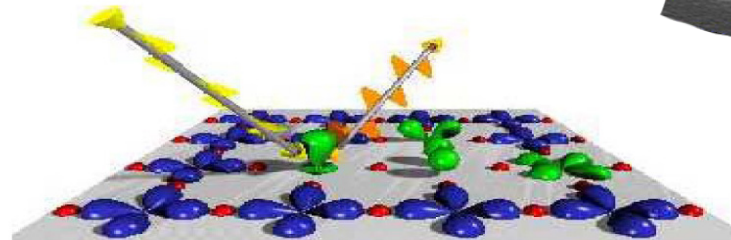
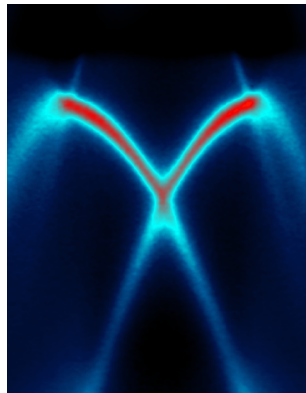
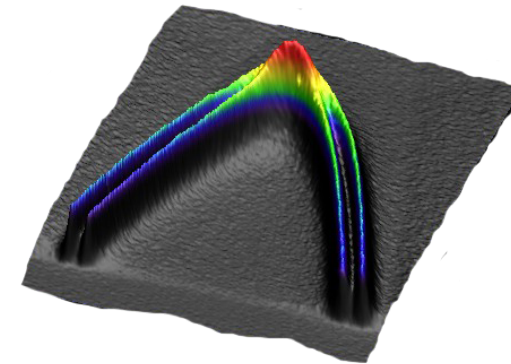
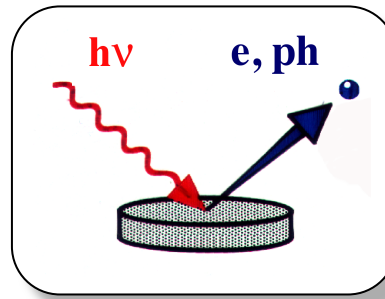
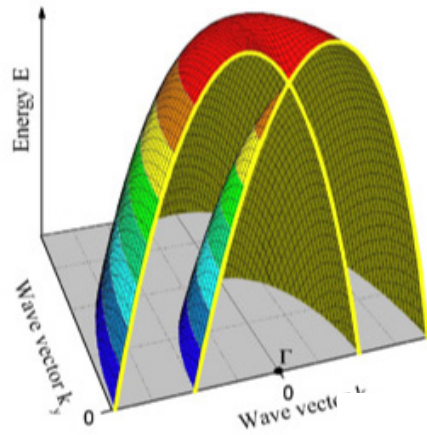
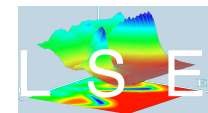


# Spin-orbit and spectroscopies from atoms to solids



Marco Grioni  
ICMP - EPFL





**EPFL**  
ÉCOLE POLYTECHNIQUE  
FÉDÉRALE DE LAUSANNE

# Plan of the lectures

## Lecture 1

- Generalities on SOI
- Techniques: XAS, PES, RIXS
- Polarized electrons: XMCD and SP-PES
- Exemples

## Lecture 2

- The Rashba effect
- From Rasba to TI's
- SOI and RIXS

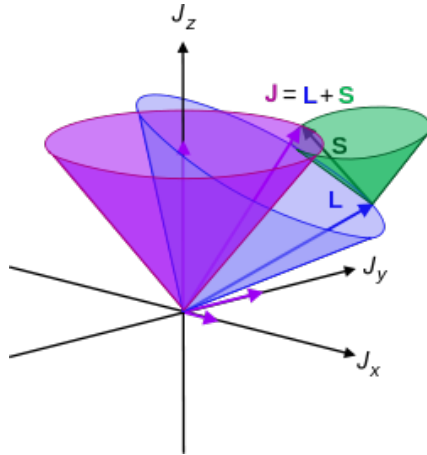


# General consideration





# Spin-orbit: where from?



**From Dirac to Schrödinger up to  $(v/c)^2$   
Extra terms**

$$H = H_0 - \frac{\mathbf{P}^4}{8m_e^3c^2} + \frac{1}{2m_e^2c^2} \frac{1}{r} \frac{dV(r)}{dr} \vec{\mathbf{L}} \cdot \vec{\mathbf{S}} + \frac{\hbar^2}{8m_e^2c^2} \nabla^2 V(r) + \dots$$



**Mass-velocity**



**Spin-orbit**



**Darwin**

$$\frac{H_{MV}}{H_0} \simeq \frac{H_{SO}}{H_0} \simeq \frac{H_D}{H_0} \simeq \alpha^2 \simeq \frac{1}{(137)^2}$$

# Spin-orbit: hydrogen atom

**A magnetic field appears in the electron's rest frame:**

$$\vec{B} = -\frac{1}{c^2} \vec{v} \times \vec{E} = \frac{1}{m_e c^2} \vec{p} \times \frac{1}{q} \frac{dV(r)}{dr} \frac{\vec{r}}{r} = -\frac{1}{q m_e c^2} \vec{r} \times \vec{p} \frac{1}{r} \frac{dV(r)}{dr} = \frac{1}{q m_e c^2} \frac{1}{r} \frac{dV(r)}{dr} \vec{L},$$

**Which entails the interaction term**  
(Thomas's precession yields the extra 1/2):

$$E_{SO} = -\frac{1}{2} \vec{M} \cdot \vec{B} = -\frac{q}{m_e} \vec{S} \cdot \vec{B} \quad \rightarrow \quad H_{SO} = \frac{1}{m_e^2 c^2} \frac{1}{r} \frac{dV(r)}{dr} \vec{L} \cdot \vec{S}$$

$$V = -\frac{Z e^2}{r}; \quad \alpha = \frac{e^2}{\hbar c} = \frac{1}{137}; \quad H_{SO} = \frac{Z \hbar^2 \alpha^2}{2 m_e^2 e^2} \frac{1}{r^3} \vec{L} \cdot \vec{S} = \xi(r) \vec{L} \cdot \vec{S}$$

$$\vec{L} \cdot \vec{S} = \frac{1}{2} (\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2)$$

# Spin-orbit: hydrogen(oid) atom (Z protons, 1 electron)

**Without spin-orbit:**  $L^2$ ;  $L_z$ ;  $S^2$ ;  $S_z$ ;  $J^2$ ;  $J_z$  commute. Eigenstates:

$$|n; l; m_l; \pm\rangle ; \quad j = l \pm \frac{1}{2}$$

**With spin-orbit:**  $H_{SO}$ ;  $J_z$ ;  $J^2$  commute. The eigenstates  $|n; j; m_j\rangle$

are appropriate combinations of the  $|n; l; m_l; \pm\rangle$

$$E_{nlj} = \langle jm_j | H_{SO} | jm_j \rangle = \frac{Z\hbar^2\alpha^2}{2m_e^2 e^2} \left\langle \frac{1}{r^3} \right\rangle \langle jm_j | \vec{S} \cdot \vec{L} | jm_j \rangle = \underset{\substack{\text{radial} \\ \downarrow}}{\zeta_{nl}} \langle jm_j | \vec{S} \cdot \vec{L} | jm_j \rangle$$

angular  
↓

$$E_{nlj} = \frac{\zeta_{nl}}{2} \begin{cases} l & (j = l + 1/2) \\ -(l+1) & (j = l - 1/2) \end{cases}$$

$$\Delta E_{SO} = \frac{(2l+1)}{2} \zeta_{nl} = \frac{|E_n| Z^2 \alpha^2}{nl(l+1)} \approx \frac{Z^2}{n^2} \frac{Z^2 \alpha^2}{nl(l+1)} \approx \frac{Z^4}{n^3}$$



