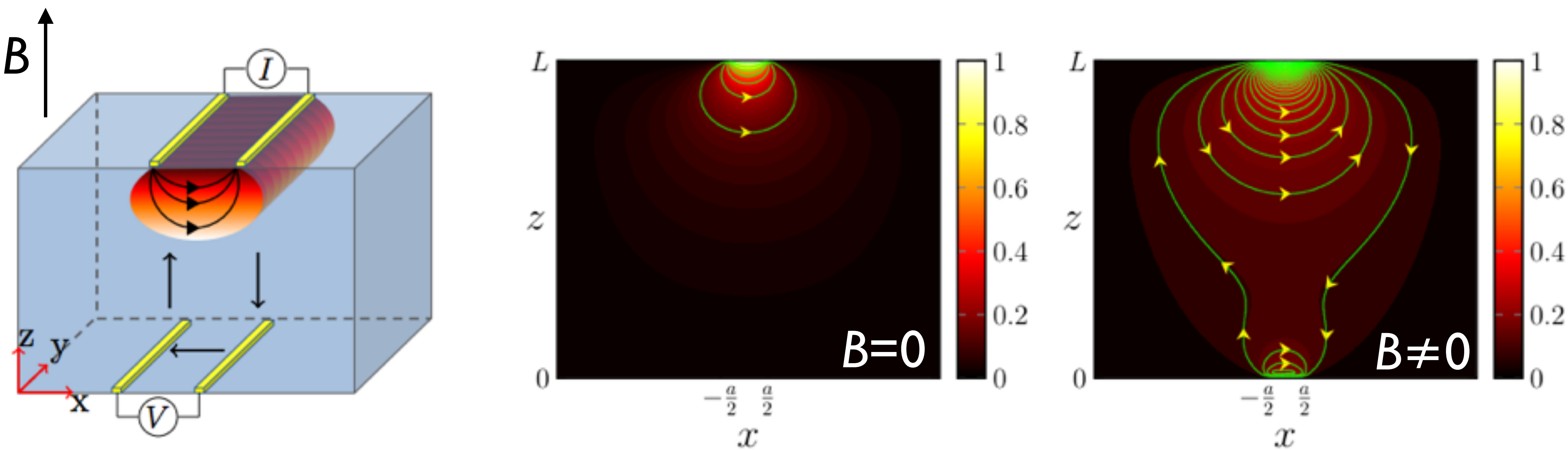


[J.W. Moll *et. al.* arXiv:1505.02817]

3. Current-at-a-Distance & Resonant Transparency

Weyl cyclotron orbits transfer charge between surfaces parallel to B

DC: Nonlocal resistance between surfaces



[Baum, Berg, S.P., Stern, arXiv:1508.03047]

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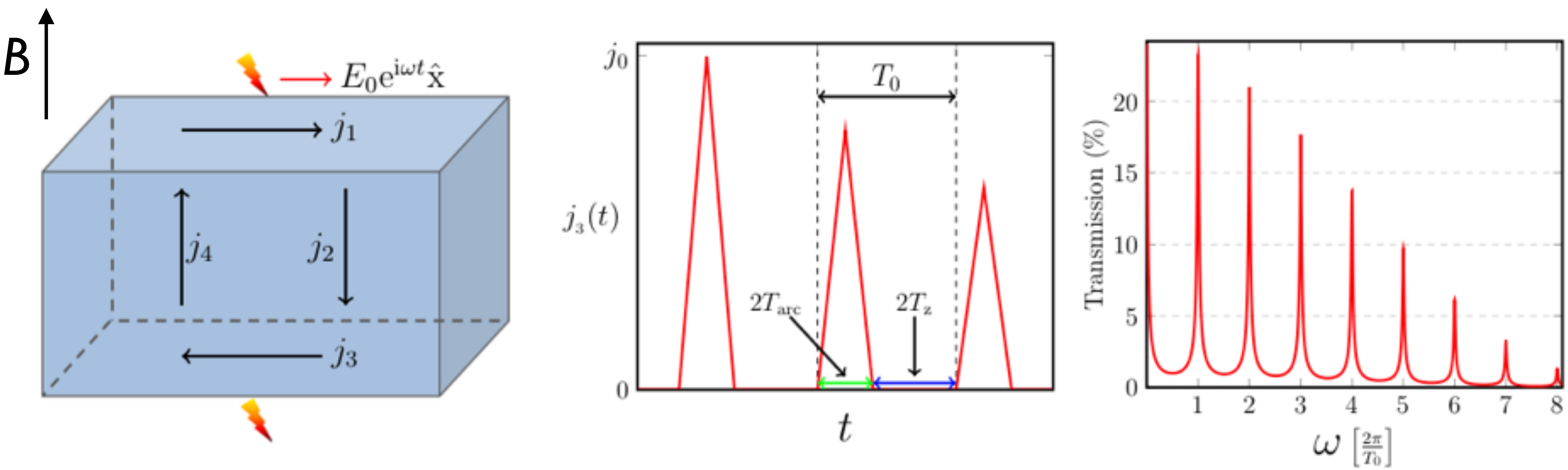
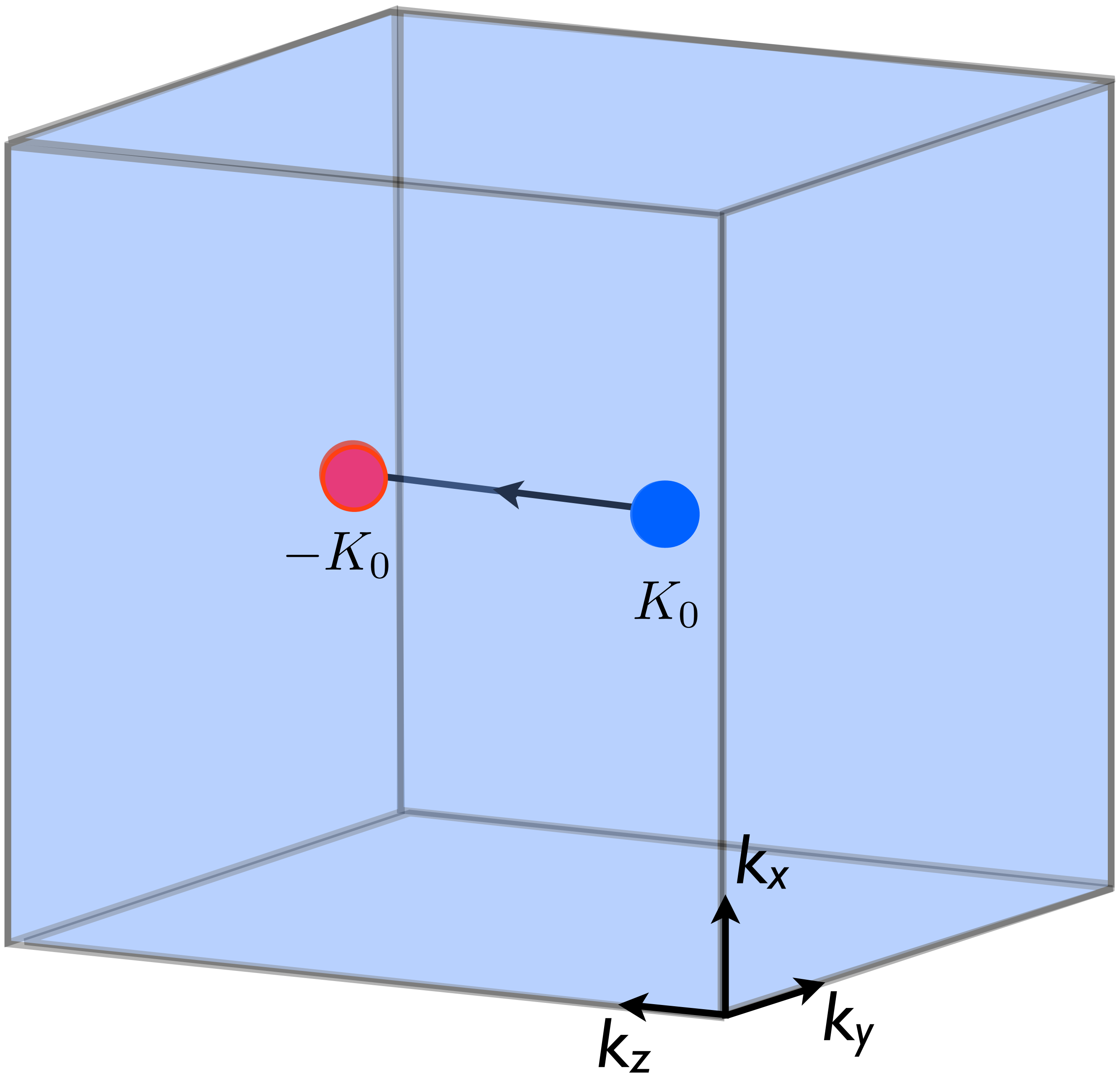
AC: resonant transmission of microwaves in anomalous skin regime

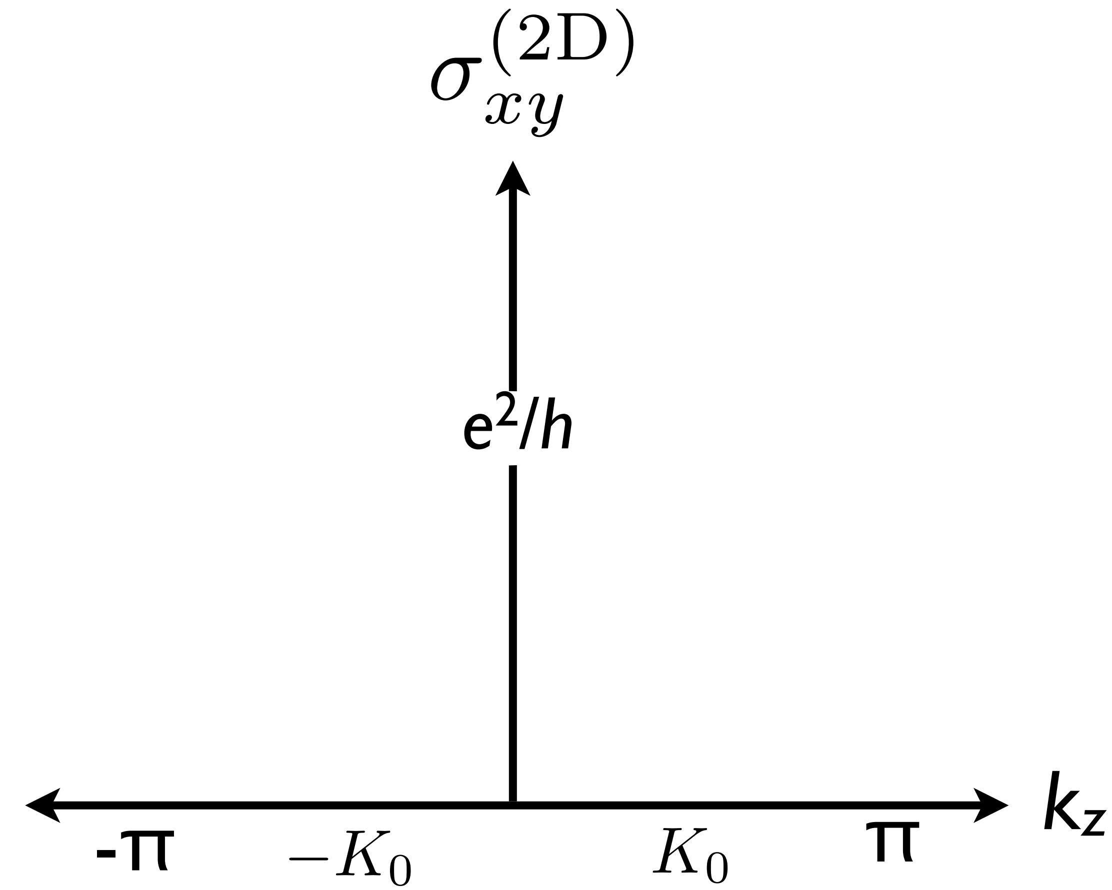


[Baum, Berg, S.P., Stern, arXiv:1508.03047]

Bulk Effects

1. Quantized Anomalous Hall Conductivity





$$\sigma_{xy}^{(3D)} = \frac{e^2}{h}(2K_0)$$

[Th.: K.-Y. Yang, Y.-M. Lu, Y. Ran, PRB'11)]

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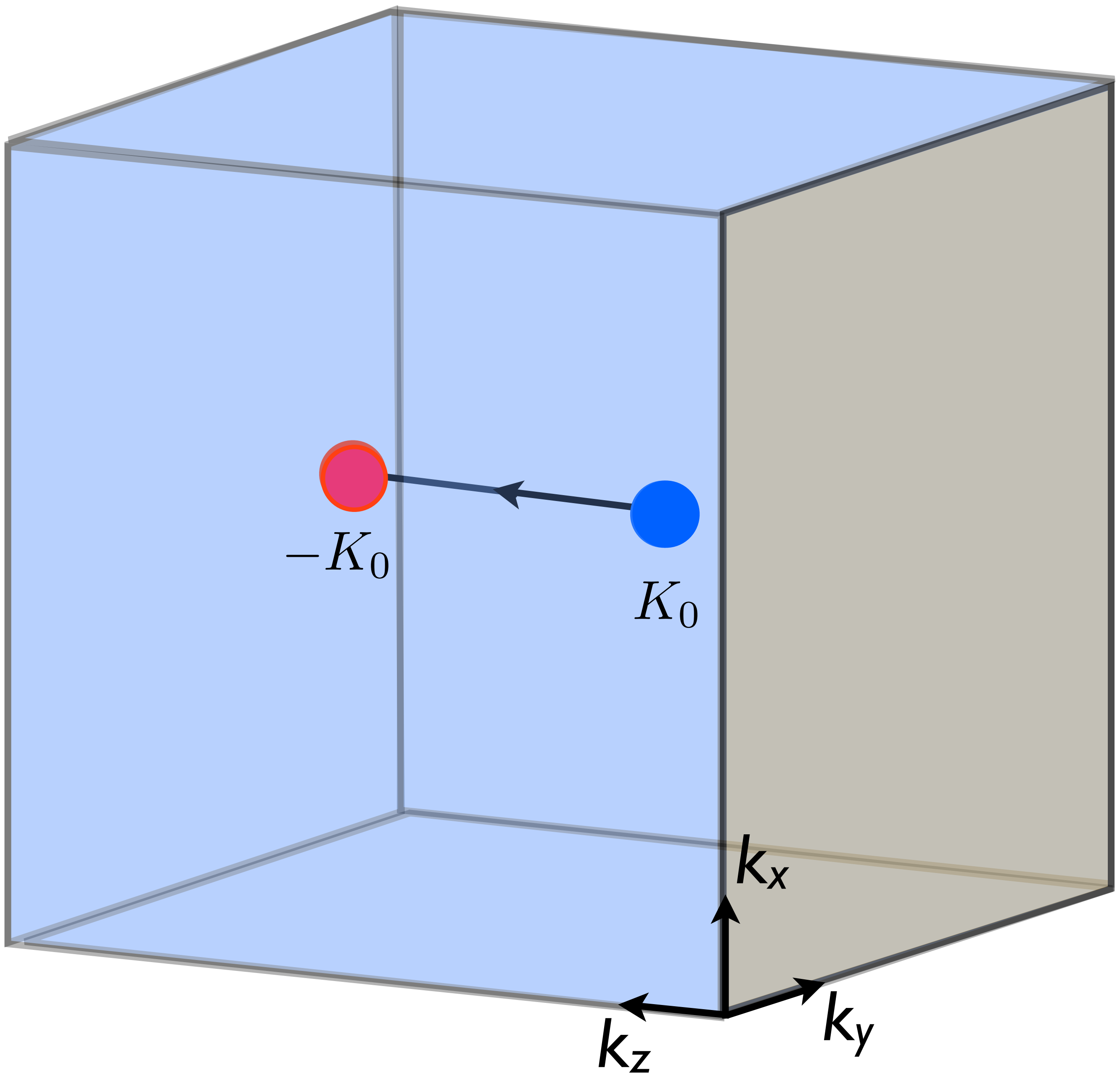

