Max Planck Institute for Solid State Research

Leggett modes and the Anderson-Higgs mechanism in superconductors without inversion symmetry

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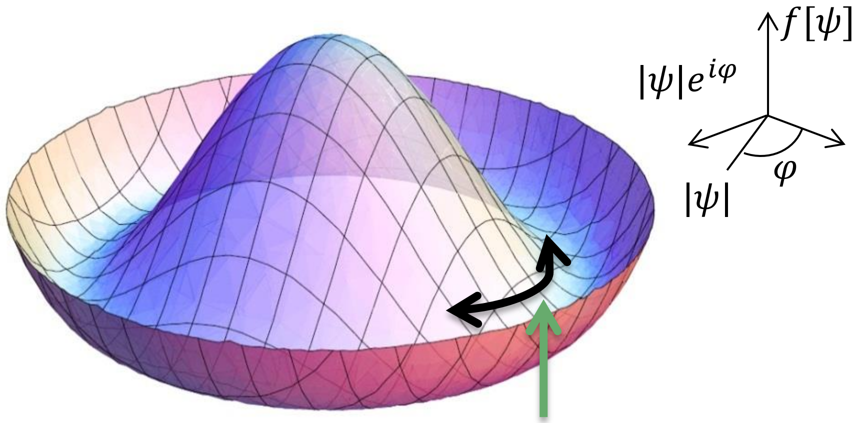
N. Bittner et al., **arXiv:1503.08133 (accepted to PRL)**

I) introduction & motivation

→ non-centrosymmetric superconductors

→ collective mode in a nutshell



II) theory

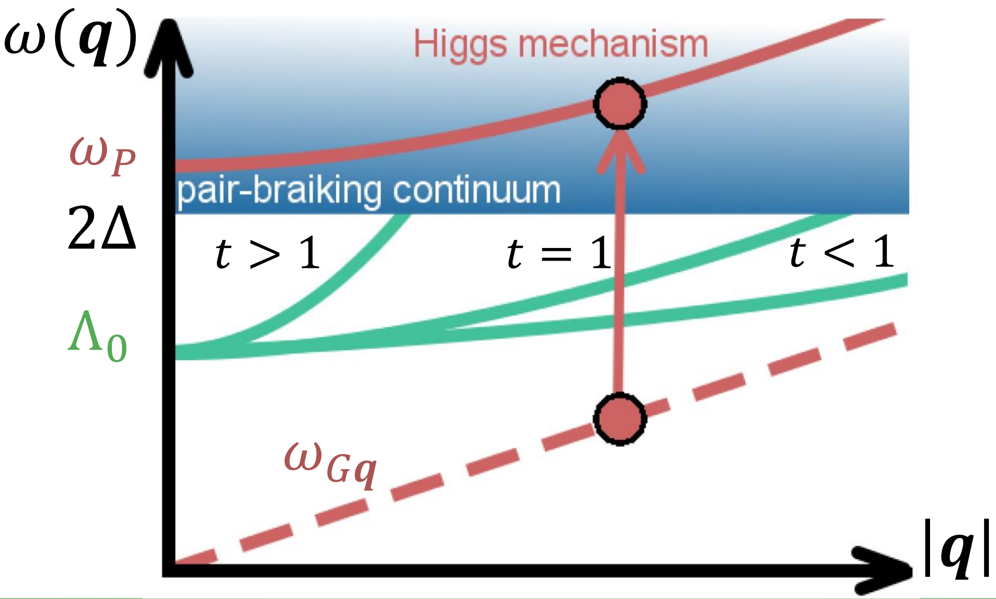
→ model description of NCS

→ matrix kinetic equation

III) results

→ New Leggett modes

→ Anderson-Higgs mechanism



Introduction : Inversion symmetry

» Singlet superconductors

$T\,|\,\boldsymbol{k},\uparrow\rangle = |-\boldsymbol{k},\downarrow\rangle$ Time reversal symmetry

» Triplet superconductors

$T\,|\,\boldsymbol{k},\uparrow\rangle = |-\boldsymbol{k},\downarrow\rangle$ Time reversal symmetry