# Textbook example: the $2p$ states of hydrogen

$$H_{so} = \frac{\hbar^2 \zeta_{nl}}{2} \begin{matrix} m_j & 3/2 & 1/2 & -1/2 & -3/2 \\ \left( \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & \sqrt{2} & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \end{matrix}$$

$$|m_l; m_s\rangle$$

$$|1; +\rangle \quad m_j = 3/2$$

$$|1; -\rangle |0; +\rangle \quad m_j = 1/2$$

$$|0; -\rangle |-1; +\rangle \quad m_j = -1/2$$

$$|-1; -\rangle \quad m_j = -3/2$$

$$\vec{S} \cdot \vec{L} = S_z L_z + \frac{1}{2}(S^+ L^- + S^- L^+)$$

$$H_{so} = \frac{\hbar^2 \zeta_{nl}}{2} \begin{matrix} j & 3/2 & 1/2 \\ \left( \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{array} \right) \end{matrix}$$

$$|\frac{3}{2}; \frac{3}{2}\rangle = |1; +\rangle ; \quad |-\frac{3}{2}; -\frac{3}{2}\rangle = |-1; -\rangle$$

$$|\frac{3}{2}; \frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|0; +\rangle + \sqrt{\frac{1}{3}}|1; -\rangle \quad \langle S_z \rangle = \frac{\hbar}{6}; \quad \langle L_z \rangle = \frac{\hbar}{3}$$

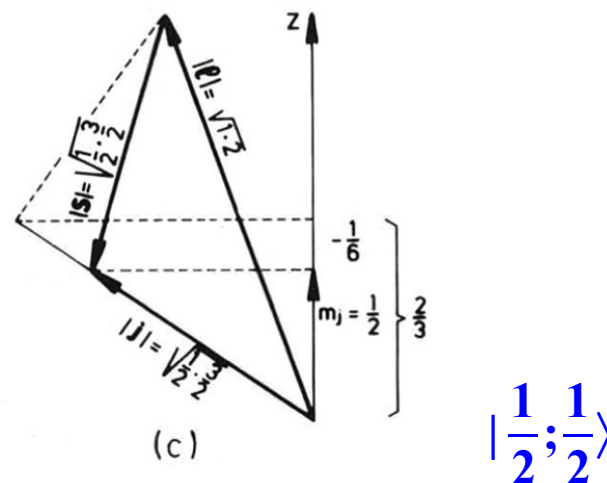
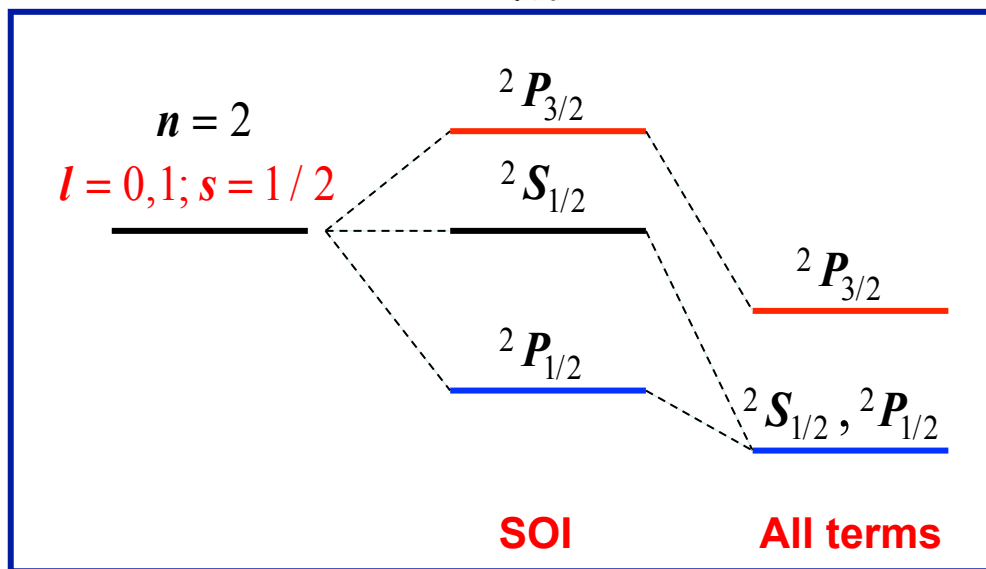
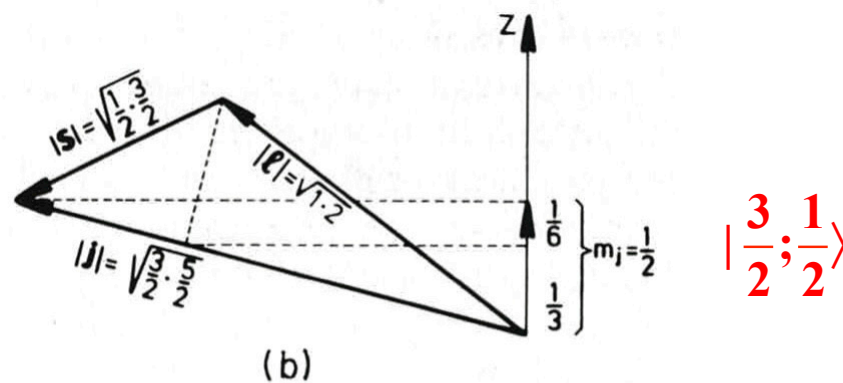
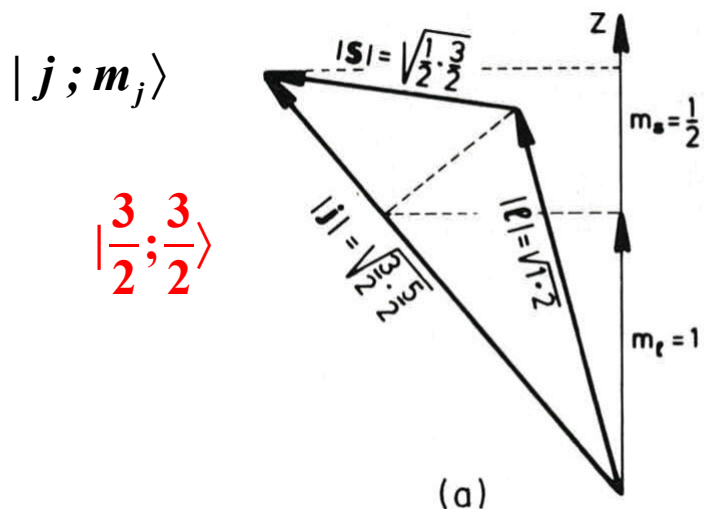
$$|\frac{3}{2}; -\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|-1; +\rangle + \sqrt{\frac{2}{3}}|0; -\rangle \quad \langle S_z \rangle = -\frac{\hbar}{6}; \quad \langle L_z \rangle = -\frac{\hbar}{3}$$

$$|\frac{1}{2}; \frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|1; -\rangle - \sqrt{\frac{1}{3}}|0; +\rangle \quad \langle S_z \rangle = -\frac{\hbar}{6}; \quad \langle L_z \rangle = \frac{2\hbar}{3}$$

$$|\frac{1}{2}; -\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|0; -\rangle - \sqrt{\frac{2}{3}}|-1; +\rangle \quad \langle S_z \rangle = \frac{\hbar}{6}; \quad \langle L_z \rangle = -\frac{2\hbar}{3}$$

# Textbook example: the $2p$ states of hydrogen

## VECTOR MODEL



# Preliminary considerations

The SOI couples the orientation of the magnetic moments to the crystal directions

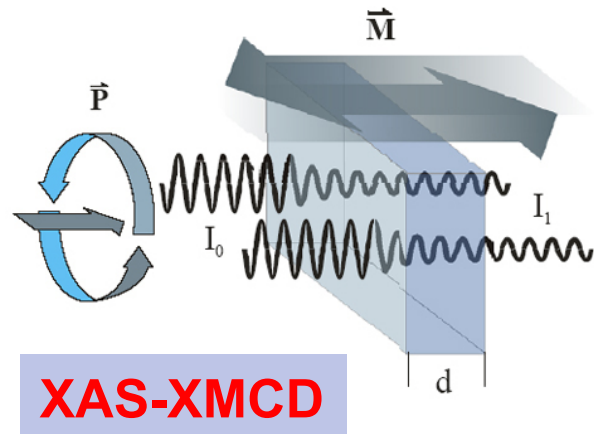
→ magnetic anisotropy, new quantum states...