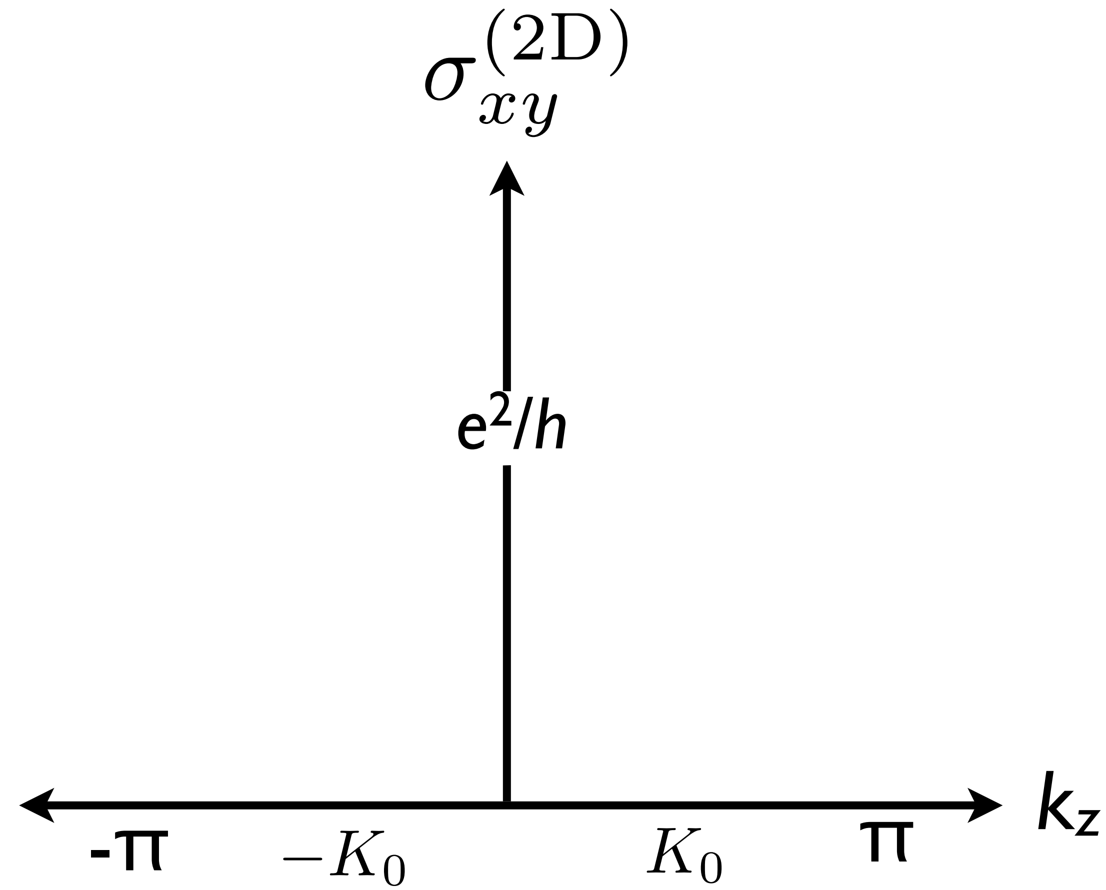

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[Th.: K.-Y. Yang, Y.-M. Lu, Y. Ran, PRB'11)]

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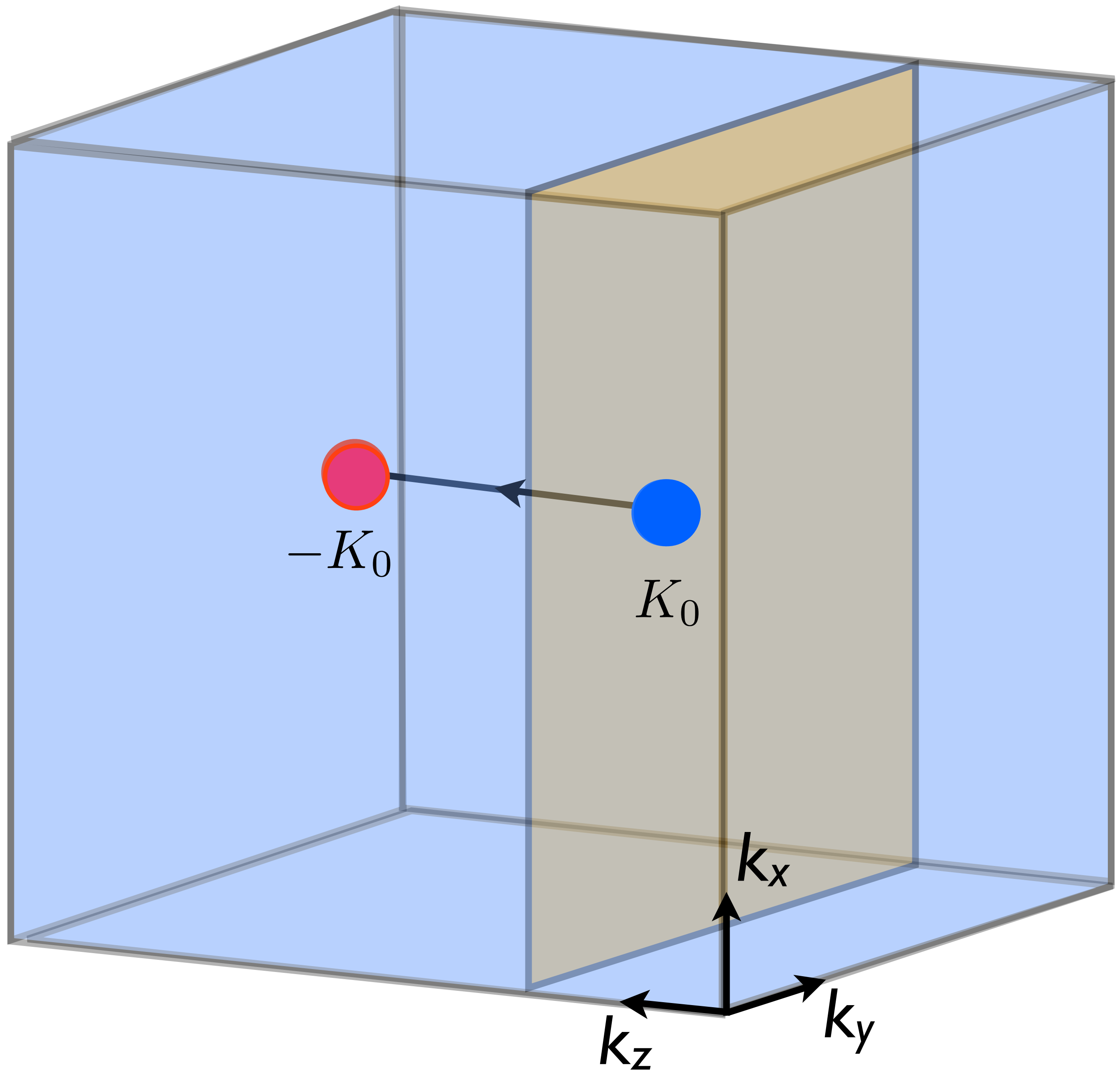

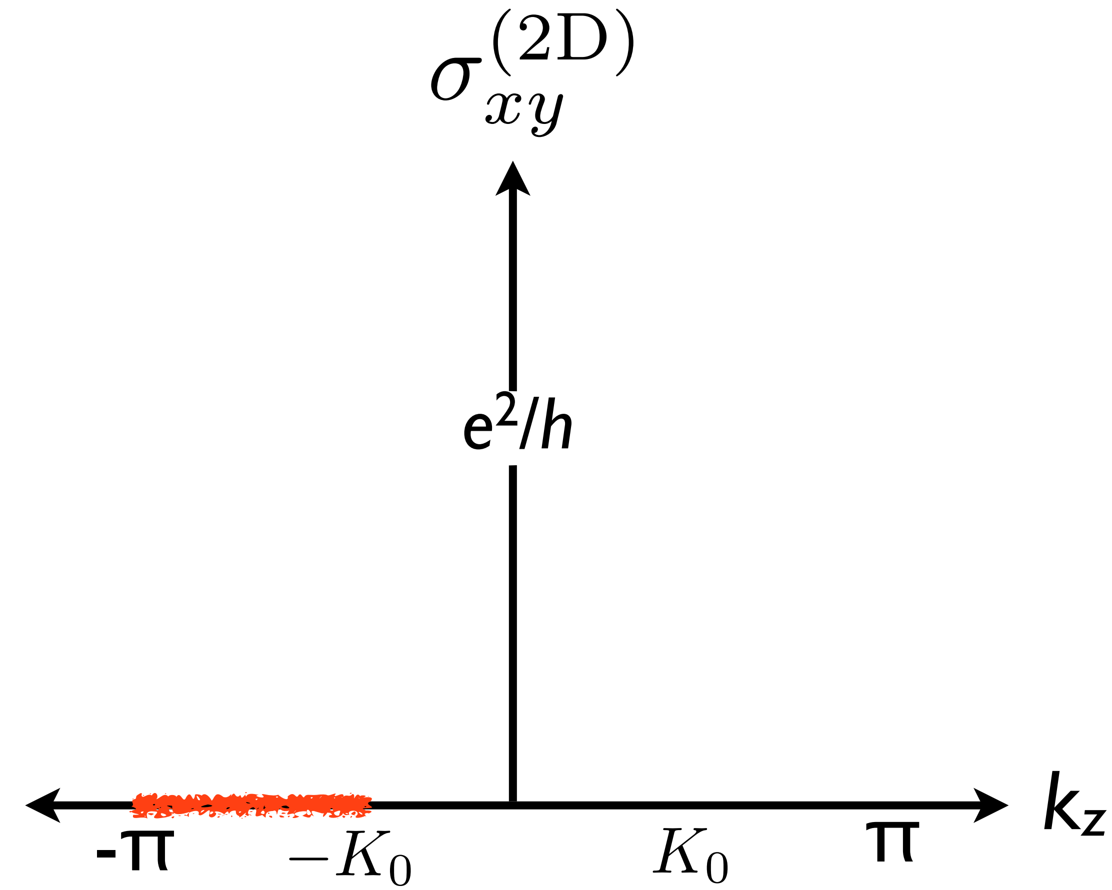

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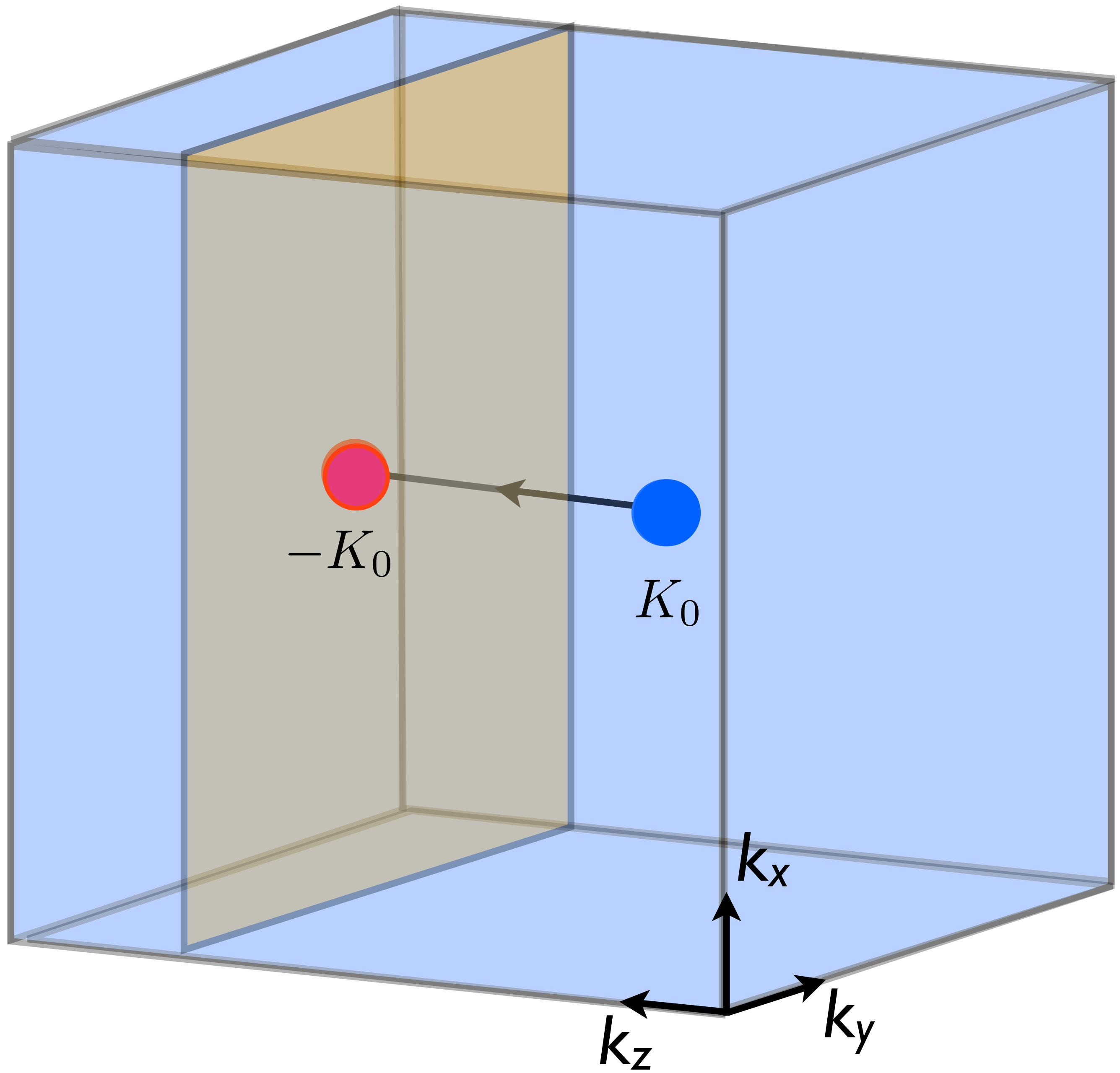