$I\,|\,\boldsymbol{k},\uparrow\rangle = |-\boldsymbol{k},\uparrow\rangle$ Inversion symmetry

} Anderson's theorem *)

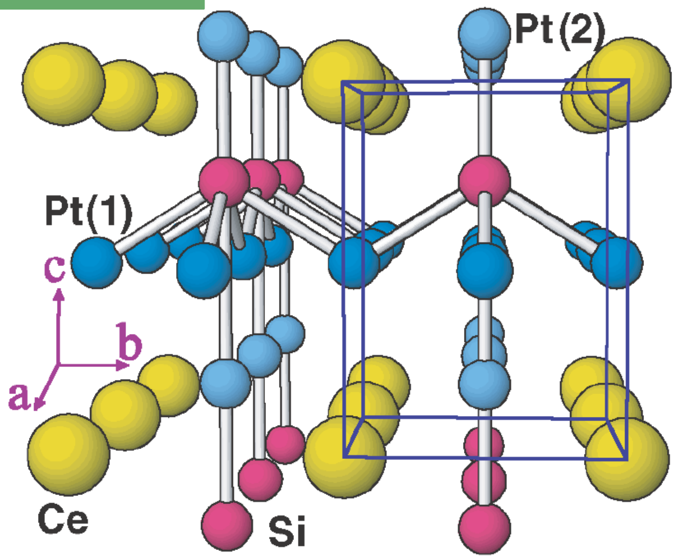
Superconductors without inversion center?

» Non-centrosymmetric superconductors

*) *P. W. Anderson, Phys. Rev. B* **30** (1984)

NCS: Strong Rashba-type SOC, γ_k

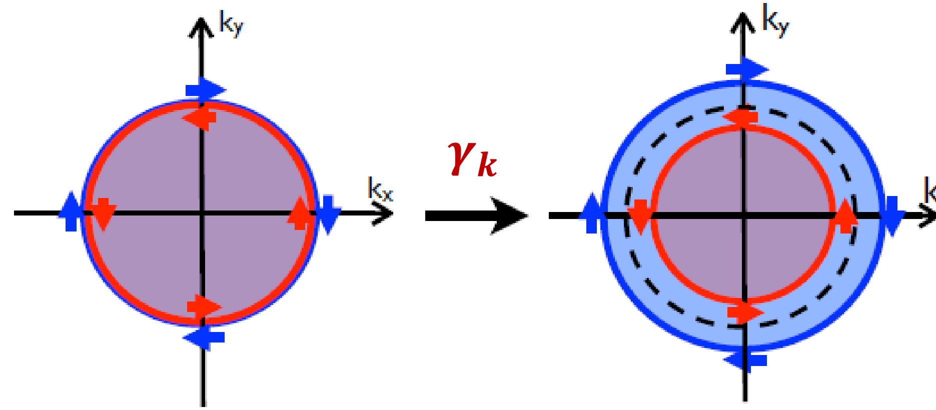
$CePt_3Si$



E. Bauer et al. (2004)

- point group C_{4V}
- evidence for a singlet as well as a triplet order parameter
- triplet-to-singlet ratio $t = \Delta_{tr}/\Delta_s$ is unknown

→ Lifting of the band degeneracy



Two-band splitting

K. V. Samokhin (2007) *P. A. Frigeri et.al. (2004)*

→ Mixing of the singlet and the triplet components of the order parameter

$$\Delta_{k\mu} = \Delta_s + \mu \, \Delta_{tr} \, \|\gamma_k\|$$

→ weak-coupling pairing interaction ansatz:

○ Singlet Γ_s ○ Triplet Γ_{tr} ○ mixing Γ_m (Dzyaloshinskii-Moriya)