Moreover, e.m. waves interact with L, not with S. **However SOI couples L and S, so that we can probe S with light!**

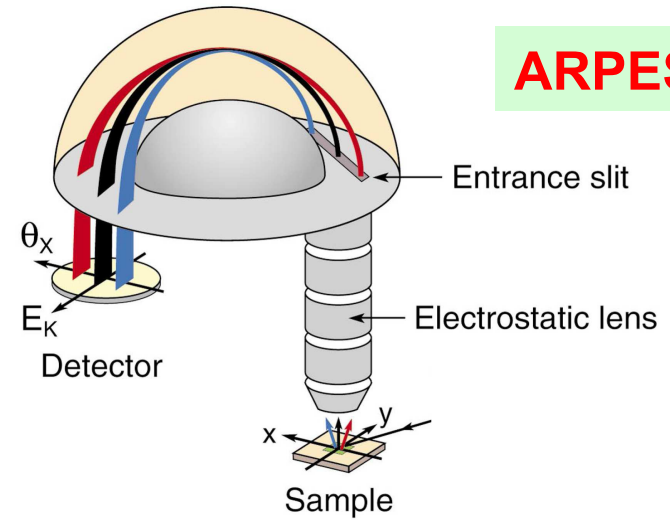
- **Circular magnetic dichroism: absorption (XMCD); photoemission (SP-PES)**
- **Resonant inelastic x-ray scattering**



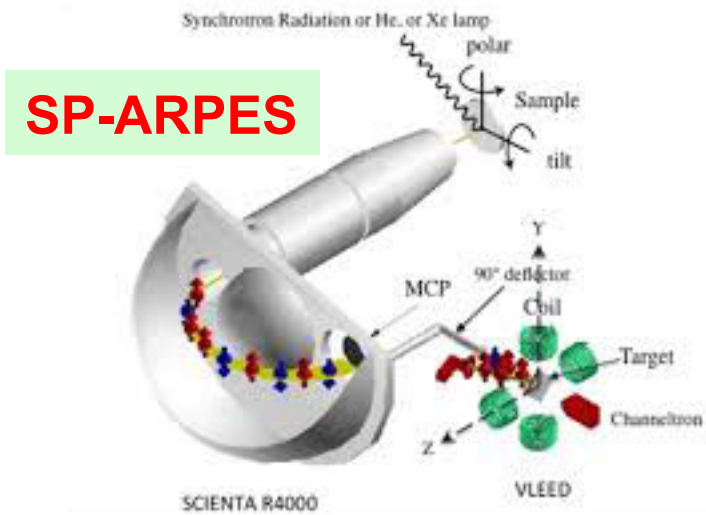
# Spectroscopic probes



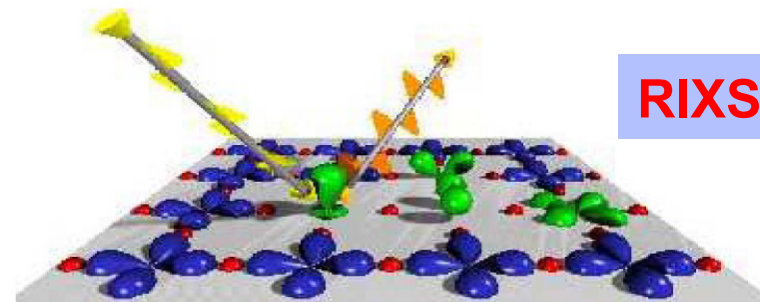
**XAS-XMCD**



**ARPES**

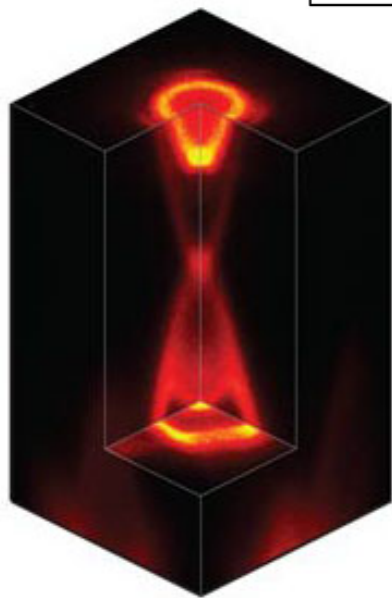
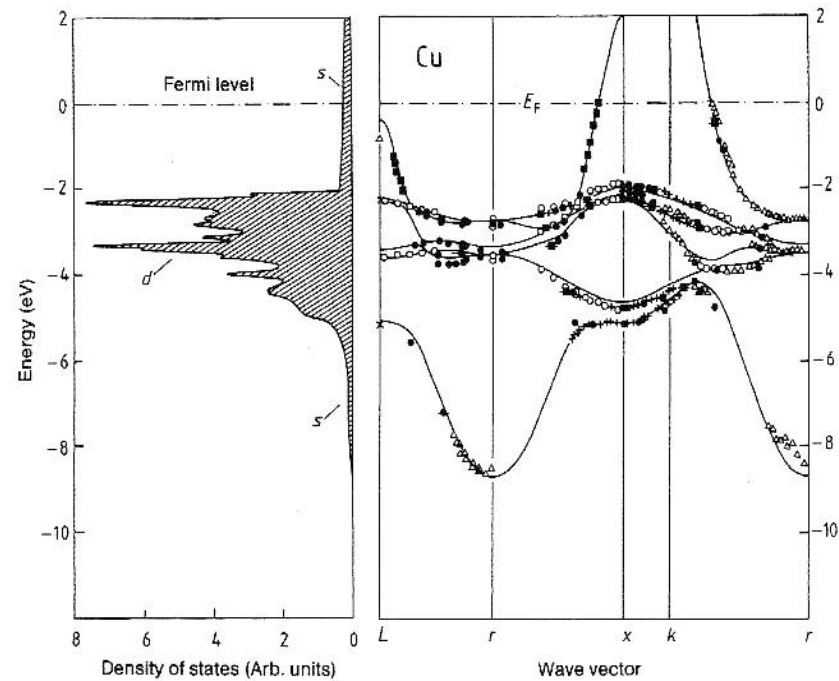
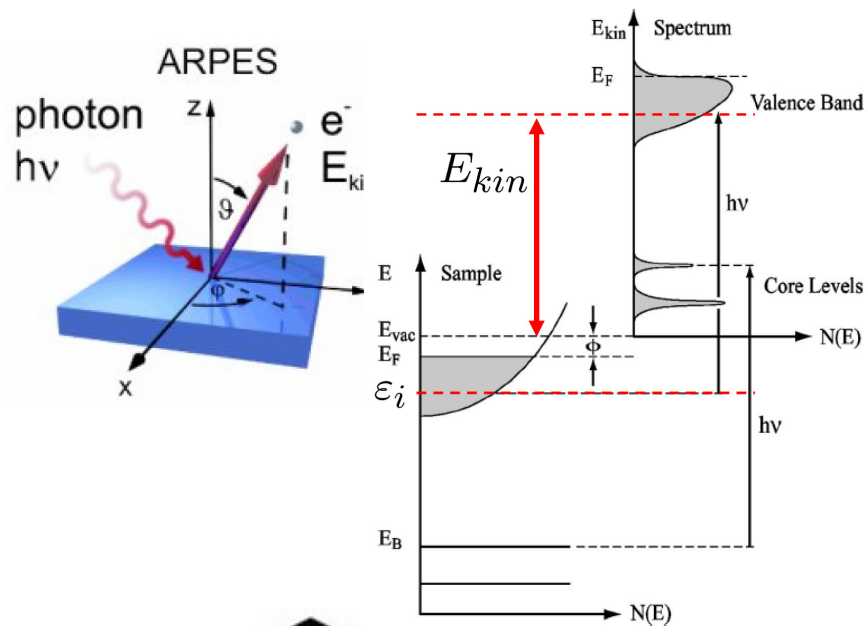