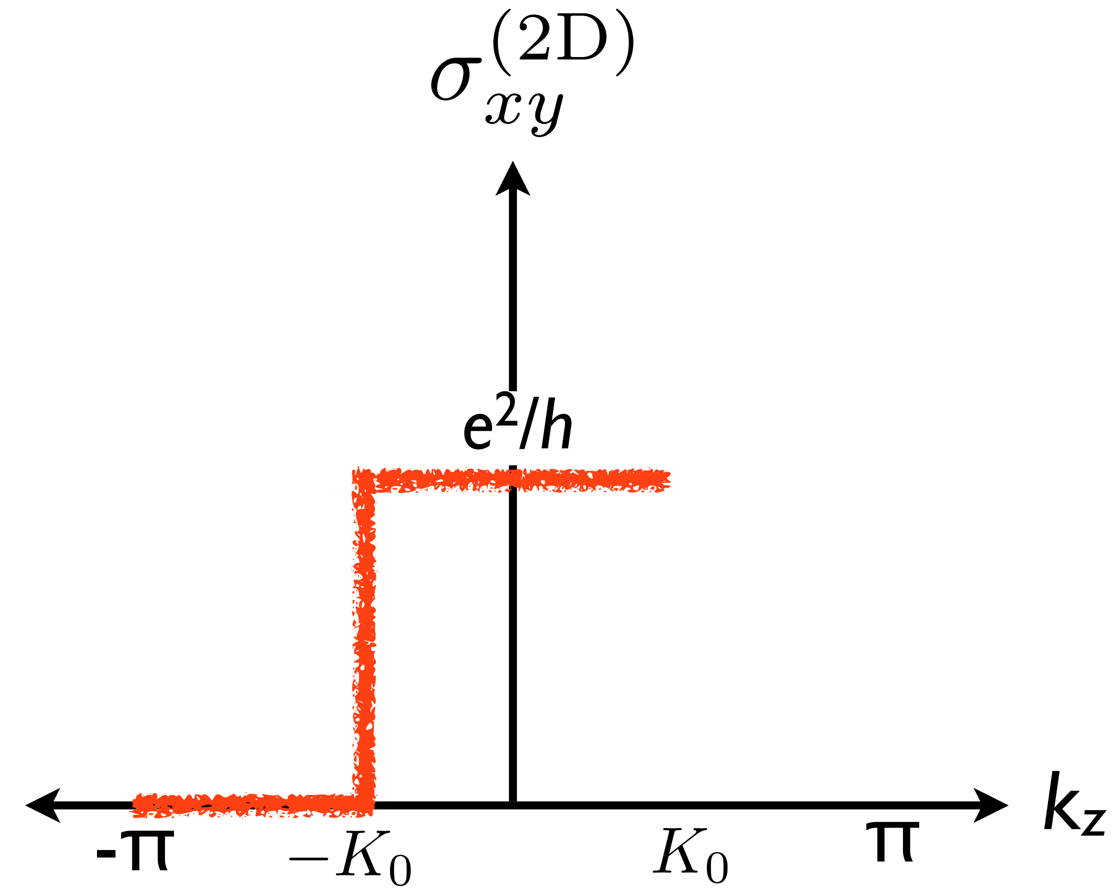

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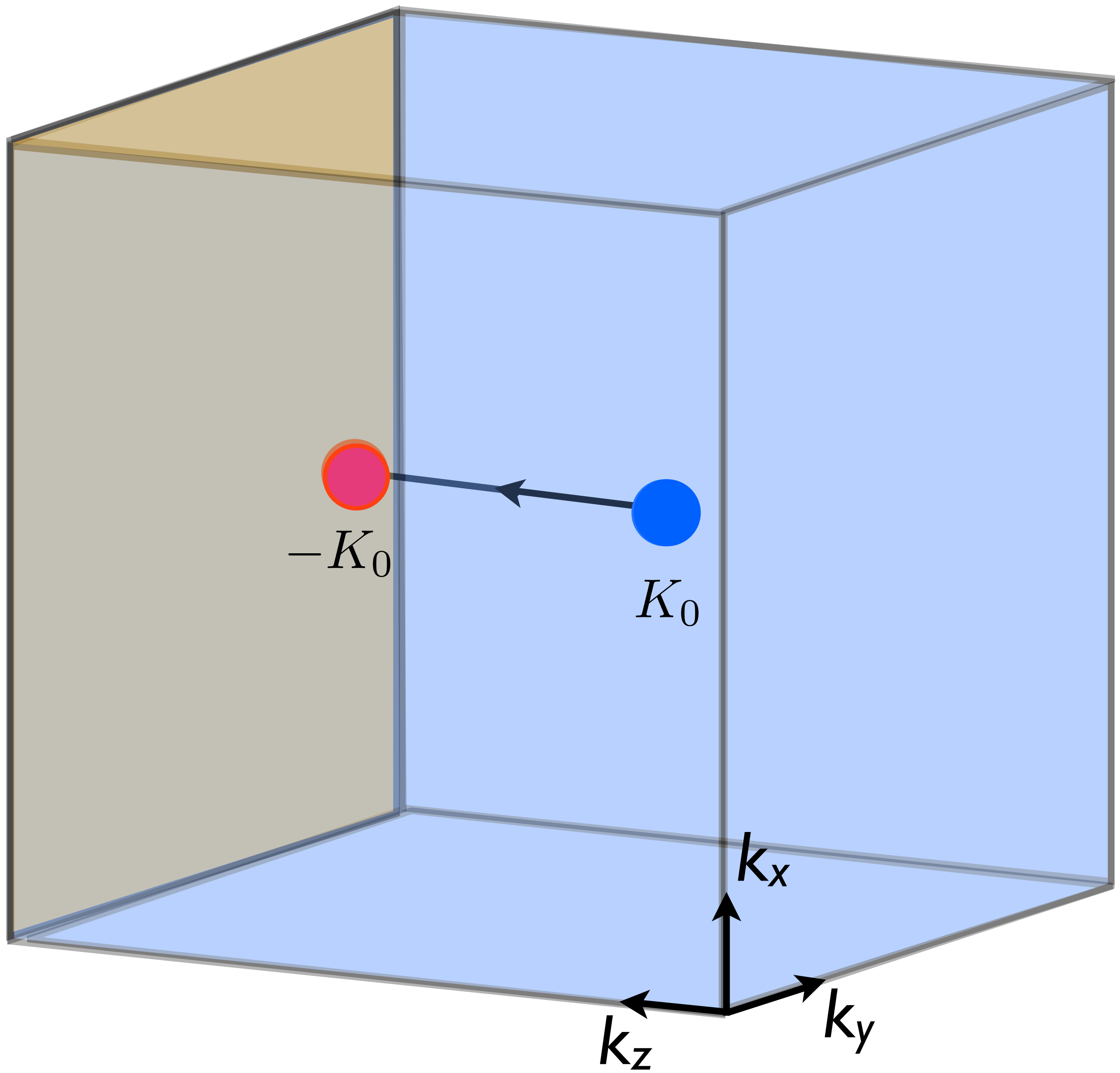

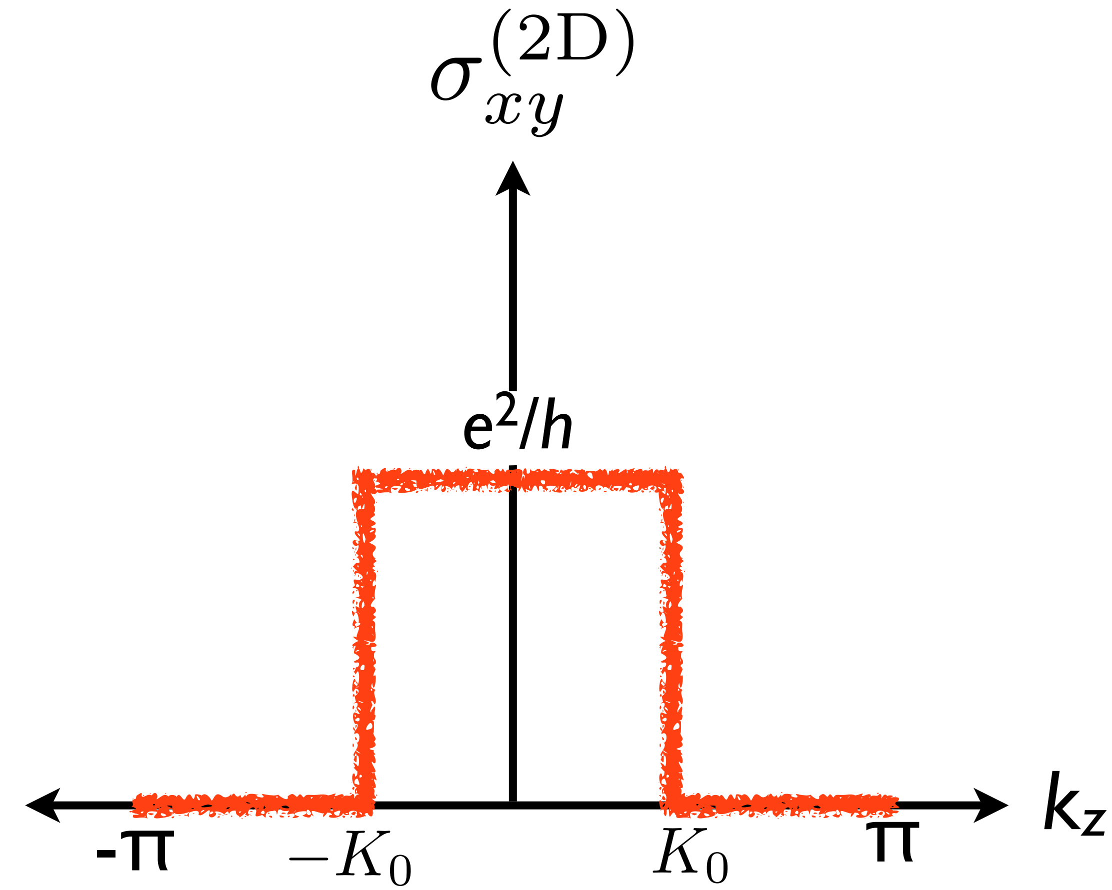

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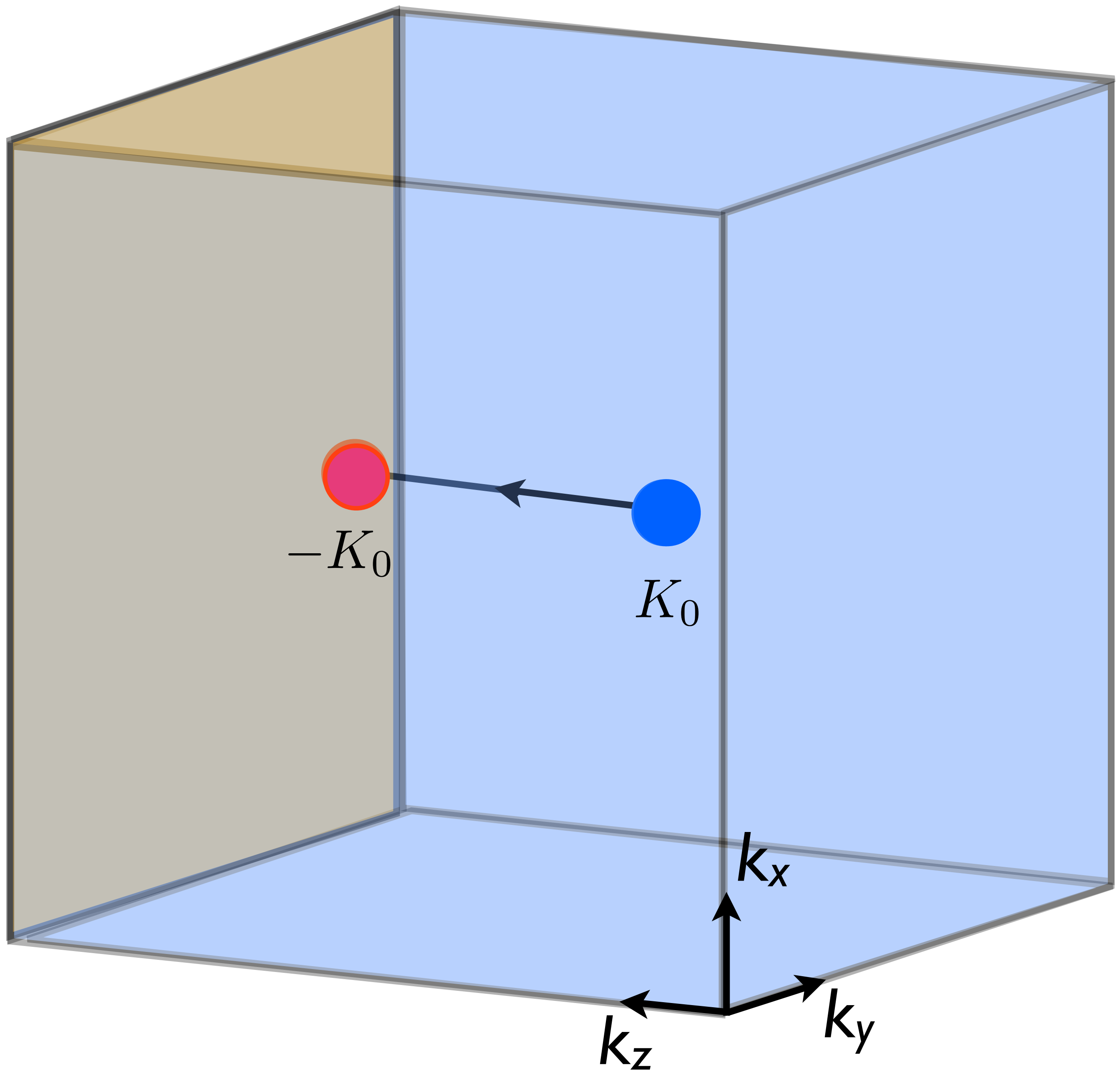

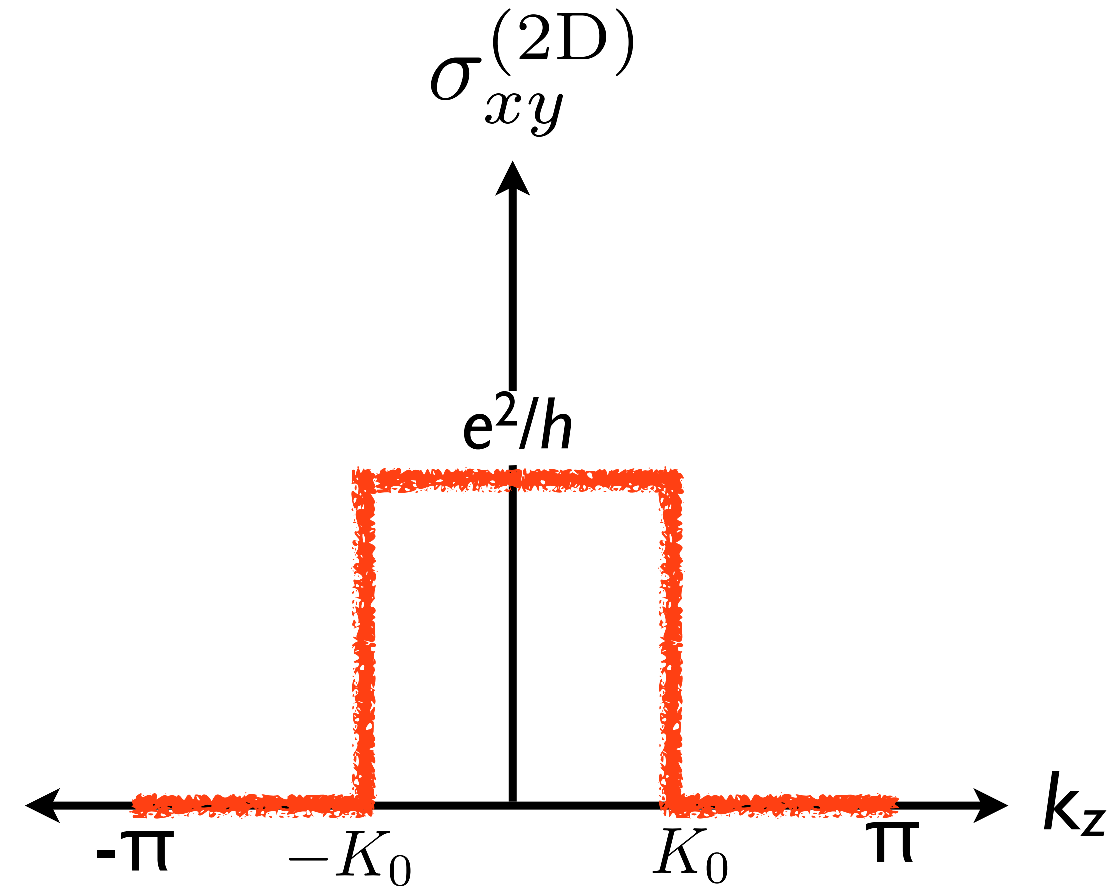

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to verify quantization, need input: location of nodes in BZ
contributions from several pairs of nodes can cancel