Examples for γ_k

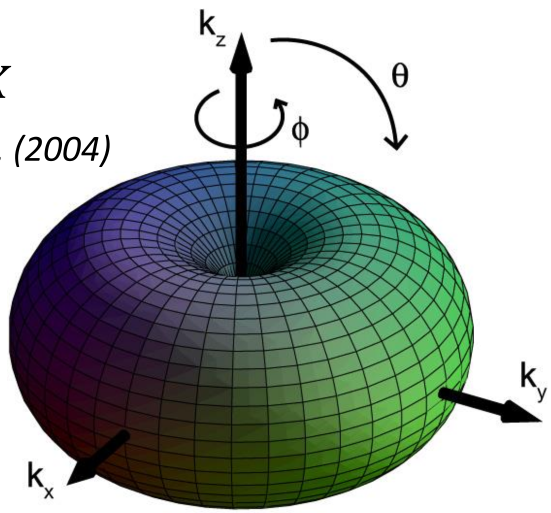
tetragonal point group, C_{4V}

cubic point group, O

$CePt_3Si$

$T_c = 0.75\ K$

E. Bauer et al. (2004)

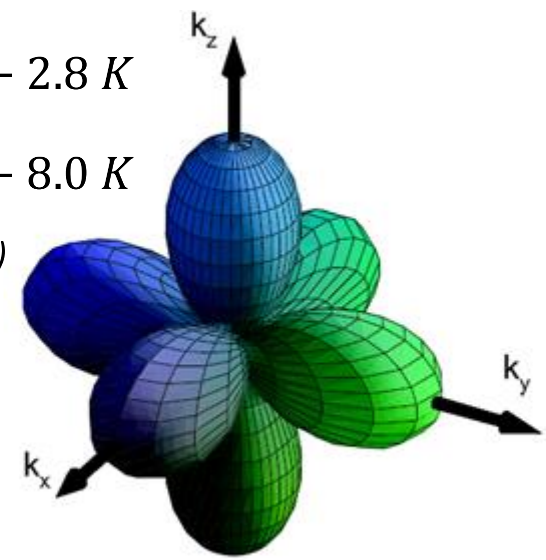


$Li_2Pd_xPt_{3-x}B$

$T_c(x=0) = 2.2 - 2.8\ K$

$T_c(x=3) = 7.2 - 8.0\ K$

T. Badica et al. (2005)



$$\gamma_k = \gamma_\perp(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{e}}_z) \ldots$$

Rashba coupling (axial)

$$\gamma_k = \gamma_1 \hat{\boldsymbol{k}} - \gamma_3 \hat{k}_x^2(\hat{k}_y^2 + \hat{k}_z^2)\hat{\boldsymbol{e}}_x + \cdots$$

pseudo-isotropic

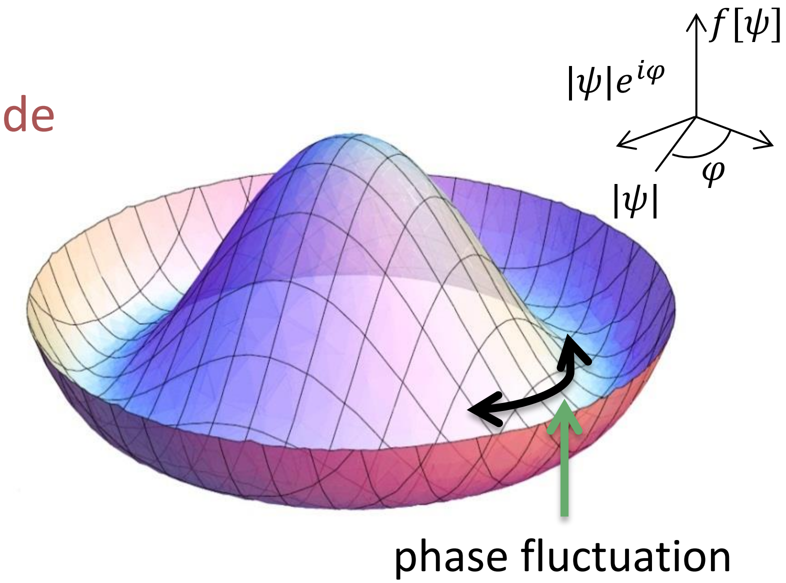
Collective modes in a nutshell

One-band superconductor

broken gauge-symmetry → Gauge mode

Gauge mode gets shifted to the plasma frequency (Anderson-Higgs mechanism)