**SP-ARPES**

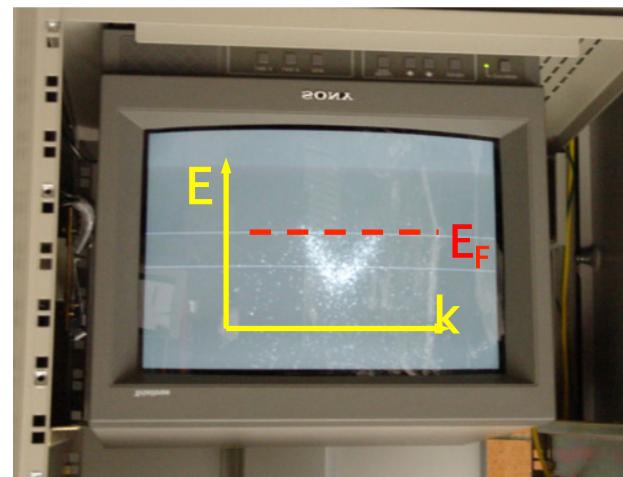


**RIXS**

# PES – ARPES – SP-ARPES

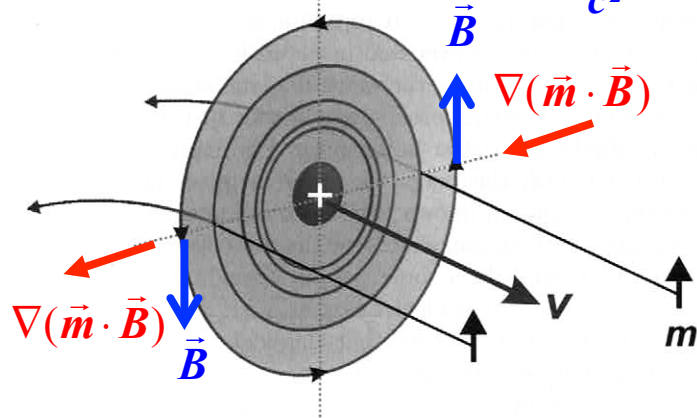


**ARPES:  
k-resolution**



# The Mott polarimeter – spin-orbit, again !

(heavy) atom  $\vec{B} = -\frac{1}{c^2} \vec{v} \times \vec{E}$ ;  $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$

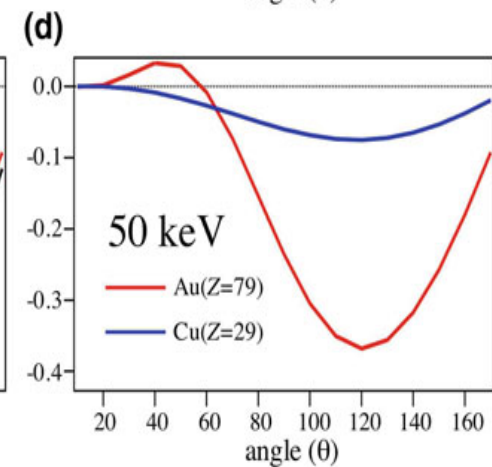
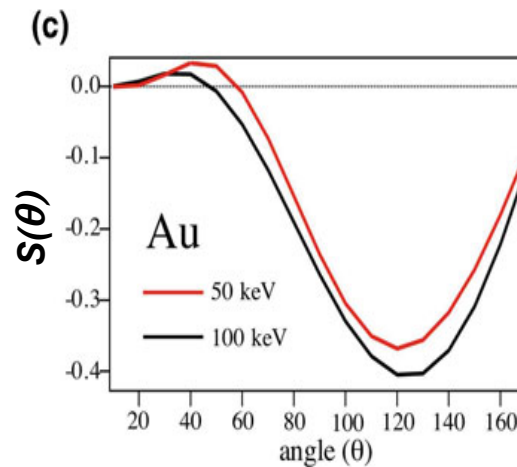
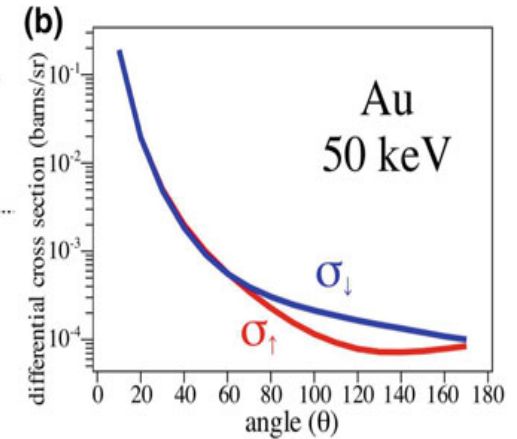
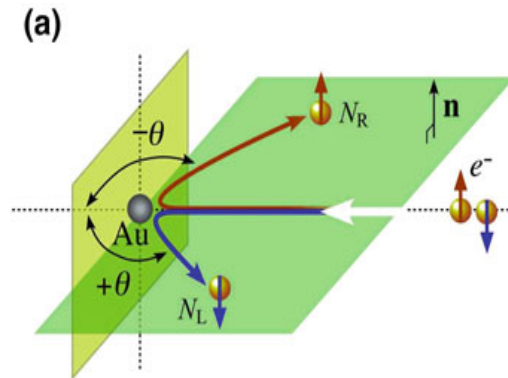