2. Adler-Bell-Jackiw Anomaly [Adler '69; Bell & Jackiw, '69]

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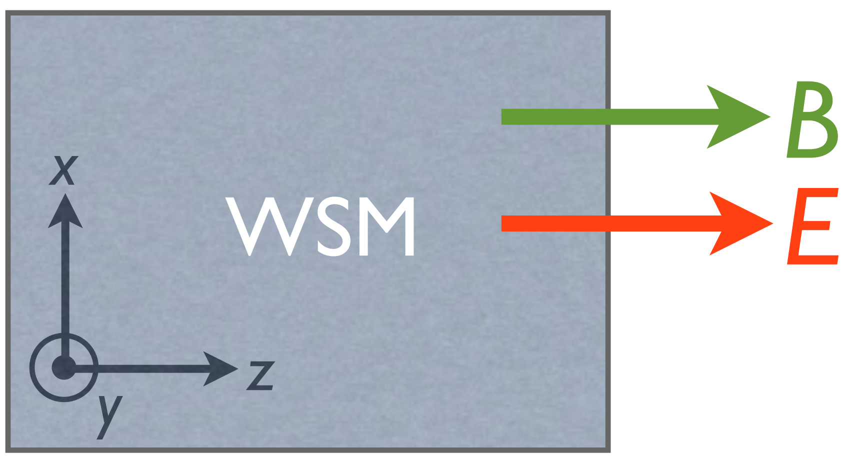
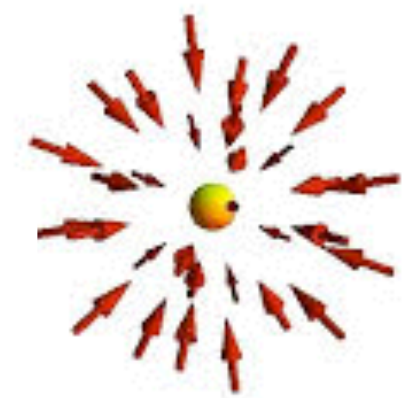


WSM: *two* Weyl fermions, opposite chirality:

'left'



$$\partial_\mu j^\mu_L = -\frac{e^2}{4\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}$$

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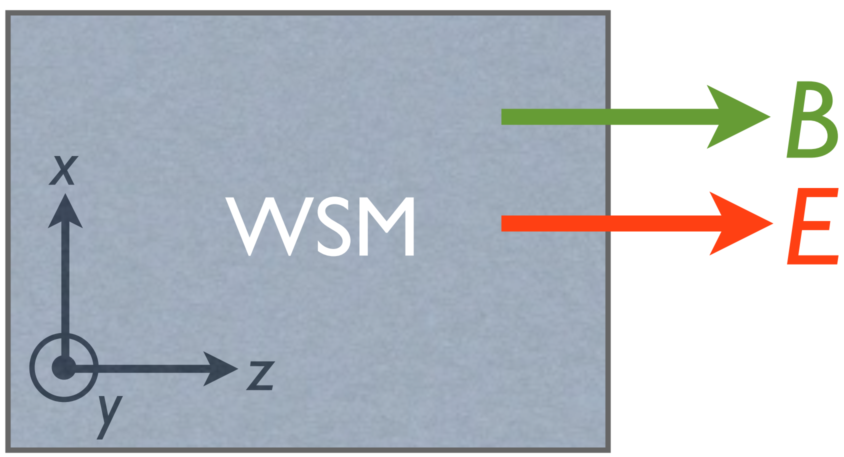

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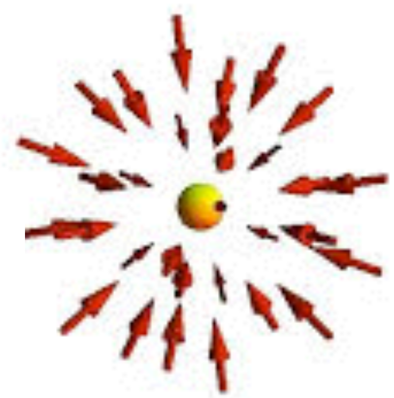

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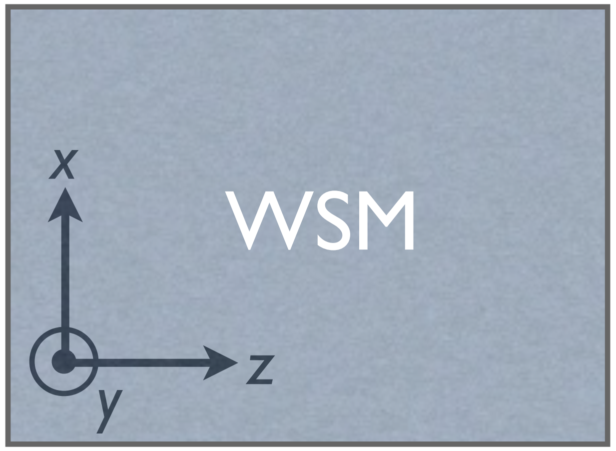
$$\partial_\mu j^\mu_R = +\frac{e^2}{4\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

electric current $j^\mu_R + j^\mu_L$ conserved

'valley' current ('$j^{\mu 5}$') $j^\mu_R - j^\mu_L$ not conserved

Microscopic Origin of ABJ Anomaly



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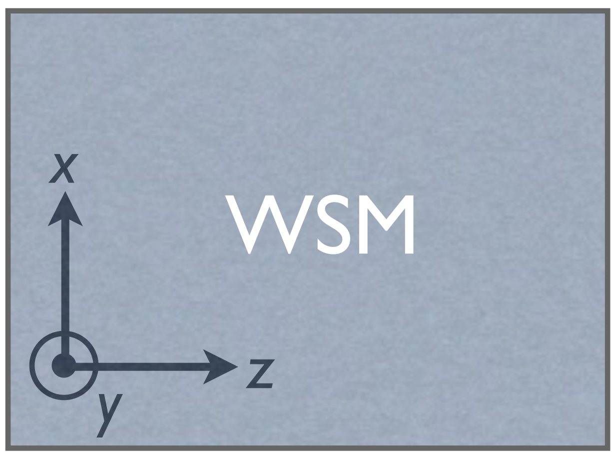

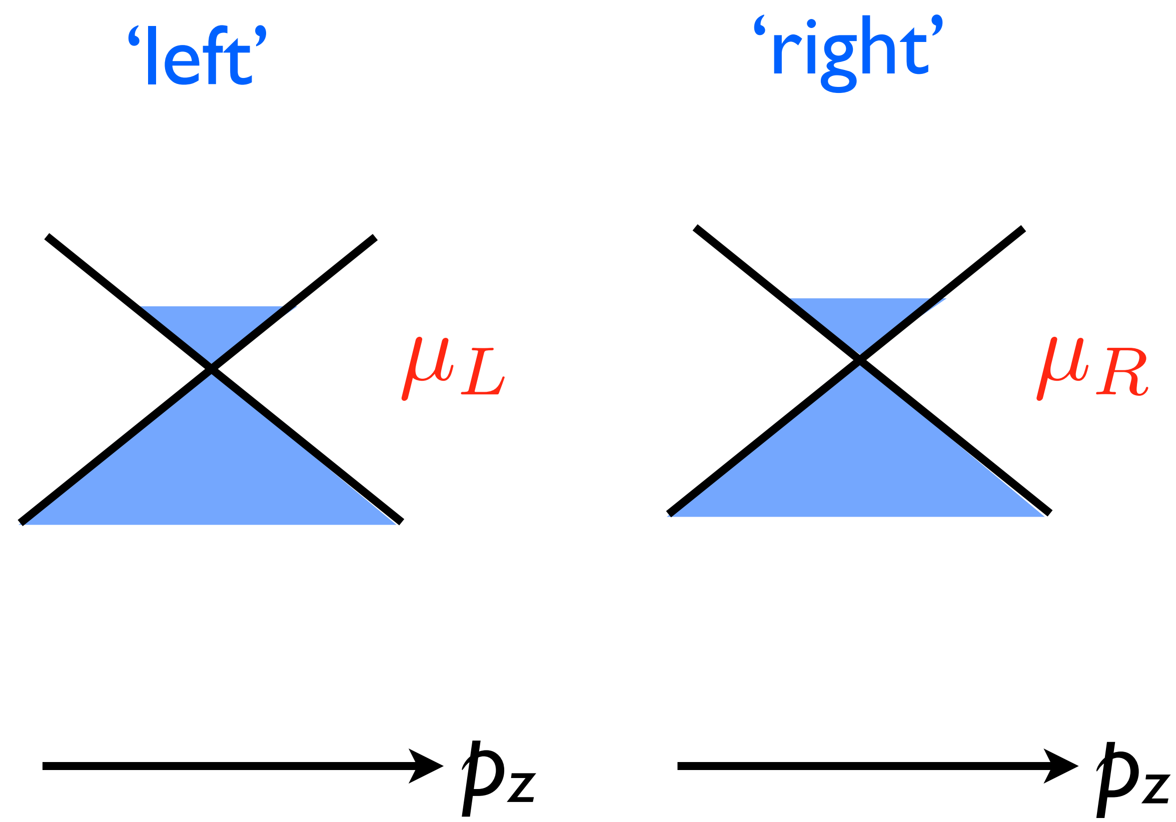

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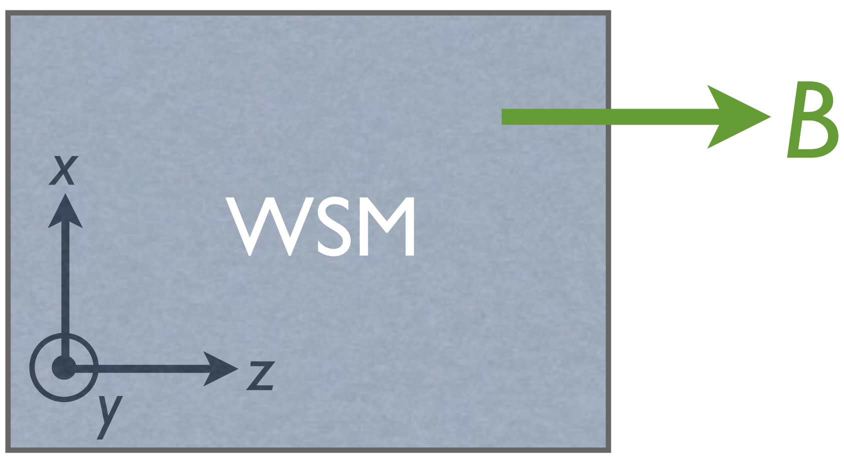

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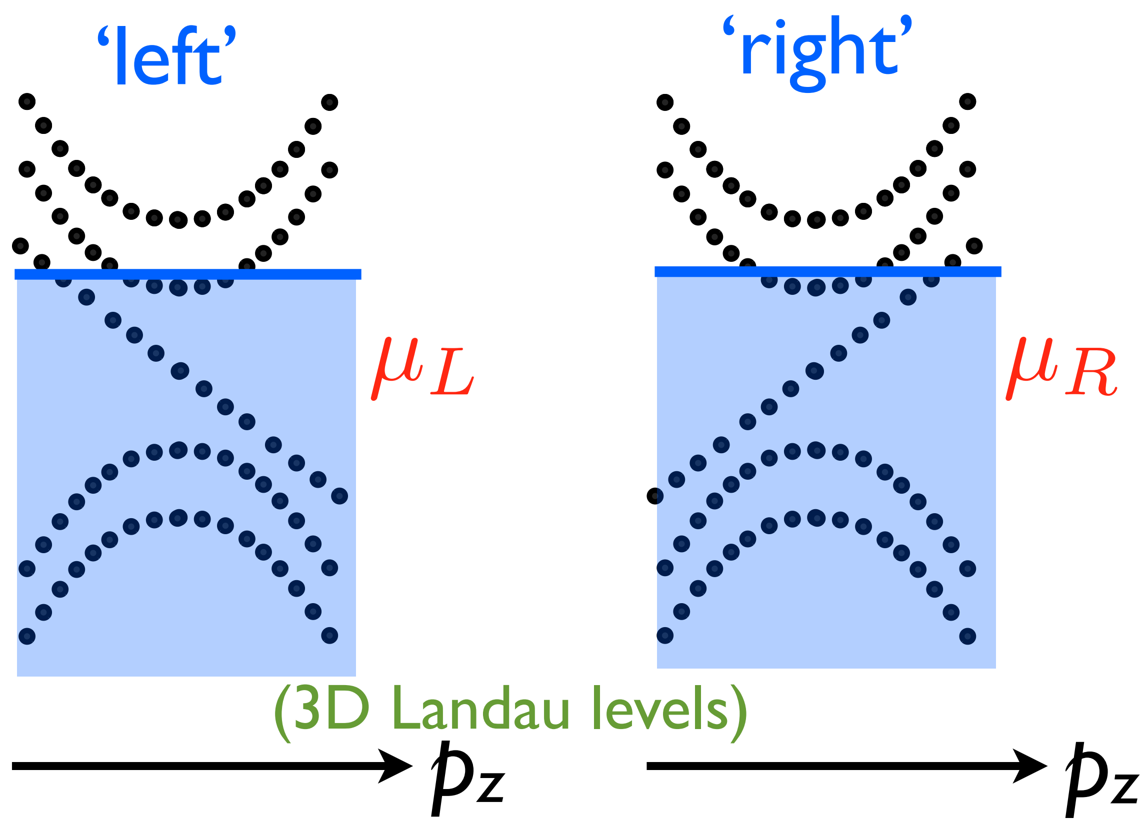

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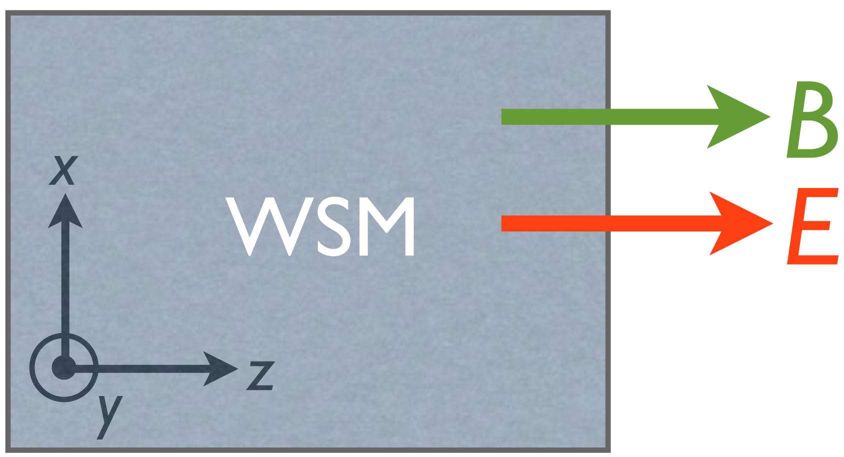