P. W. Anderson (1963), P. W. Higgs (1964)



Collective modes in a nutshell

One-band superconductor

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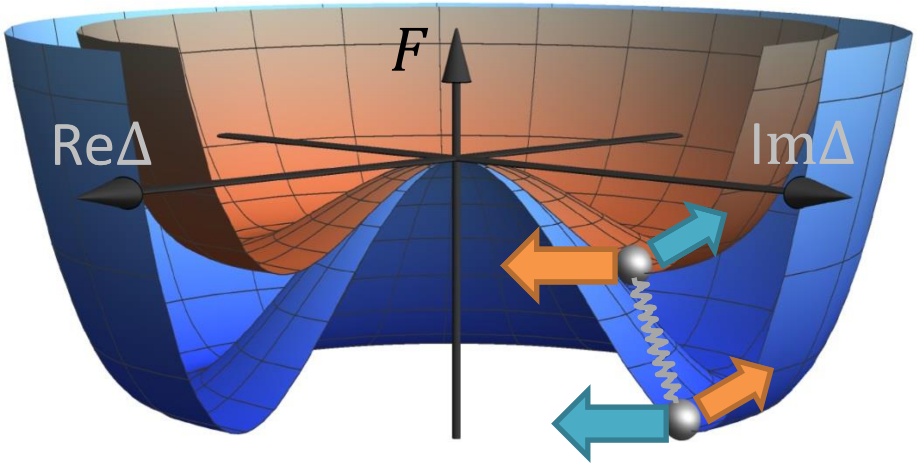
P. W. Anderson (1963), P. W. Higgs (1964)



Two-band superconductor

Leggett's collective mode, ω_L

$$\omega_L^2 = 4\,|\Delta_1|\,|\Delta_2|\,\frac{V_{12}}{V_{11}V_{22} - V_{12}^2}$$

A. J. Leggett (1966)

» no analogue in the one band case

» intrinsic Josephson effect

» Experimental evidence in Raman response for MgB_2

G. Blumberg et al., (2007)

Collective modes in **non-centrosymmetric** superconductors?

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Model description of ncs

Hamiltonian (spin basis):

spin-orbit coupling

→ antisymmetric $\boldsymbol{\gamma}_{-\boldsymbol{k}} = -\boldsymbol{\gamma}_{\boldsymbol{k}}$

→ reflects the crystal symmetry

$$\hat{H}_{ncs} = \sum_{\boldsymbol{k}\sigma\sigma'} \hat{c}^{\dagger}_{\boldsymbol{k}\sigma} \left[\xi_{\boldsymbol{k}} \delta_{\sigma\sigma'} + \boldsymbol{\gamma}_{\boldsymbol{k}} \cdot \boldsymbol{\tau}_{\sigma\sigma'} \right] \hat{c}_{\boldsymbol{k}\sigma'} + \text{Cooper pairing terms}$$

→ Two band description in the helicity band basis

Unitary transformation → Hamiltonian in the **band basis**:

$$\hat{H}_{ncs} = \begin{pmatrix} \xi_{\boldsymbol{k}+} & 0 & 0 & \Delta_{\boldsymbol{k}+} \\ 0 & \xi_{\boldsymbol{k}-} & -\Delta_{\boldsymbol{k}-} & 0 \\ 0 & -\Delta^*_{\boldsymbol{k}-} & -\xi_{\boldsymbol{k}-} & 0 \\ \Delta^*_{\boldsymbol{k}+} & 0 & 0 & -\xi_{\boldsymbol{k}+} \end{pmatrix} \quad \text{with} \quad \xi_{\boldsymbol{k}\mu} = \xi_{\boldsymbol{k}} + \mu \|\boldsymbol{\gamma}_{\boldsymbol{k}}\|$$