In the electron's rest frame

$$\sigma(\theta) = \sigma_0 (1 + S(\theta) \vec{P} \cdot \vec{n})$$

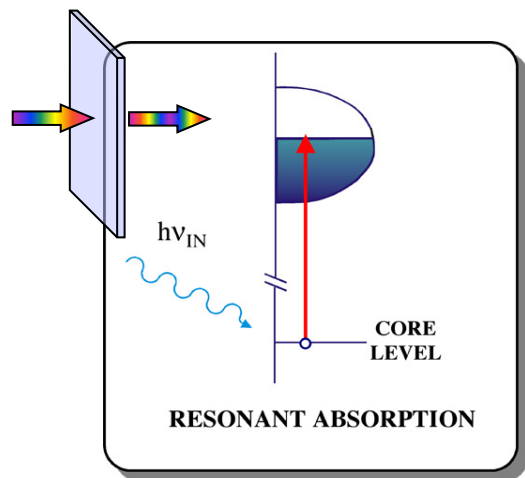
$\vec{P}$  Spin polarization

$S$  Sherman function



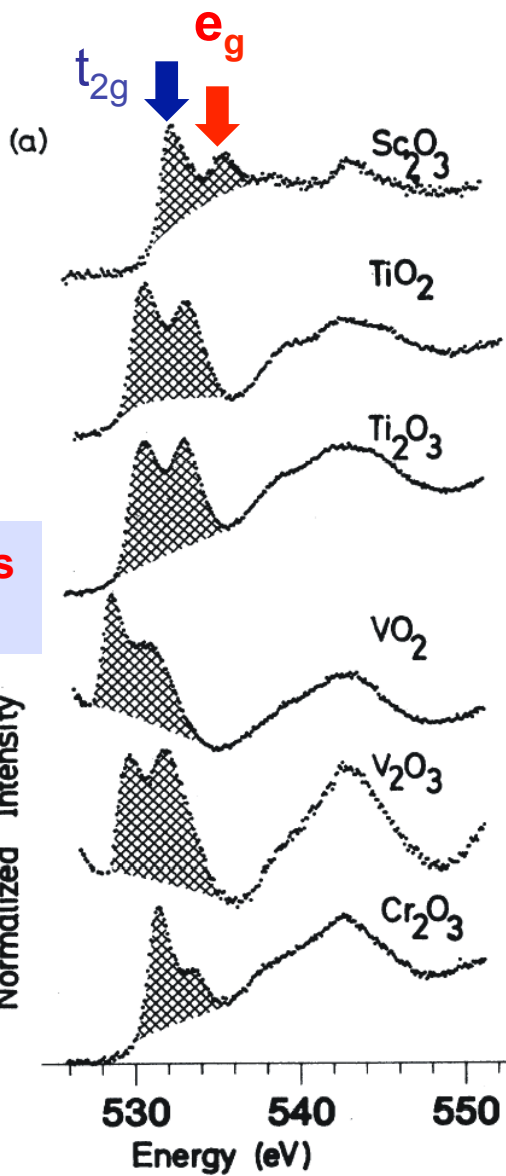
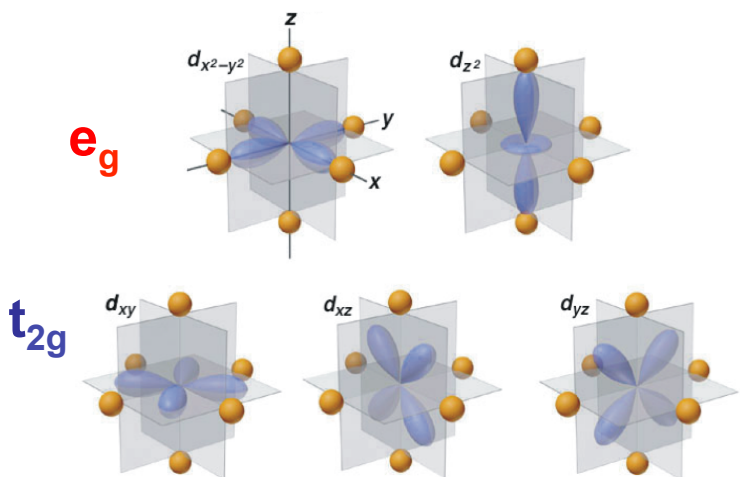


# XAS: unoccupied states

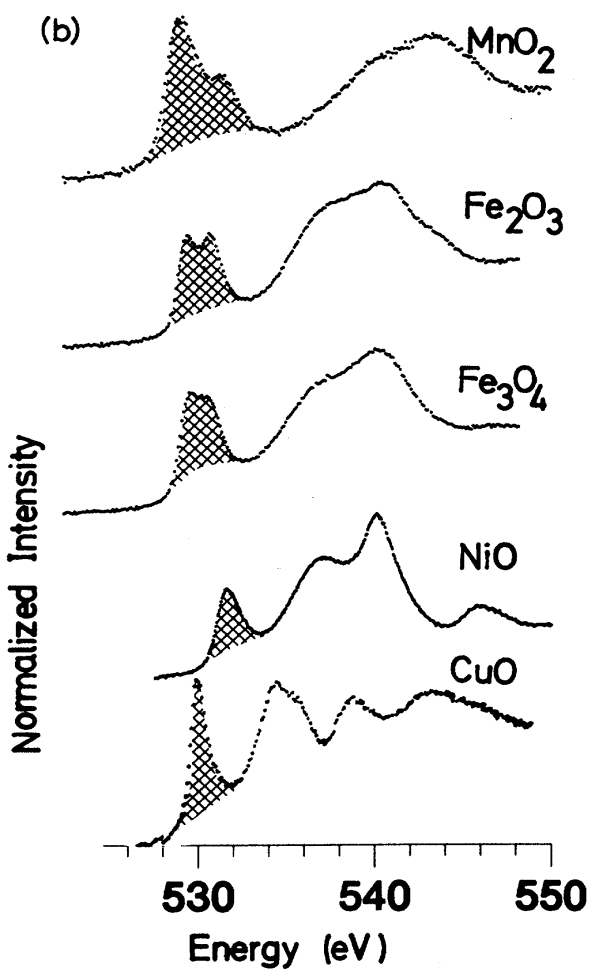


Core hole and dipole selection rules ( $\Delta l = \pm 1$ ) select the final states

TM 3d – O 2p hybrid states



The O K edge (1s → εp) of 3d TM oxides

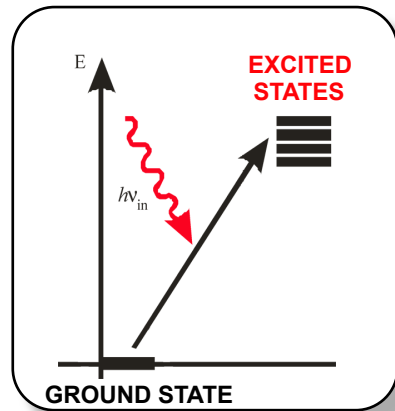


F.M.F. de Groot et al., PRB (1989)

# Beyond the single particle picture

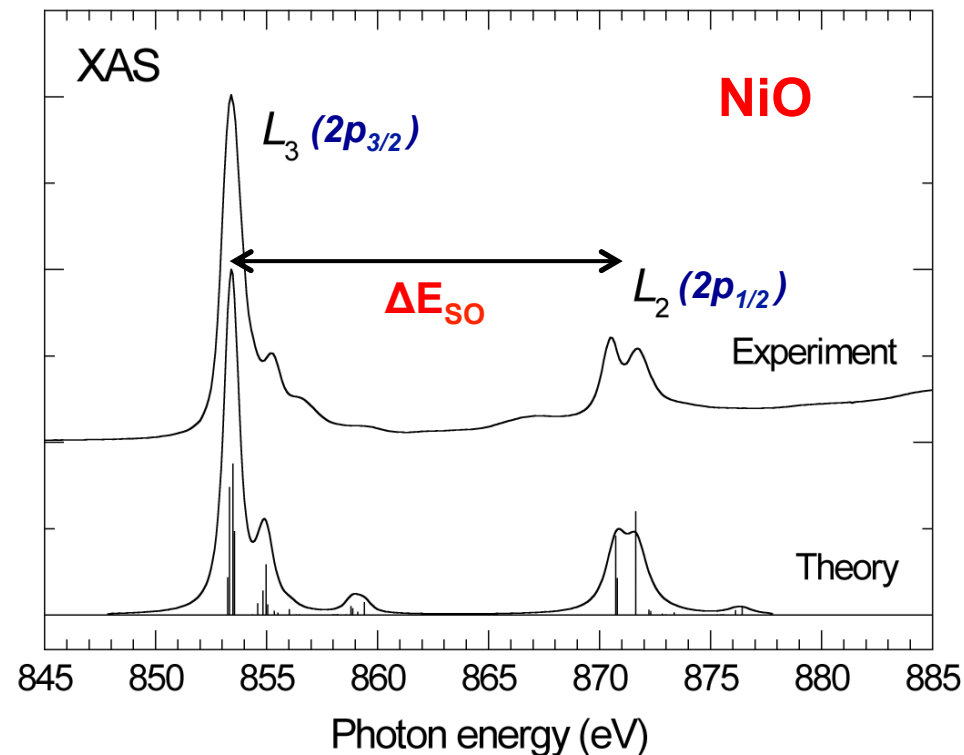
The XAS~DOS picture breaks down when the final states are strongly correlated: TM 3d states

Total energy diagram



$\Delta E_{SO}(2p)$  large  
J is the good quantum number  
 $\Delta J=0, \pm 1$  (no  $0 \rightarrow 0$ )

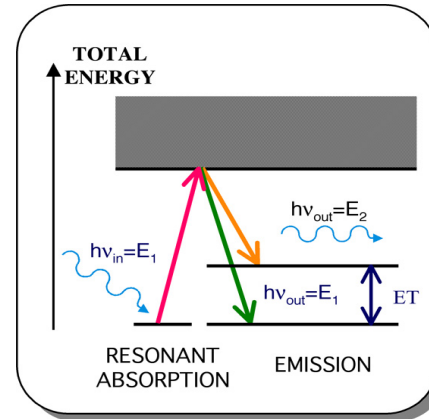
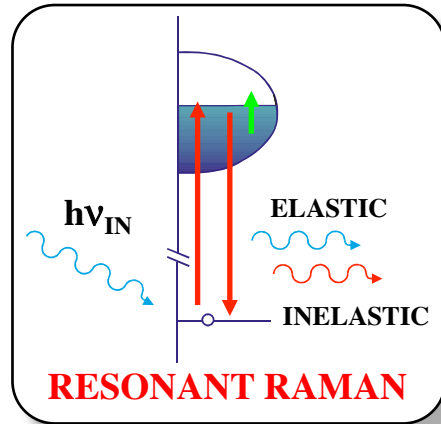
3d TM  $L_{2,3}$  edges:  $3d^N \rightarrow 2p^5 3d^{N+1}$



The atomic-like multiplet structure is a fingerprint of the *ground state* configuration...