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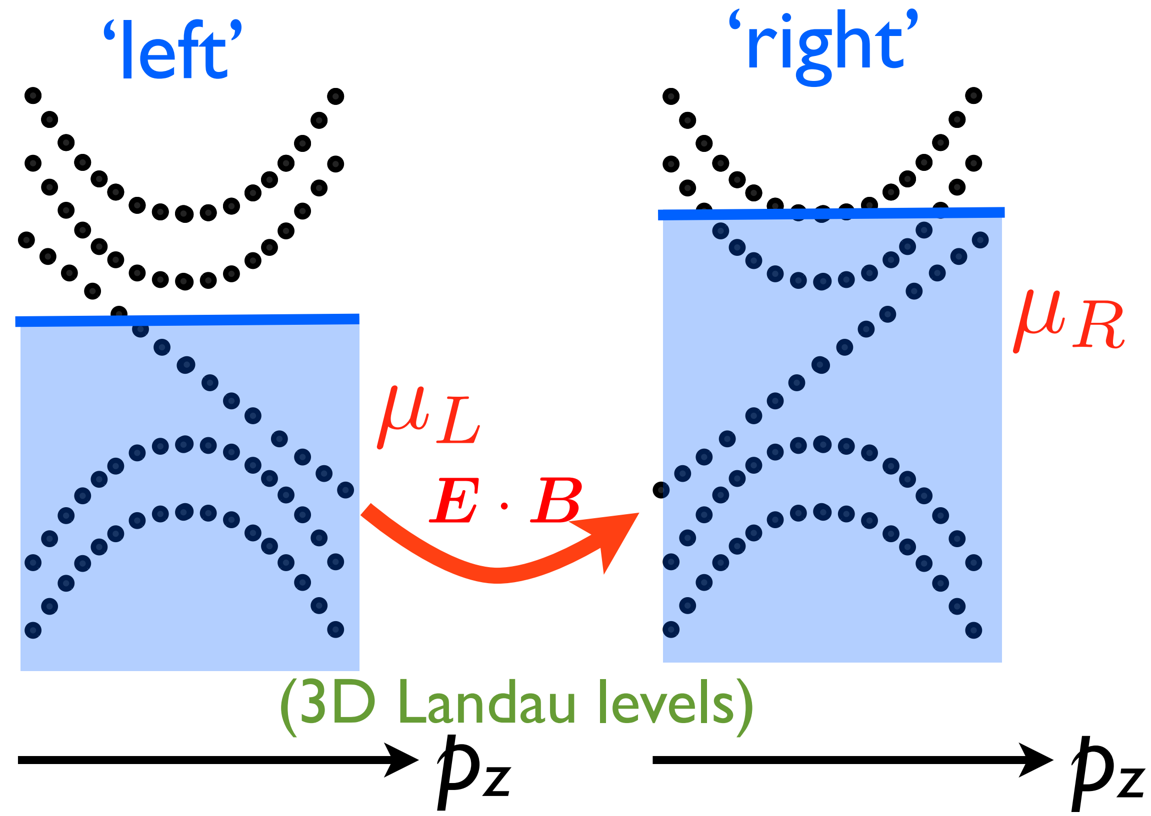

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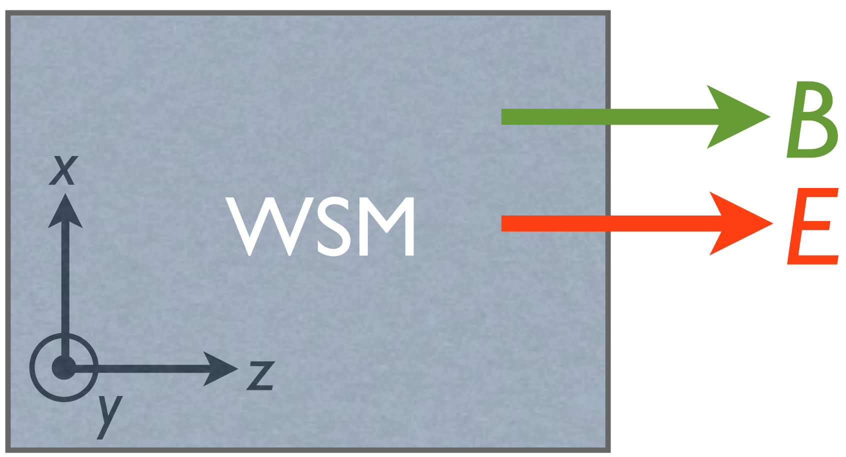

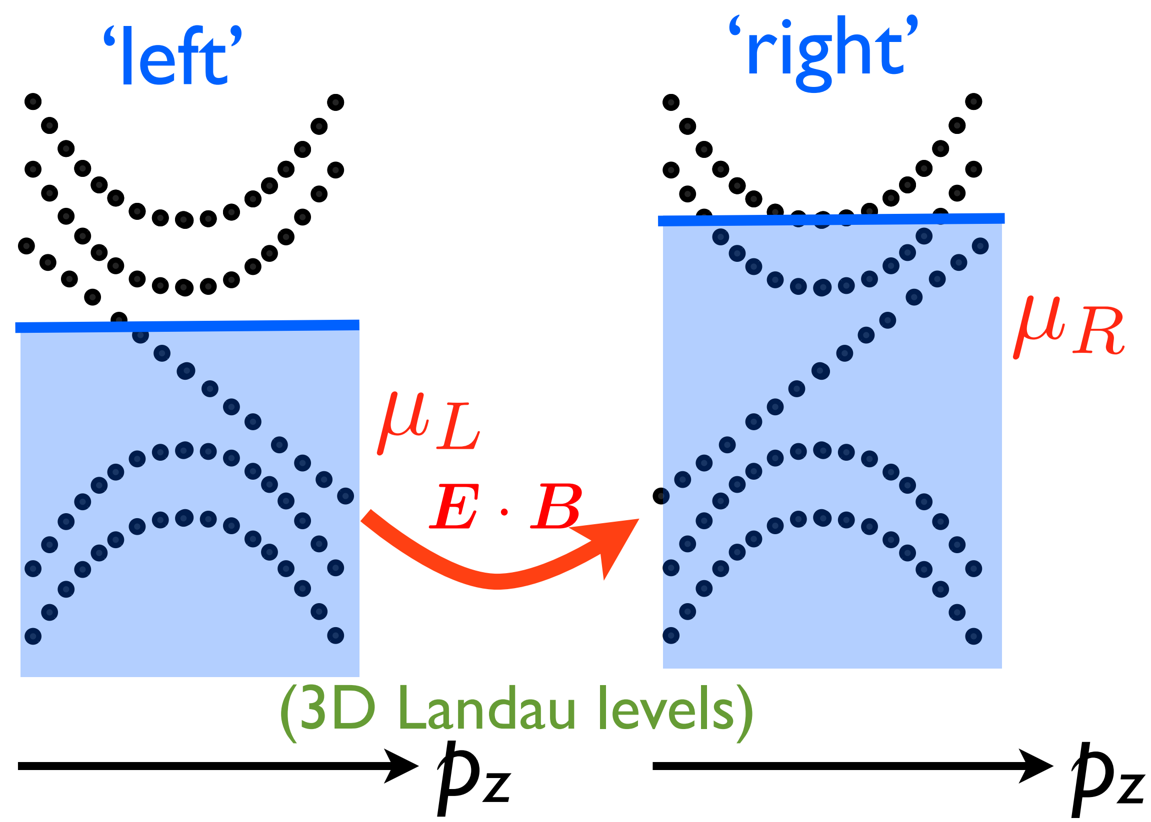

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Measurable consequences?

[Nielsen & Ninomiya, Phys. Lett. B **130**, 389 (1983); see also Vazifeh & Franz PRL '13]

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Unlike TIs, here ‘generic’ metallic transport masks topology

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T-breaking WSM from magnetic order ⇒ anom. magnon-plasmon coupling

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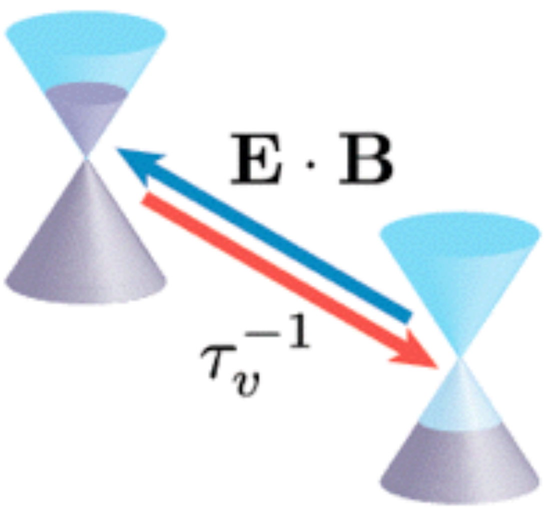
realization specific, not generic signature

Ideally: *qualitative* (yes vs. no) not *quantitative* (big vs. small) signature!

‘Seeing’ the Anomaly via Nonlocal Transport

Valley charge not conserved: $$\partial_\mu (j_R^\mu - j_L^\mu) = \frac{e^2}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$



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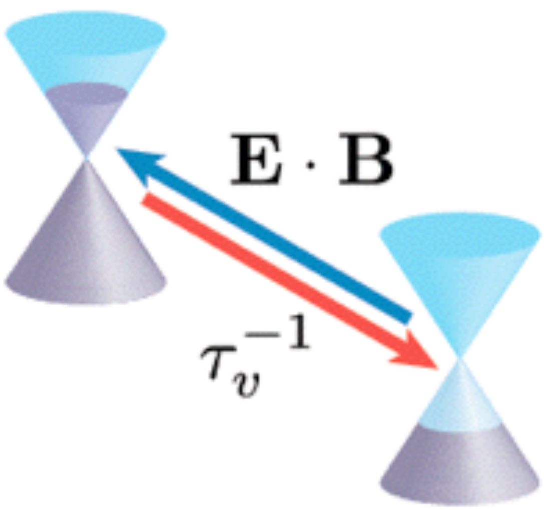

relaxing 'valley' imbalance requires inter-node scattering

but *charge* imbalance can relax at single node!

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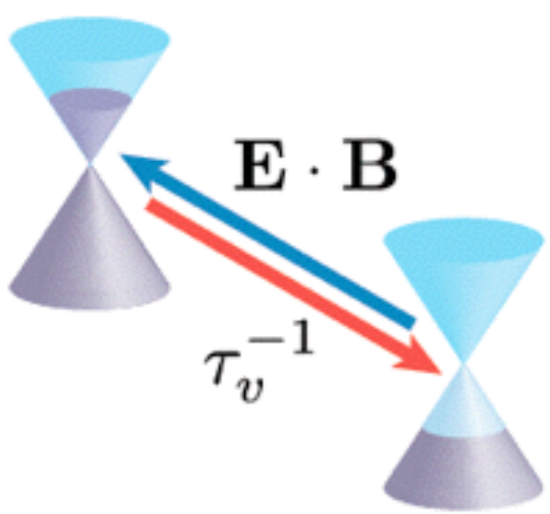

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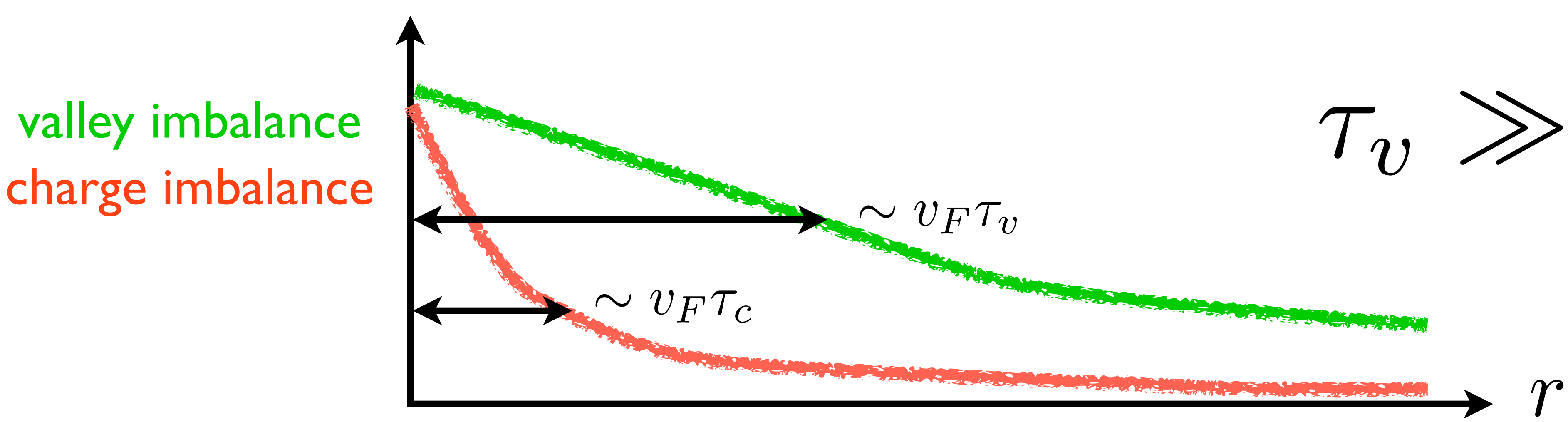
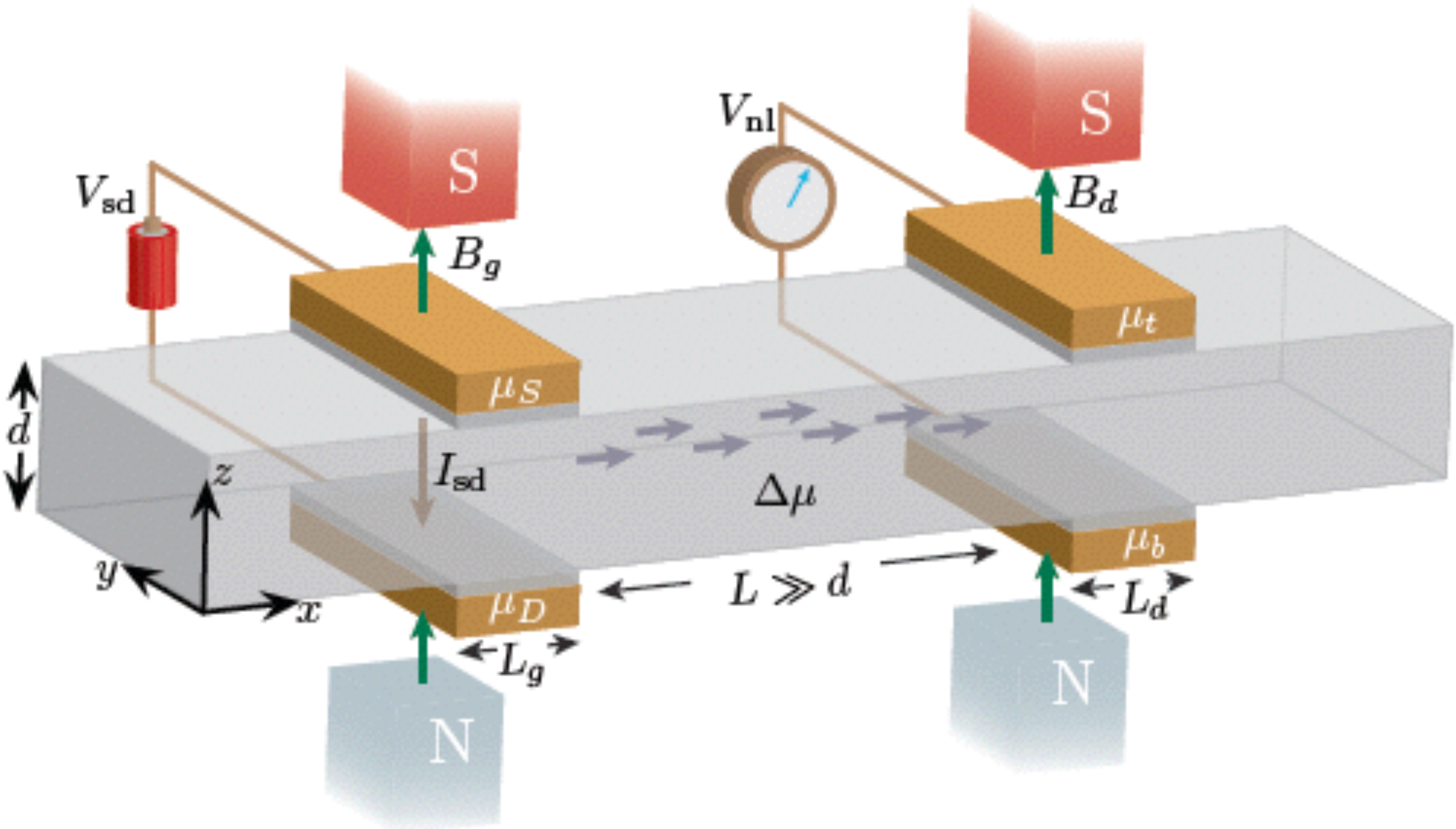
$\Rightarrow$ 'topological' transport ~ valley imbalance ~ slow relaxation