→ NEW Order parameter: $\Delta_{\boldsymbol{k}\lambda} = \Delta_s + \mu\, \Delta_{tr} \|\boldsymbol{\gamma}_{\boldsymbol{k}}\|$

singlet and triplet

Matrix Kinetic Equation

External perturbation: $\phi(\boldsymbol{q}, \omega)$

➡ deviation of the density matrix $\underline{n}_k(\boldsymbol{q}, \omega) = \underline{n}_{\boldsymbol{k}}^0 + \delta\underline{n}_k(\boldsymbol{q}, \omega)$

collisionless quantum dynamics

von Neumann equation in Nambu space:

$$i\hbar\frac{\partial}{\partial t}\underline{n}_k + [\underline{n}_k, \underline{\widehat{H}}] = 0$$

$$\underline{\widehat{H}} = \widehat{H}_{ncs} + \delta\underline{\widehat{H}}(\boldsymbol{q}, \omega)$$

→ 4x4 matrix equation in spin- and particle-hole space

→ Equivalent to the Landau-Boltzmann equation in the normal state

→ Diagonal elements ➡ analytical results for the momentum distribution function, δn_s

→ Nondiagonal elements ➡ analytical results for the phase fluctuations of the order parameter

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Results: Anderson-Higgs mechanism in NCS (I)

Condensate density response

$$\delta n_s = \frac{\omega_{G\boldsymbol{q}}^2\left(\omega^2 - \omega_{L\boldsymbol{q}}'^2\right)\sum_\alpha\langle\lambda_{\boldsymbol{p}\alpha}\rangle}{\omega^4 - \omega^2\left(\omega_P^2 + \omega_{Gq}^2 + \omega_{L\boldsymbol{q}}^2\right) + \left(\omega_P^2 + \omega_{Gq}^2\right)\omega_{L\boldsymbol{q}}'^2} e\Phi$$

Gauge (Anderson-Bogoliubov) boson

$$\omega_1^2 = \omega_{G\boldsymbol{q}}^2 + \mathcal{O}\left(\frac{\omega_{G\boldsymbol{q}}^4}{\omega_{L\boldsymbol{q}}^2}\right) \quad \text{with} \quad \omega_{G\boldsymbol{q}}^2 = \frac{\sum_\mu\left\langle\left(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{p}\mu}\right)^2\lambda_{\boldsymbol{p}\mu}\right\rangle}{\sum_\mu\langle\lambda_{\boldsymbol{p}\mu}\rangle}$$

massless order parameter collective mode

Results: Anderson-Higgs mechanism in NCS (I)

Condensate density response

$$\delta n_s = \frac{\omega_{G\boldsymbol{q}}^2 \left(\omega^2 - \omega_{L\boldsymbol{q}}'^2\right) \sum_\alpha \langle \lambda_{\boldsymbol{p}\alpha} \rangle}{\omega^4 - \omega^2 \left(\boldsymbol{\omega_P^2} + \boldsymbol{\omega_{G\boldsymbol{q}}^2} + \omega_{L\boldsymbol{q}}^2\right) + \left(\boldsymbol{\omega_P^2} + \boldsymbol{\omega_{G\boldsymbol{q}}^2}\right)\omega_{L\boldsymbol{q}}'^2} e\Phi$$

Gauge (Anderson-Bogoliubov) boson

$$\omega_1^2 = \omega_P^2 + \omega_{G\boldsymbol{q}}^2 + \mathcal{O}\left(\frac{\omega_{L\boldsymbol{q}}^2}{\omega_P^2}\right) \quad \text{with} \quad \omega_{G\boldsymbol{q}}^2 = \frac{\sum_\mu \left\langle \left(\boldsymbol{q} \cdot \boldsymbol{v}_{\boldsymbol{p}\mu}\right)^2 \lambda_{\boldsymbol{p}\mu} \right\rangle}{\langle \lambda_{\boldsymbol{p}\mu} \rangle}$$