'boring' bulk transport ~ charge imbalance ~ fast relaxation



$$\tau_v \gg \tau_c$$

Proposed Experiment



[SP, *et al.*, *PRX* **4**, 031035 (2014)]

Generating Valley Imbalance

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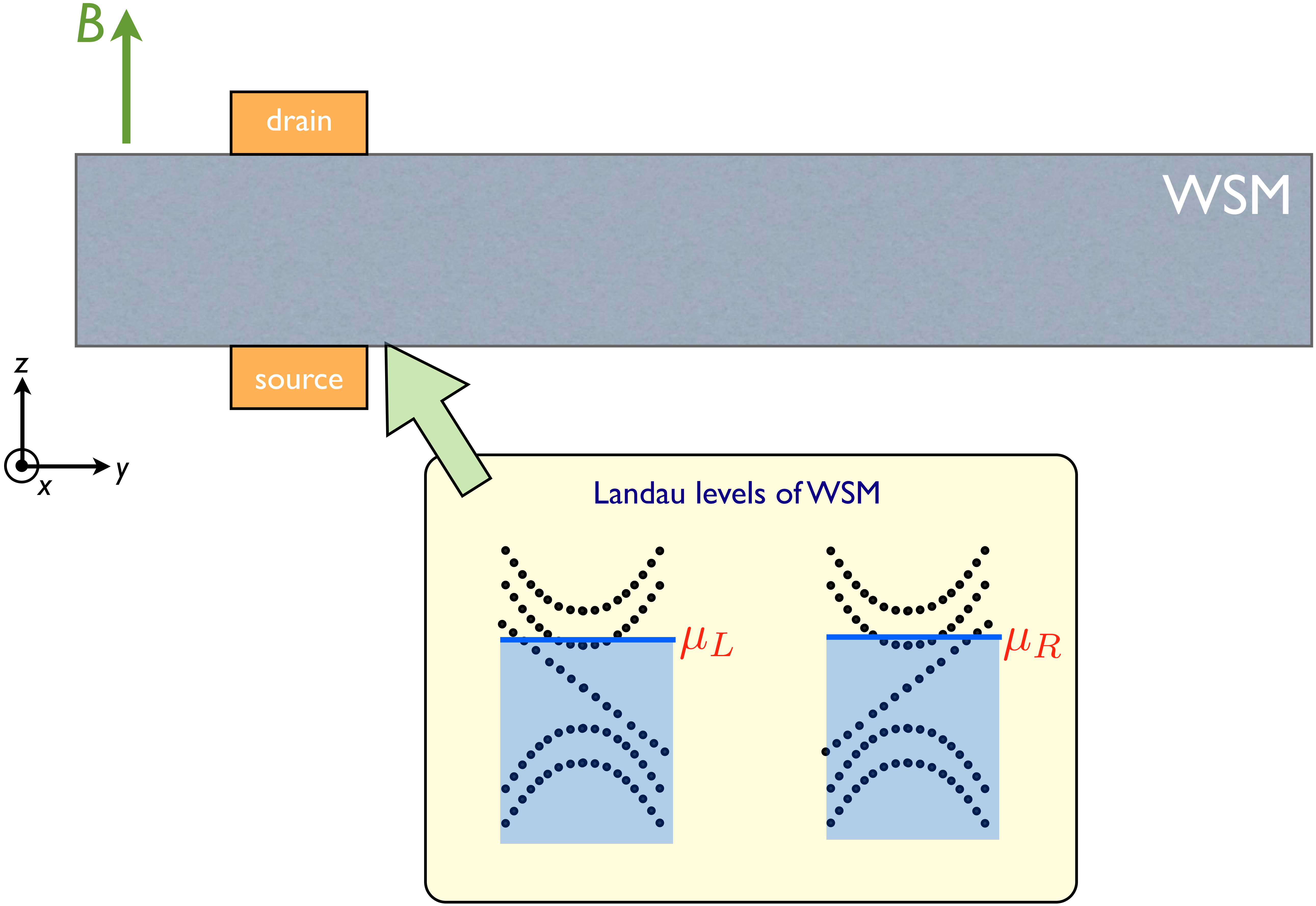

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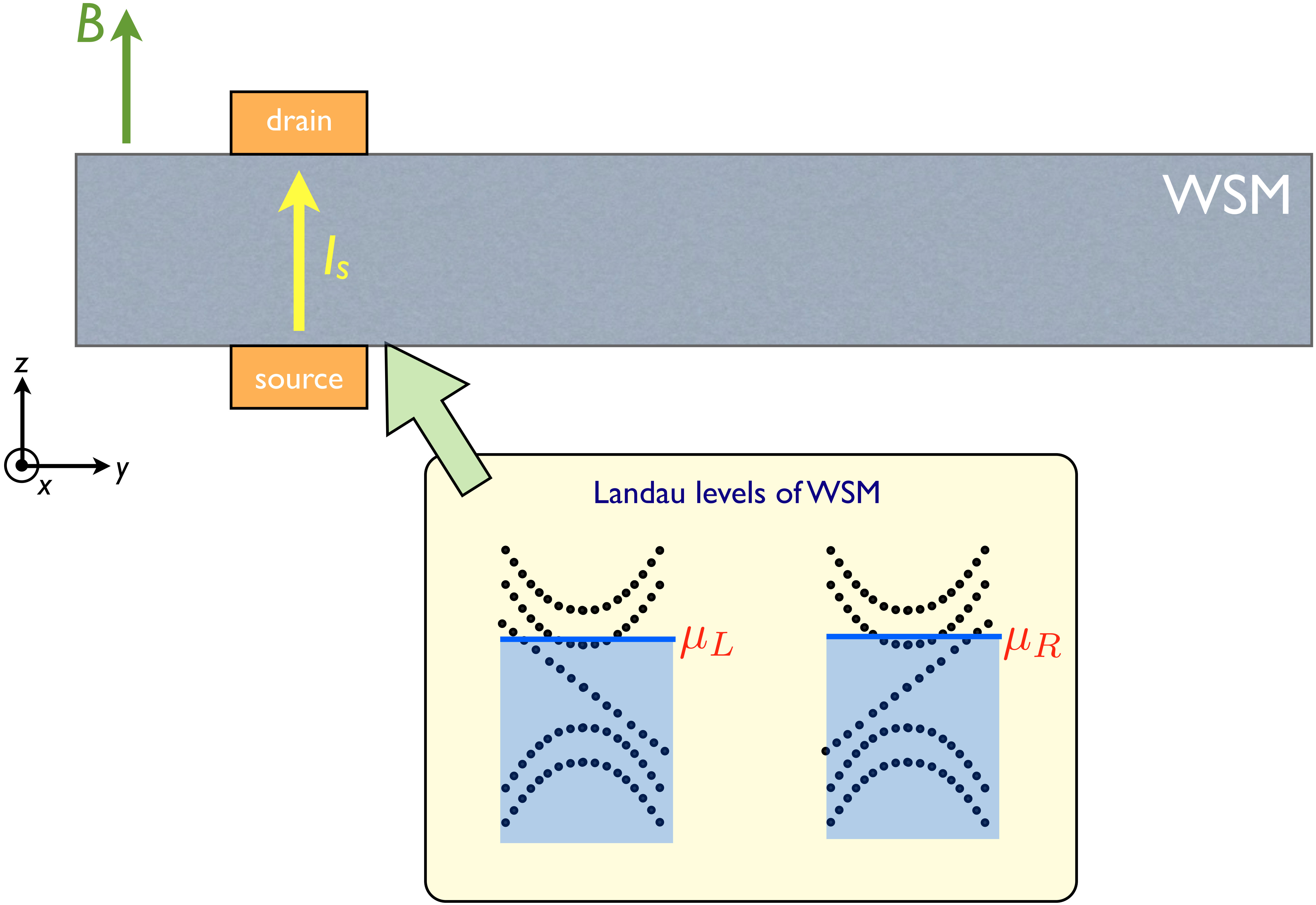

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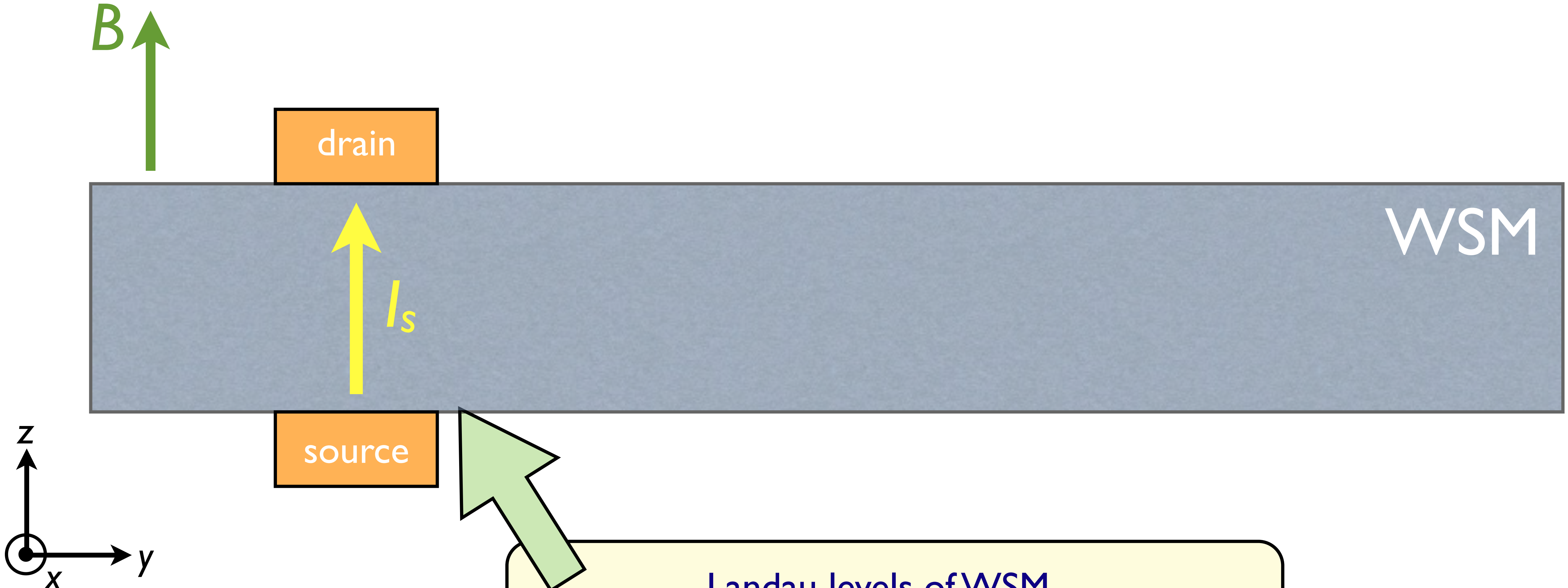

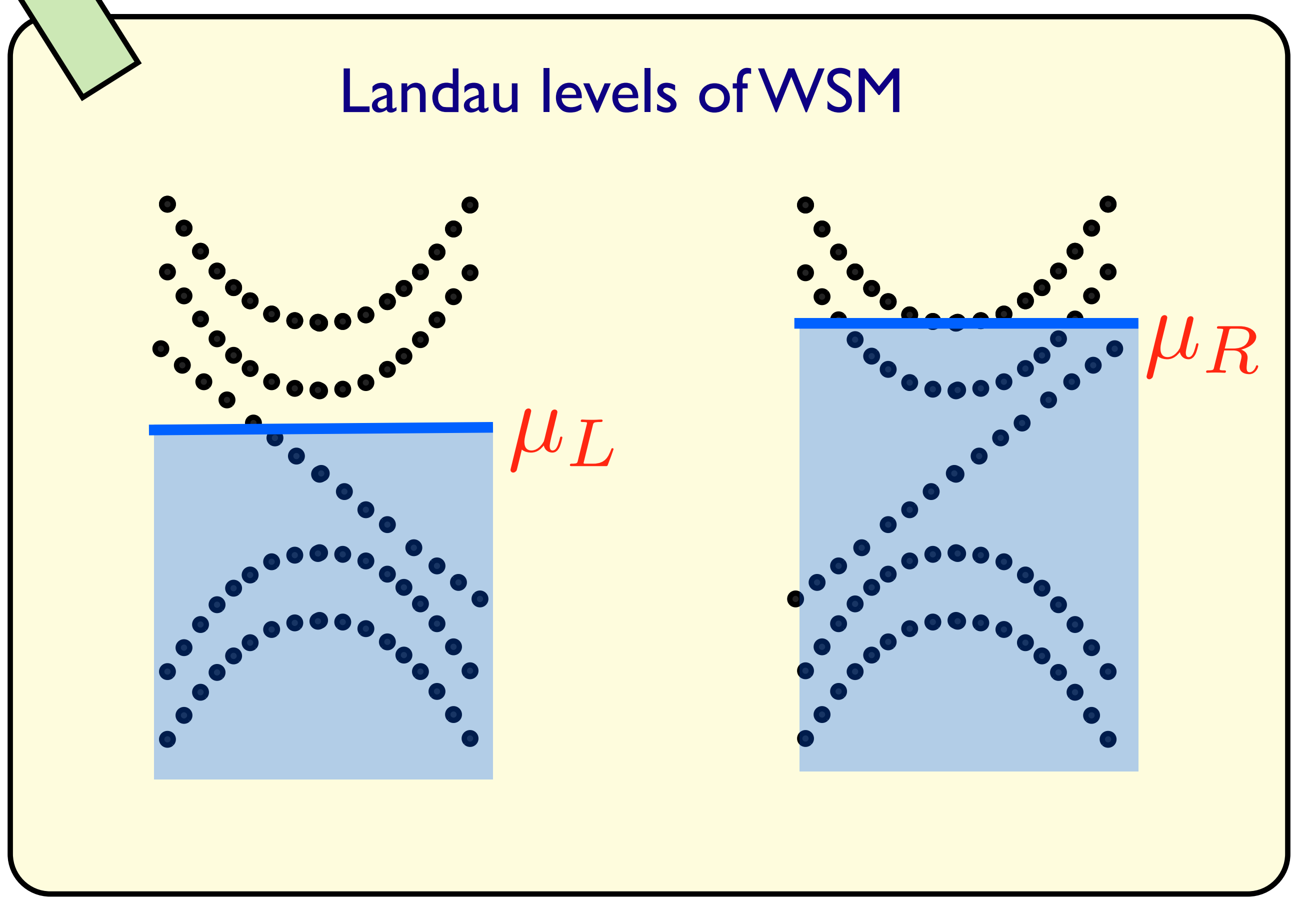

z-current
⇒ valley imbalance

$$\mu_R - \mu_L \propto I_s$$

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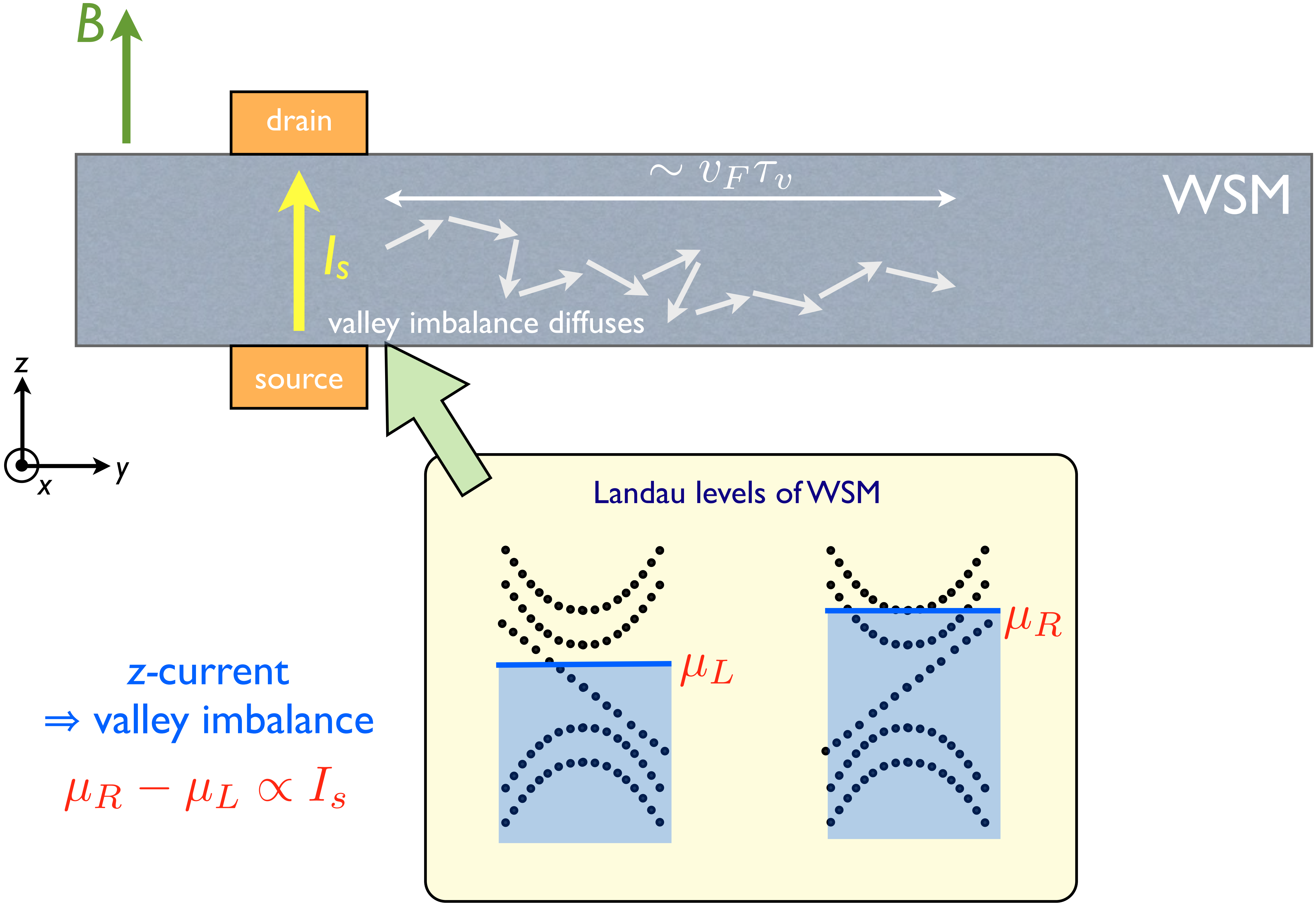

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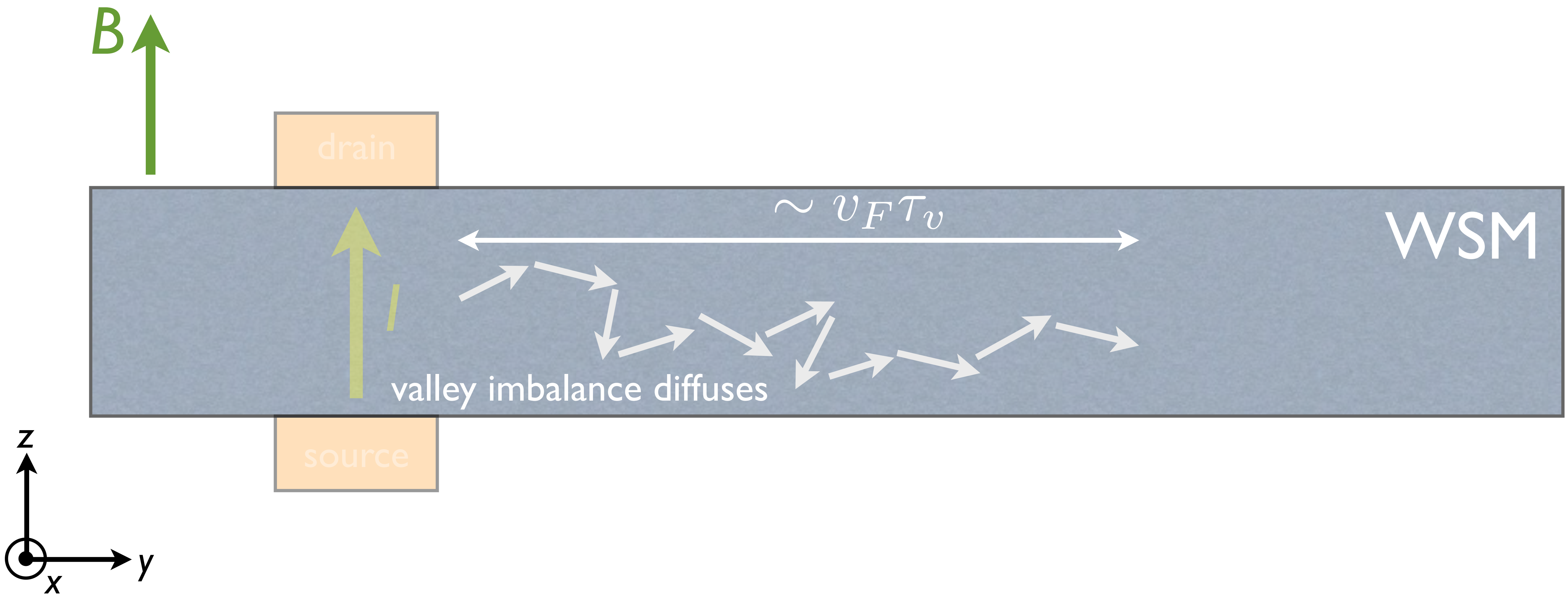
Detecting Valley Imbalance



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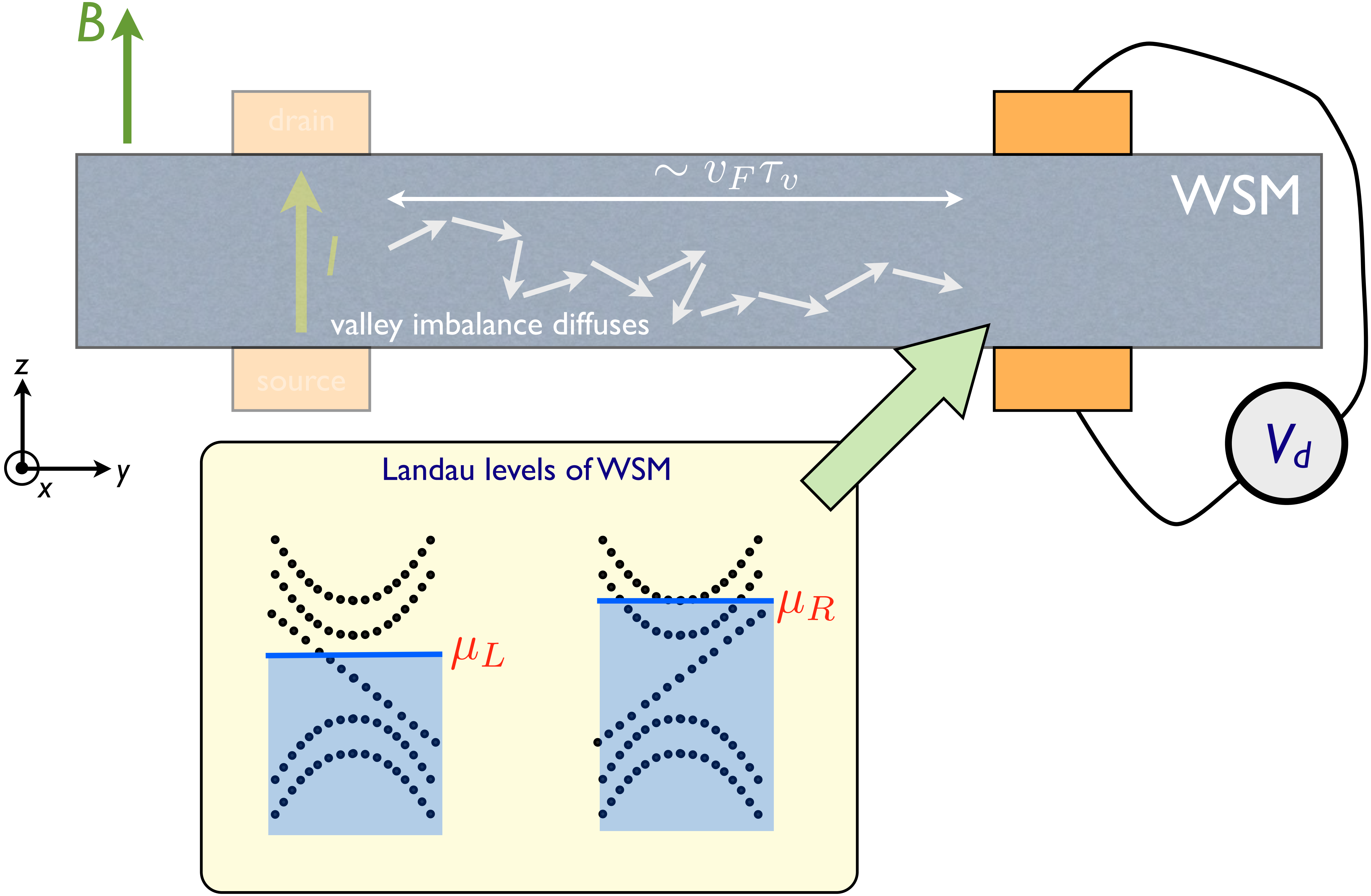

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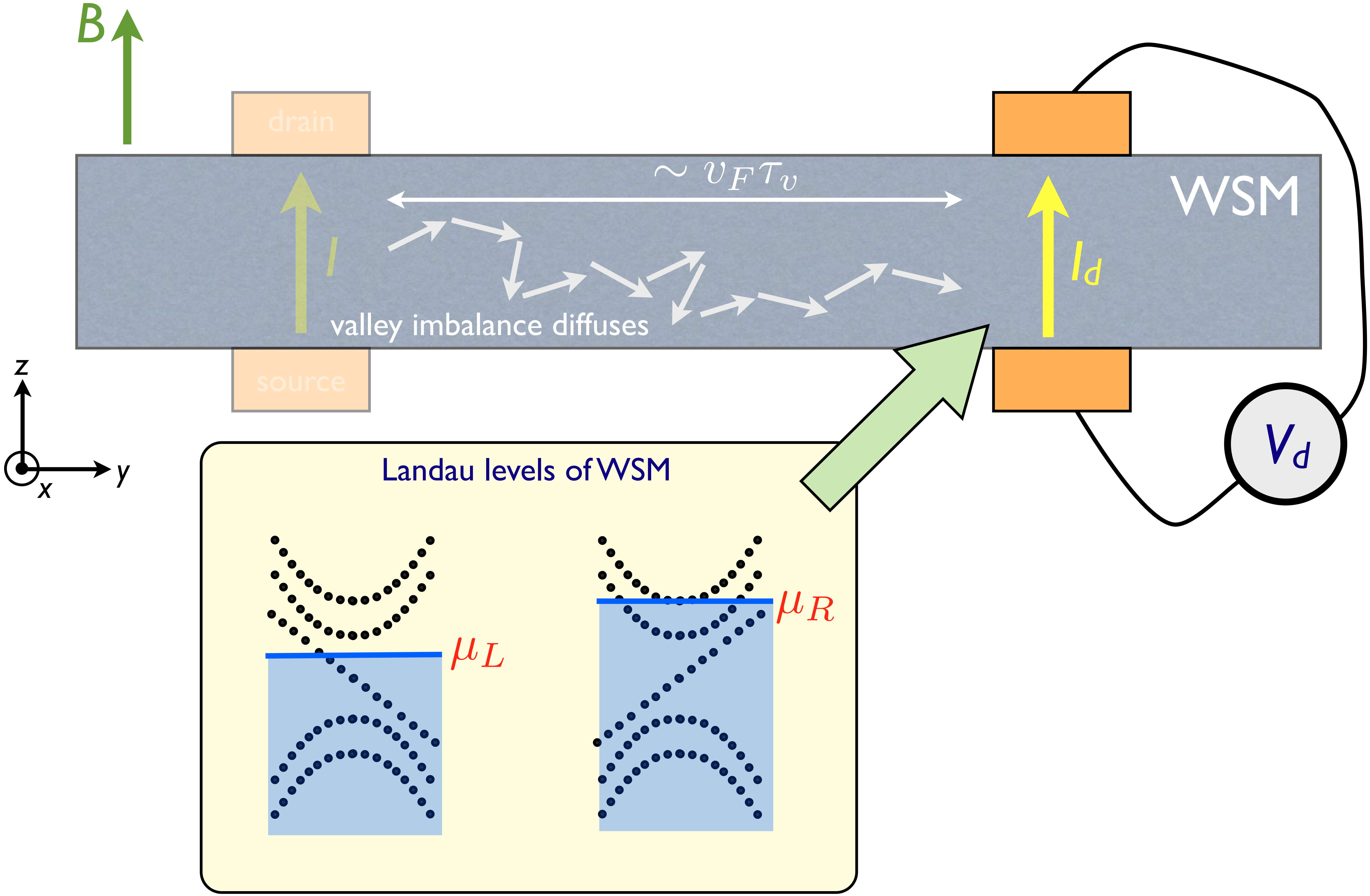