Anderson-Higgs mechanism for the gauge mode:

$$\omega_{G\boldsymbol{q}}^2 \to \omega_P^2 + \omega_{G\boldsymbol{q}}^2$$

condensate plasma frequency

$$\omega_P^2 = \frac{4\pi n e^2}{m \boldsymbol{q}^2} \sum_\mu \left\langle \left(\boldsymbol{q} \cdot \boldsymbol{v}_{\boldsymbol{p}\mu}\right)^2 \lambda_{\boldsymbol{p}\mu} \right\rangle$$

Results: Anderson-Higgs mechanism in NCS (II)

Condensate density response

$$\delta n_s = \frac{\omega_{G\boldsymbol{q}}^2\left(\omega^2 - \omega_{L\boldsymbol{q}}^{\prime 2}\right)\sum_\alpha \langle \lambda_{\boldsymbol{p}\alpha} \rangle}{\omega^4 - \omega^2\left(\omega_P^2 + \omega_{Gq}^2 + \omega_{Lq}^2\right) + \left(\omega_P^2 + \omega_{Gq}^2\right)\omega_{Lq}^{\prime 2}} e\Phi$$

Main results:

New Leggett modes

$$\omega_2^2 = \omega_{Lq}^2 + \mathcal{O}\left(\frac{\omega_{G\boldsymbol{q}}^4}{\omega_{L\boldsymbol{q}}^2}\right) \qquad \text{with} \qquad \omega_{Lq}^2 = \Lambda_0^2 + v^2\boldsymbol{q}^2$$

massive order parameter collective mode

Condensate density response

$$\delta n_s = \frac{\omega_{G\boldsymbol{q}}^2\left(\omega^2 - \omega_{L\boldsymbol{q}}'^2\right)\sum_\alpha \langle \lambda_{\boldsymbol{p}\alpha} \rangle}{\omega^4 - \omega^2\left(\omega_P^2 + \omega_{Gq}^2 + \omega_{Lq}^2\right) + \left(\omega_P^2 + \omega_{Gq}^2\right)\omega_{Lq}'^2} e\Phi$$

Main results:

New Leggett modes

$$\omega_2^2 = \omega_{Lq}'^2 + \mathcal{O}\left(\frac{\omega_{L\boldsymbol{q}}^2}{\omega_P^2}\right) \quad \text{with} \quad \omega_{Lq}'^2 = \Lambda_0^2 + v'^2 \boldsymbol{q}^2$$

Leggett mode remains almost **unaffected** by the Higgs mechanism

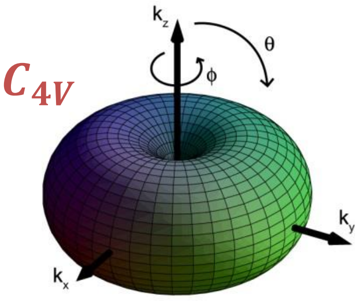
Results: New Leggett modes in ncs (I)

Dispersion

→ Propotional to $\boldsymbol{q}^2$ in leading order

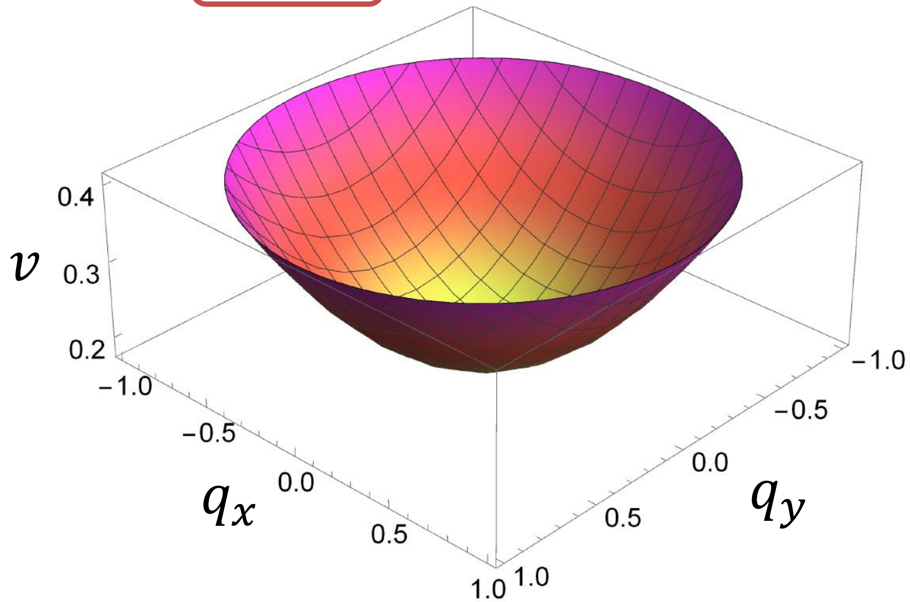
→ Depends on the triplet-to-singlet ratio $t = \Delta_{tr}/\Delta_s$

C_{4V}

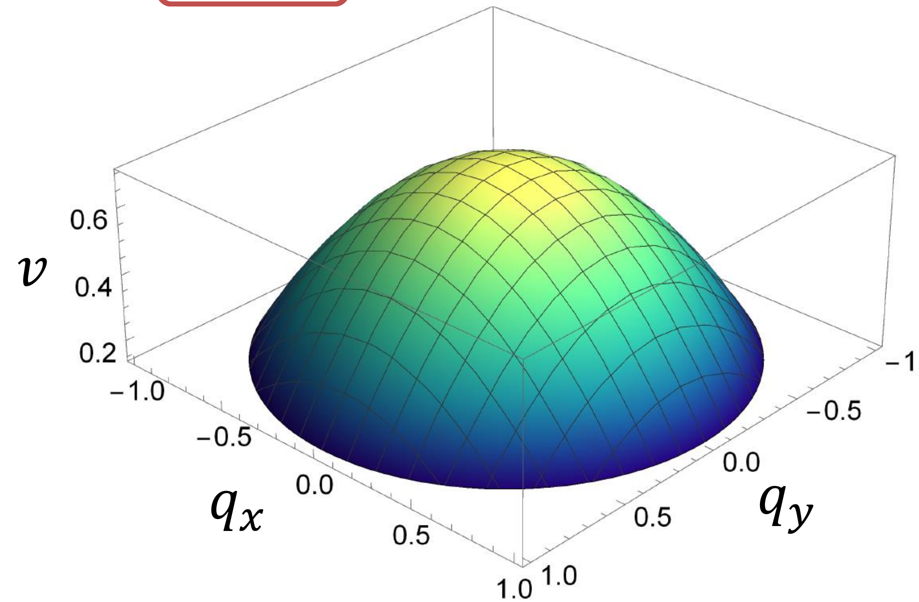


$$\gamma_{\boldsymbol{k}} = \gamma_{\perp}(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{e}}_z) \ldots$$

$t = 0.5$



$t = 1.5$



Mass

Generalized Leggett mass

$$\Lambda_0^2 = 4\gamma_{ncs}\ \Delta_s\Delta_{tr}\ \frac{\sum_\mu \langle \lambda_{p\mu} \rangle}{\lambda_0\lambda_2 - \lambda_1^2}$$

with order parameter:

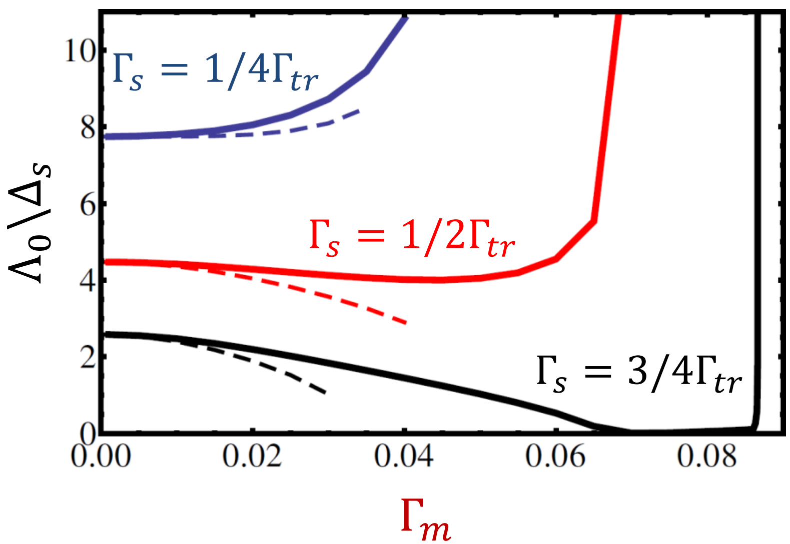
$$\Delta_{k\lambda} = \Delta_s + \mu\ \Delta_{tr}\ \|\gamma_k\|$$