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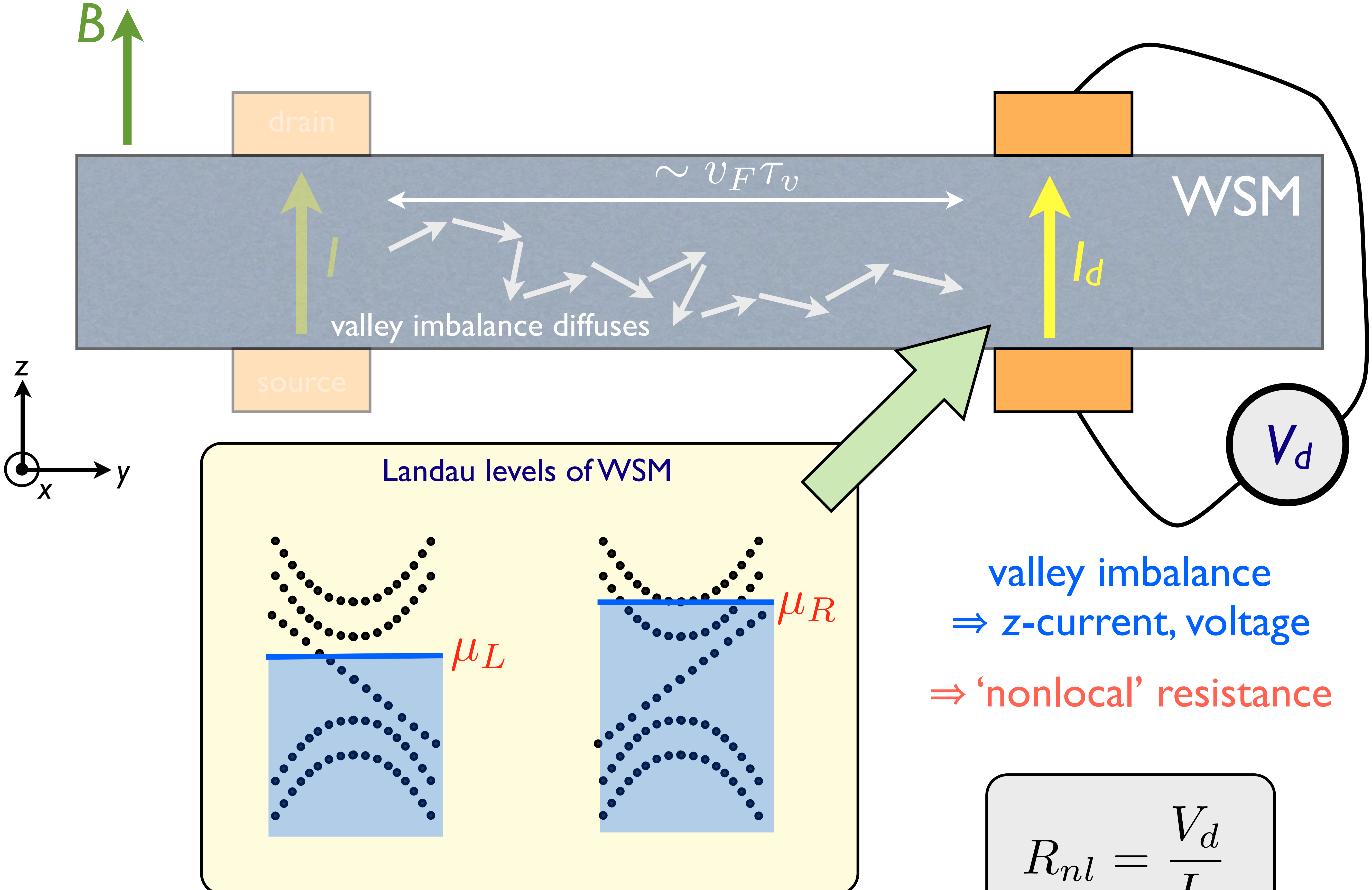


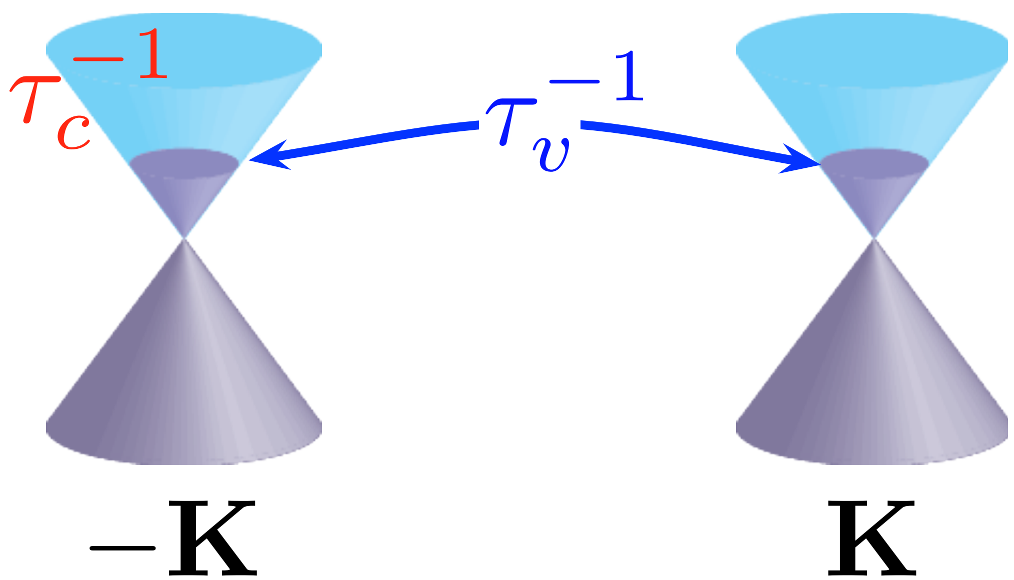
valley imbalance
⇒ z-current, voltage

⇒ 'nonlocal' resistance

$$R_{nl} = \frac{V_d}{I_s}$$

[SP, *et al.*, *PRX* **4**, 031035 (2014)]

Aside: Modeling Disorder



for a screened impurity potential,

$$v(\mathbf{q}) \approx \frac{v_0}{q^2 + k_{\rm sc}^2}$$

with $k_{\rm sc} \sim k_F$

$$\frac{\tau_c}{\tau_v} \approx \frac{|v(2\mathbf{K}_0)|^2}{|v(0)|^2} \approx \left(\frac{k_F}{2K_0}\right)^2 \sim x^{4/3}$$

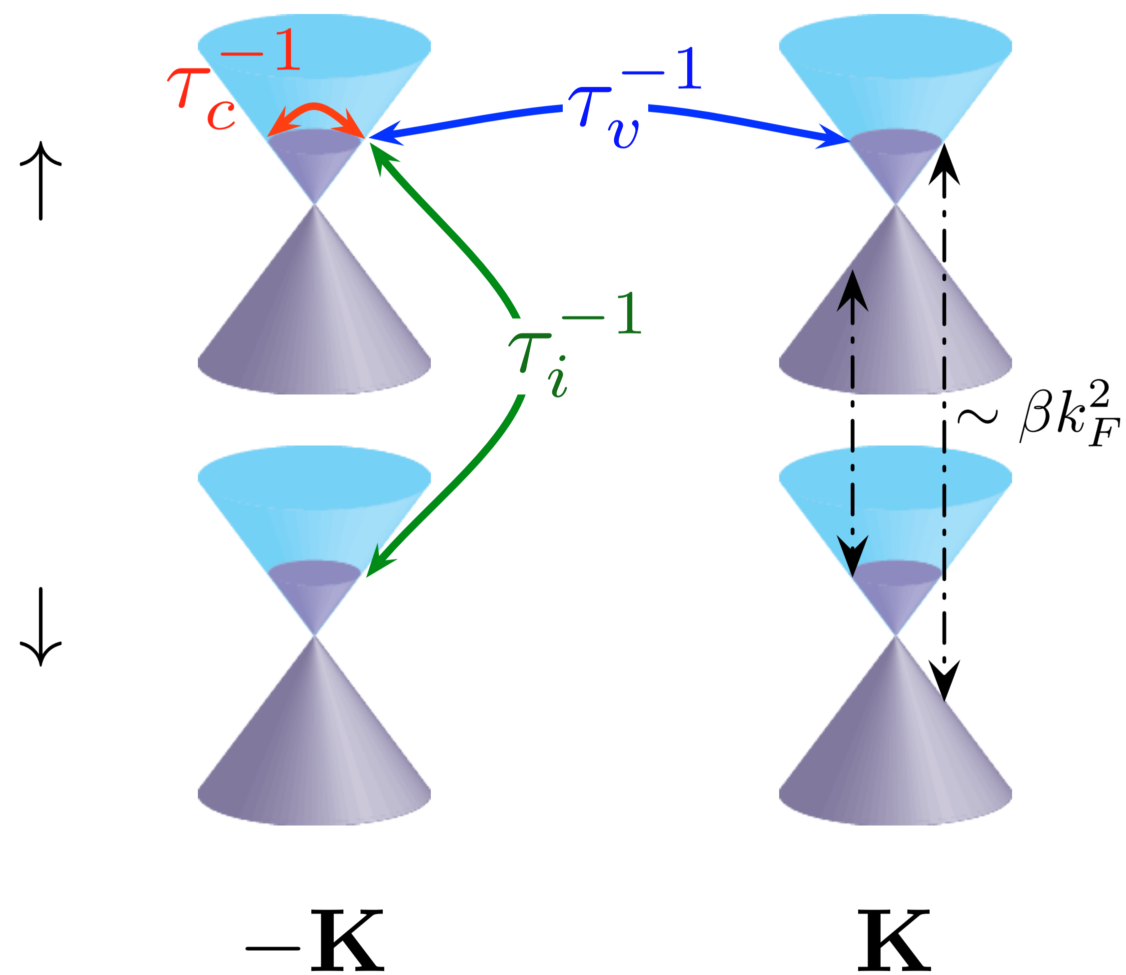
(*x* = doping level)

For x ~ 1%, and mean free path ~ 10 nm, $\ell_v \sim 5\,\mu\mathrm{m}$

Aside: Impurity Scattering in Dirac Semimetals

τ_c, τ_v as before; but new 'isospin' relaxation time τ_i

need τ_i to be long or else mixing destroys valley imbalance



τ_i depends on isospin mixing due to curvature terms in H_{eff}

$$|\mathbf{k},+,\uparrow\rangle_c \approx |\mathbf{k},+,\uparrow\rangle + \frac{\beta k}{2v_F}\sin^2\theta_{\mathbf{k}} e^{-2i\phi_{\mathbf{k}}}|\mathbf{k},-,\downarrow\rangle$$

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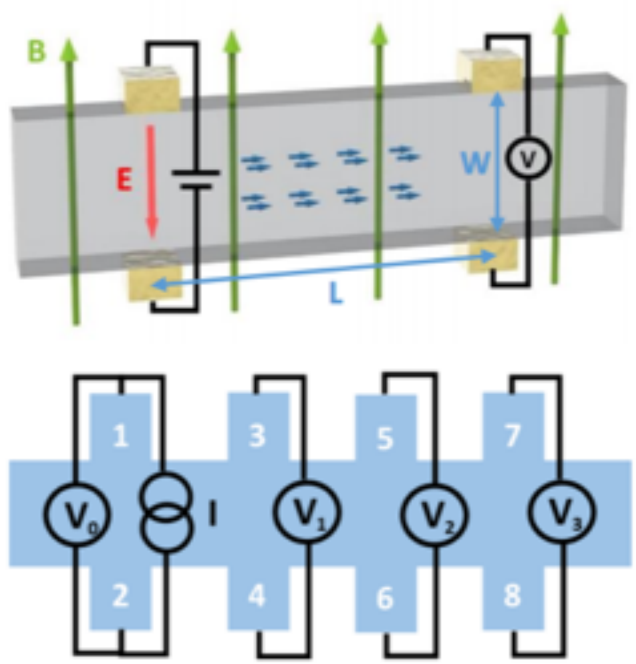
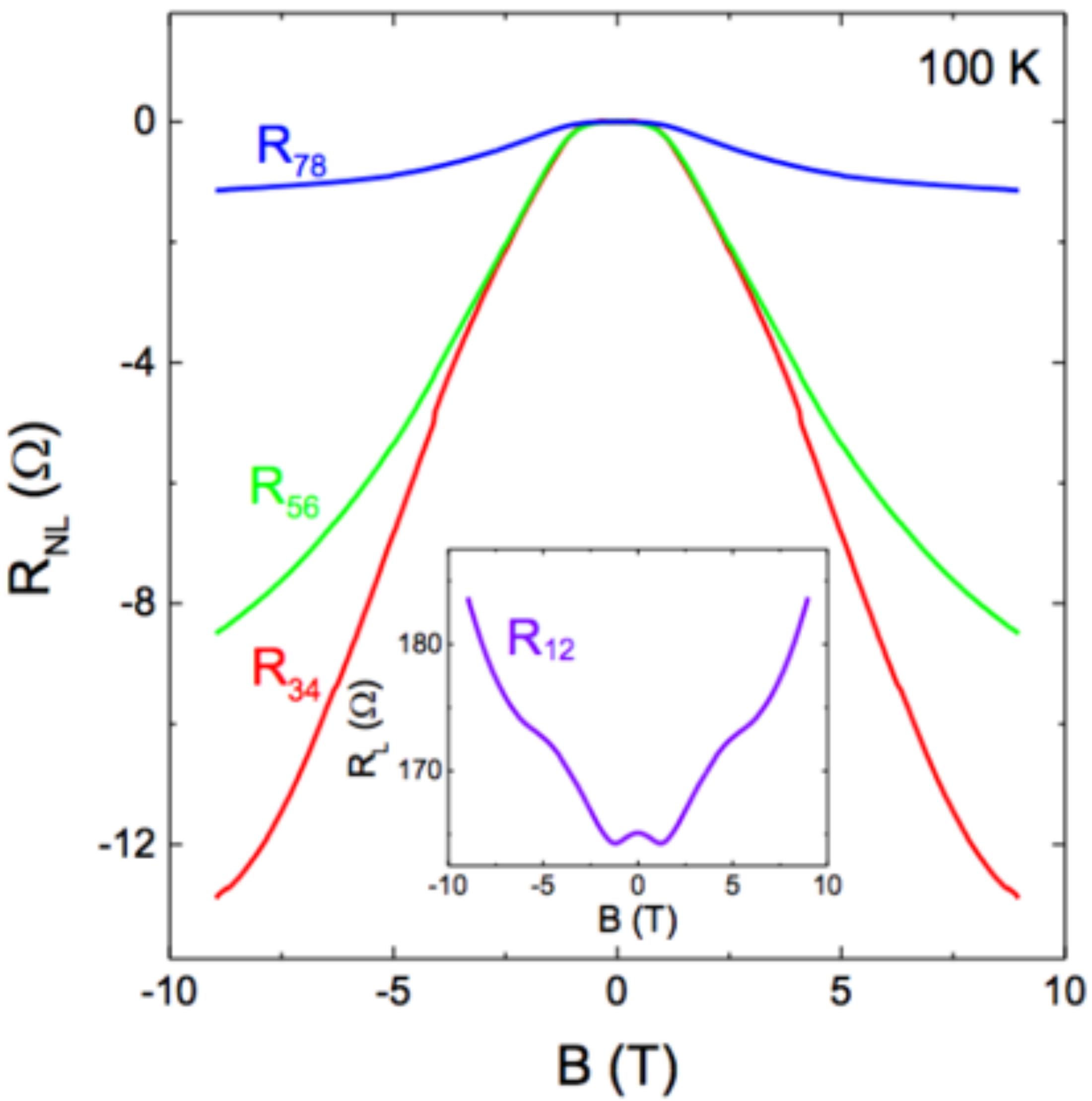
(analogous to Elliot-Yafet spin relaxation)

Estimate for Na_3Bi yields $\tau_i \sim 10^3\tau_c$

Experiments?





Small field-induced nonlocal contribution in Cd_3As_2?

Possible alternative explanations? (early days yet…)

[C. Zhang *et al.*, arXiv:1504.07698]

Summary

Topological phenomena don't need a bulk gap!

Weyl/Dirac Semimetals: robust topological transport phenomena

surface: Fermi arcs + effects on cyclotron orbits

bulk: anomaly, chiral charge pumping, …

Many proposed effects (only discussed a few)…

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Experimentalists:
We Need You!

Thanks for listening!