$$\propto \frac{\Gamma_m}{\Gamma_s\Gamma_{tr} - \Gamma_m^2}$$

$$\lambda_n \propto \|\gamma_k\|^n$$

DEPENDENCE ON THE POINT GROUP SYMMETRY !

→ Numerical results



→ Analytical solution (dashed lines) for $t = \frac{\Delta_{tr}}{\Delta_s} \to 0$

→ If $\Gamma_s \approx \Gamma_{tr} \Rightarrow \Lambda_0 \to 0$

Summary

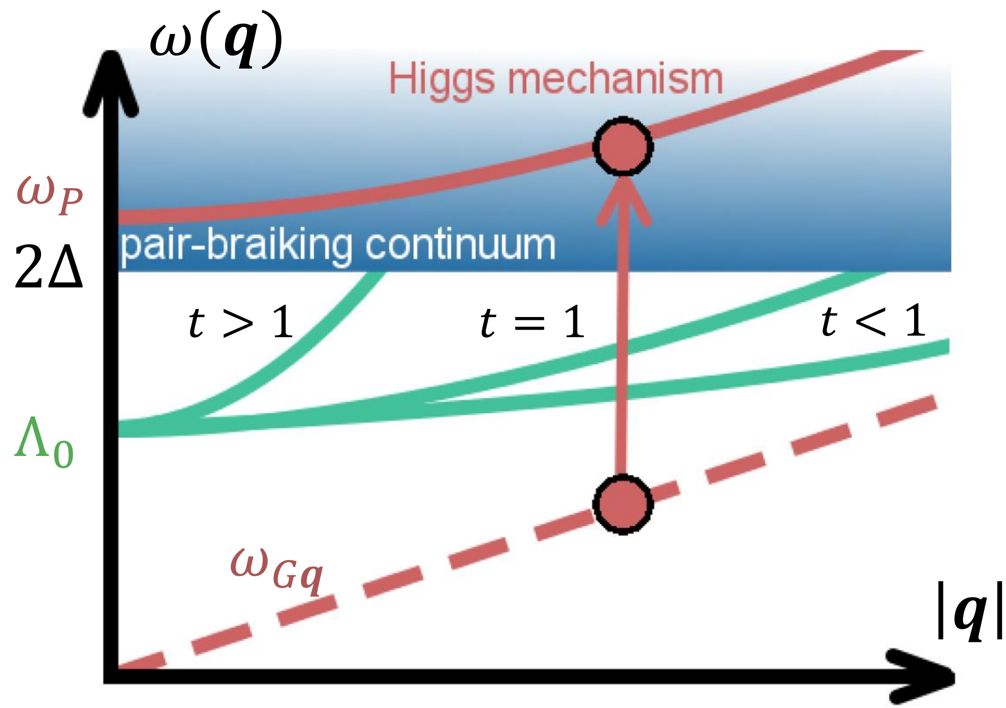
Leggett mode

→ **exists** in non-centrosymmetric superconductors

→ depends on the **triplet-to-singlet ratio** $t = \Delta_{tr}/\Delta_s$ via $\boldsymbol{\gamma_k}$

→ under certain conditions **massless**

→ remains almost **unaffected by the Higgs mechanism**



N. Bittner et al., arXiv:1503.08133 (accepted to PRL)

Gauge mode

→ depends on the **point group symmetry**

→ gets shifted to the plasma frequency **by the Higgs mechanism**