# Photons do it all: resonant inelastic x-ray spectroscopy (RIXS)

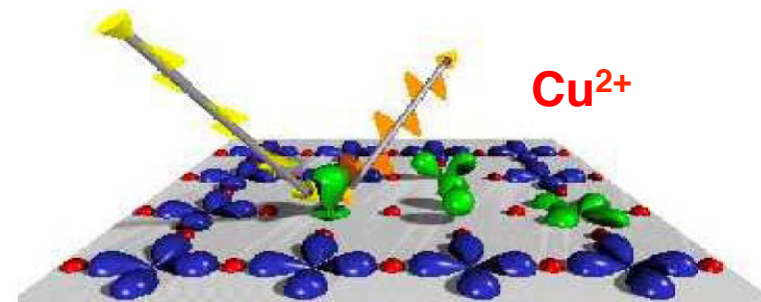
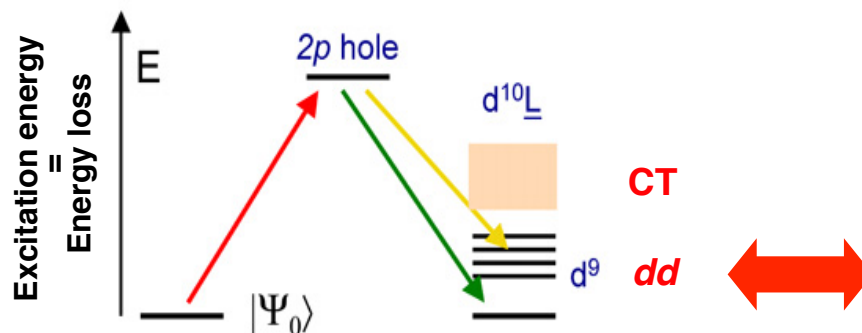
Threshold  
excitation  
of a core  
state



Energy loss  
(energy transfer)

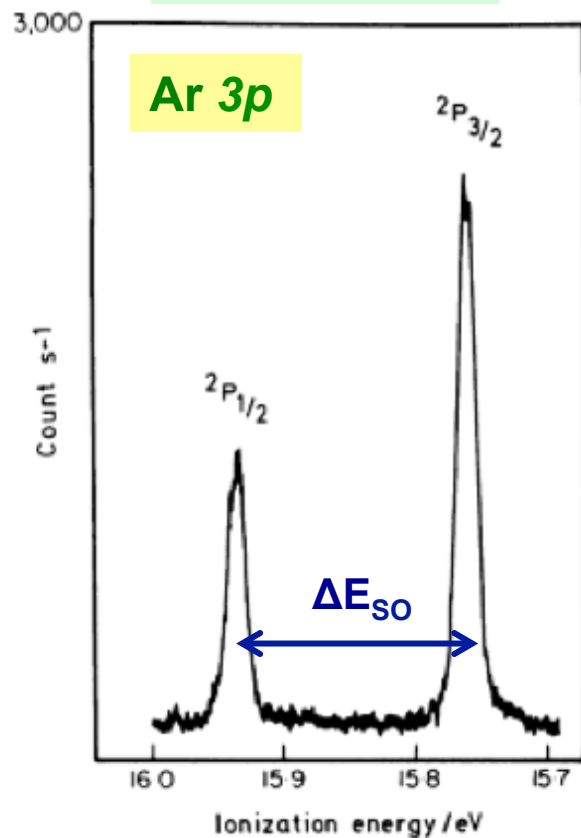


Example - Cu L3 RIXS:  $3d^9 \rightarrow 2p^5 3d^{10} \rightarrow 3d^9$

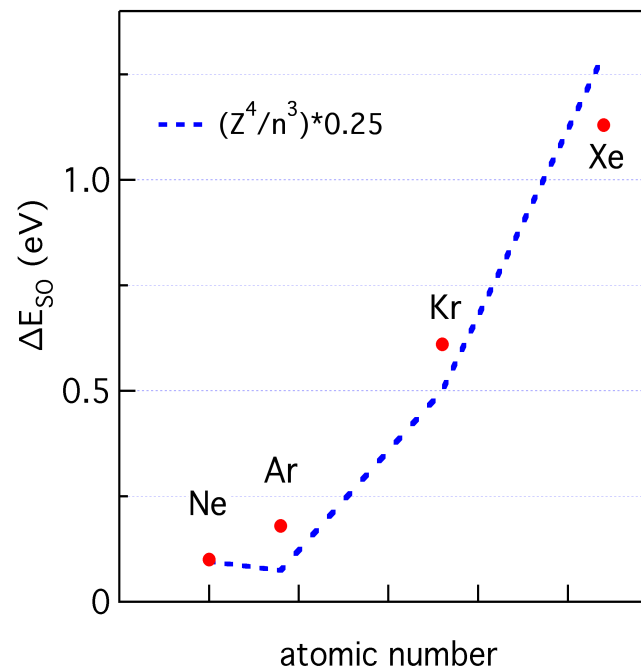


# Spin-orbit: rare gases

## Photoemission



## Rare gases - $np$



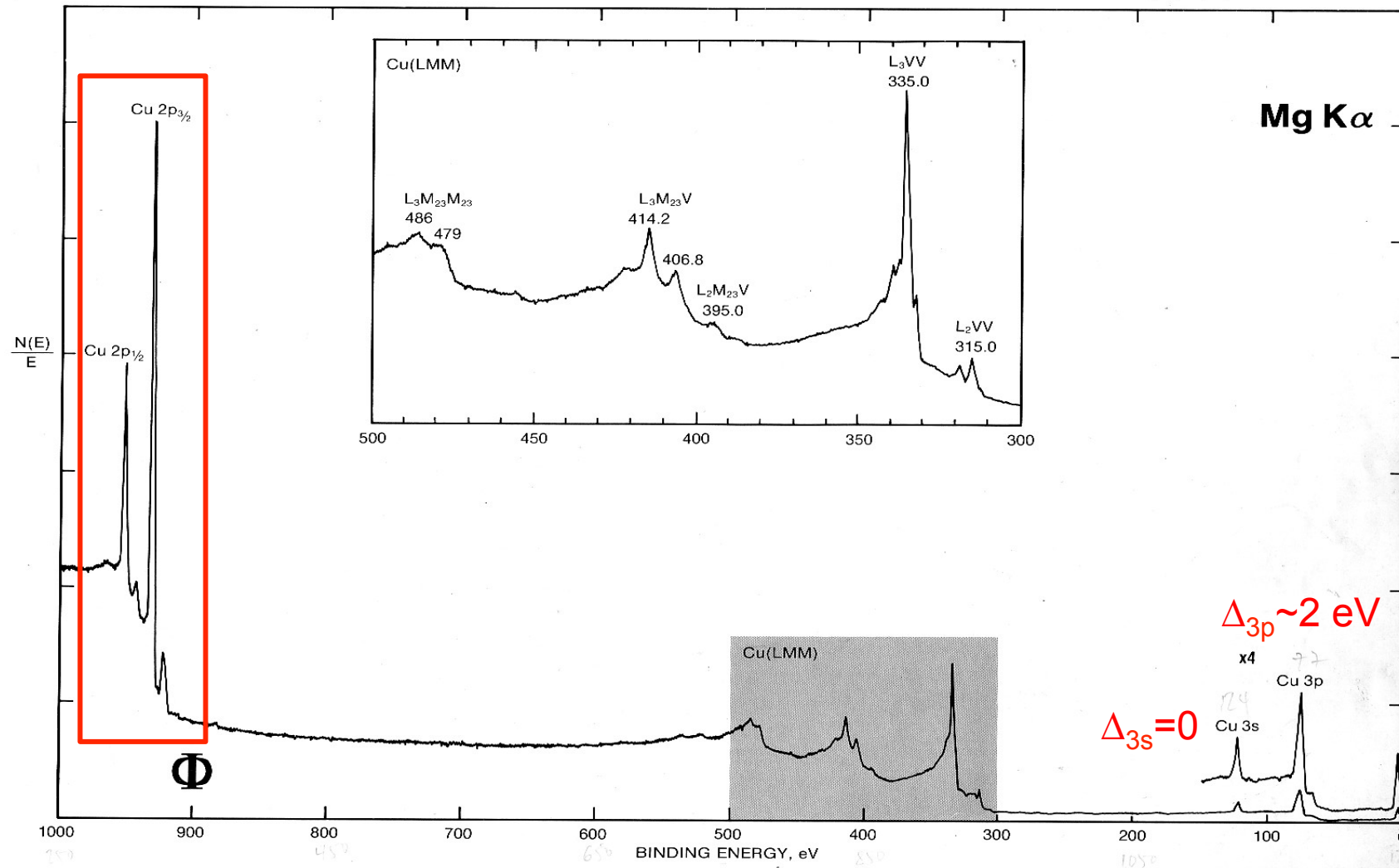
Semi-empirical:

$$\zeta_{n,p} = 0.405 Z^{2.33} / (E_n)^{1.5}$$

# Solids: core levels - PES

HANDBOOK OF X-RAY PHOTOELECTRON SPECTROSCOPY

Copper, Cu Atomic Number 29



$\Delta_{2p} \sim 20 \text{ eV}$

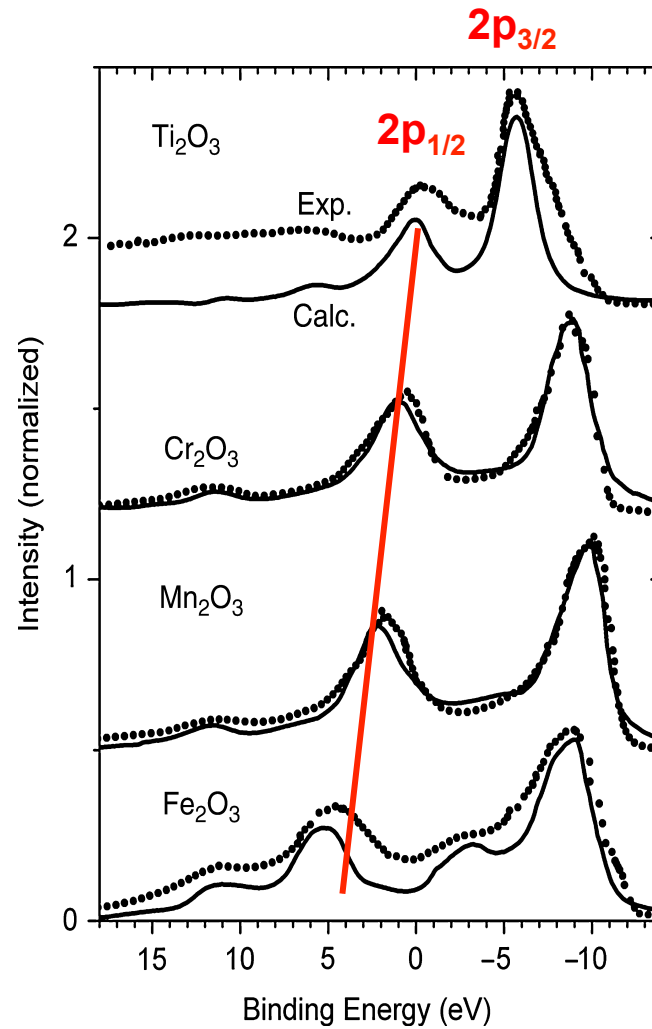
$\Delta_{3p} \sim 2 \text{ eV}$   
 $\Delta_{3s} = 0$



# Solids: core levels - PES

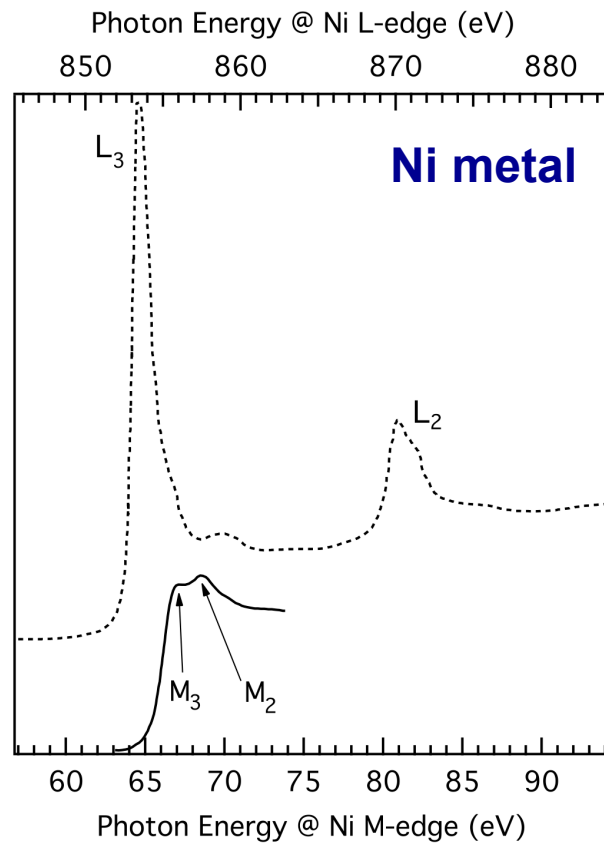
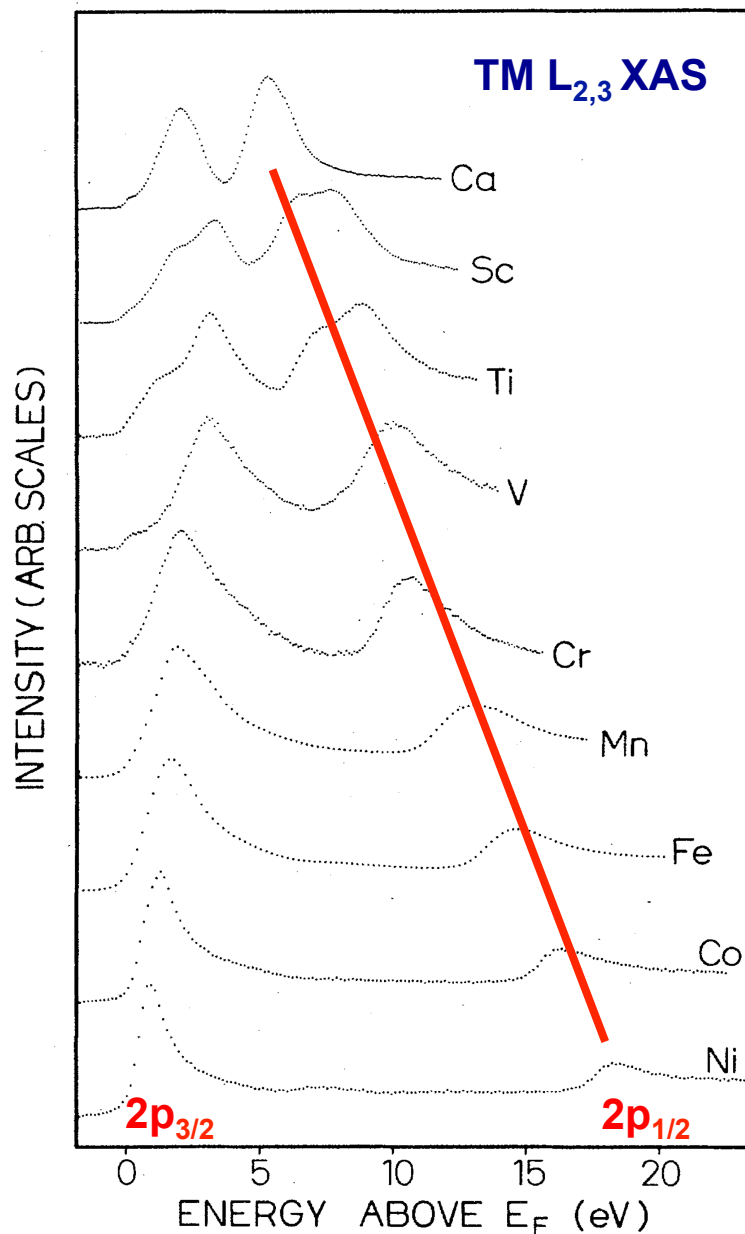
Early 3d TM oxides:

**TM 2p PES**

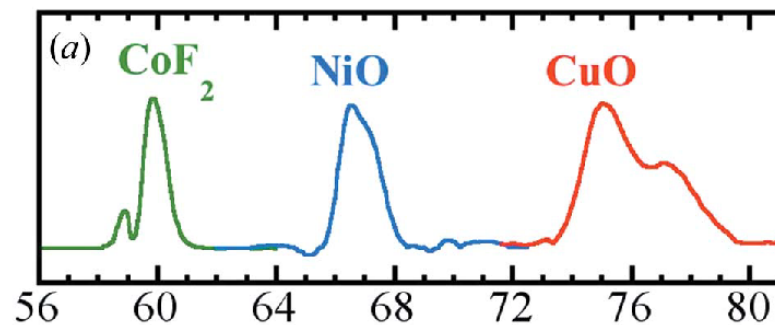


De Groot and Kontani (2008)

# Solids: core levels - XAS

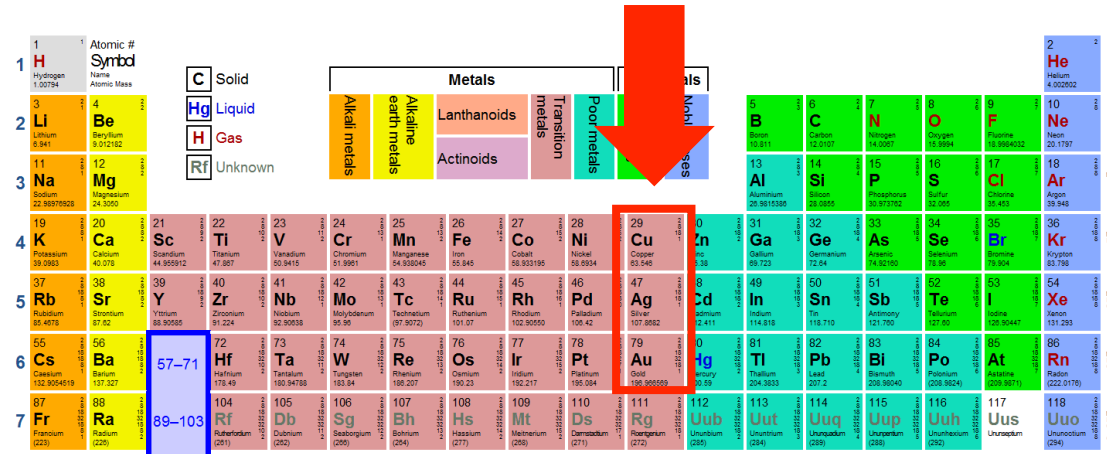
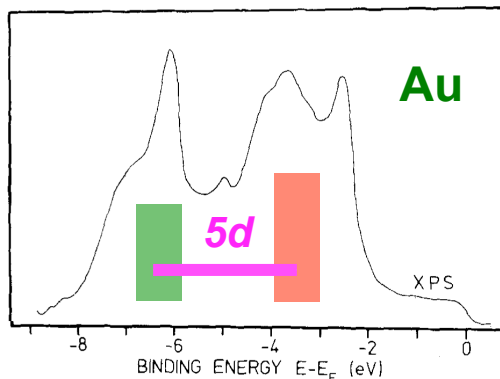
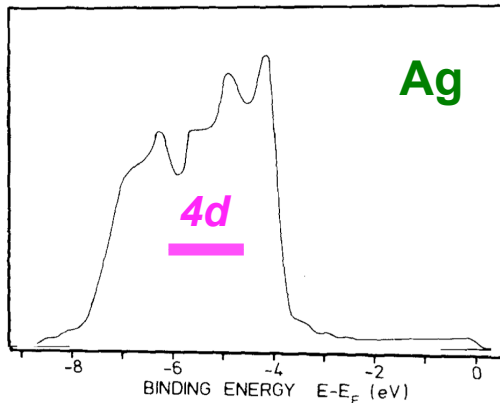
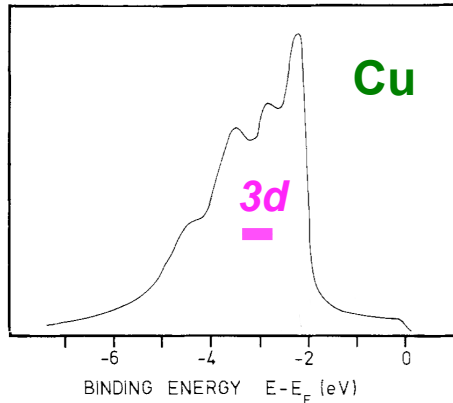


**$M_{23}$  ( $3p \rightarrow 3d$ ) edges**

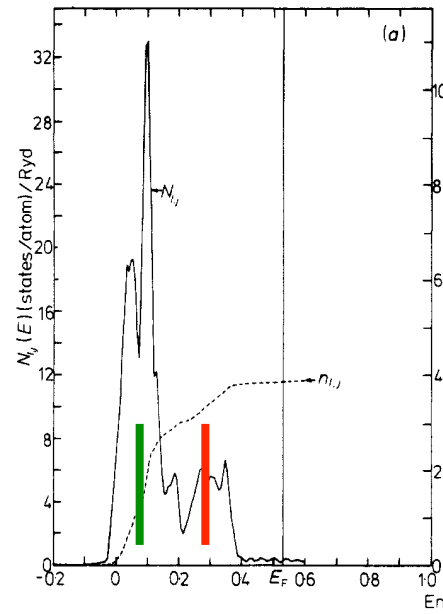


# Solids: valence bands - PES

## PES – noble metals



### Au – $5d_{3/2}$



### Au – $5d_{5/2}$

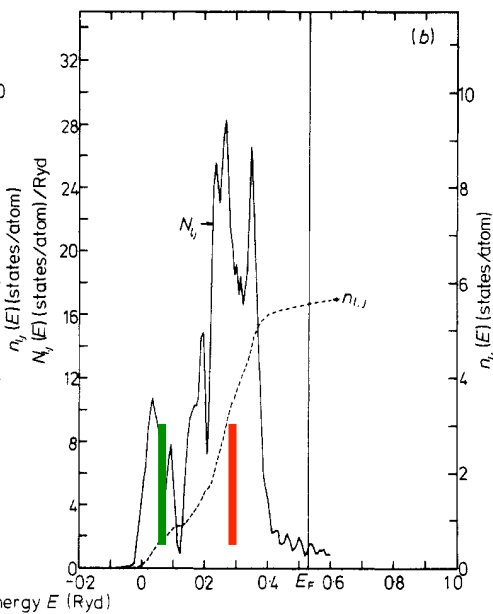
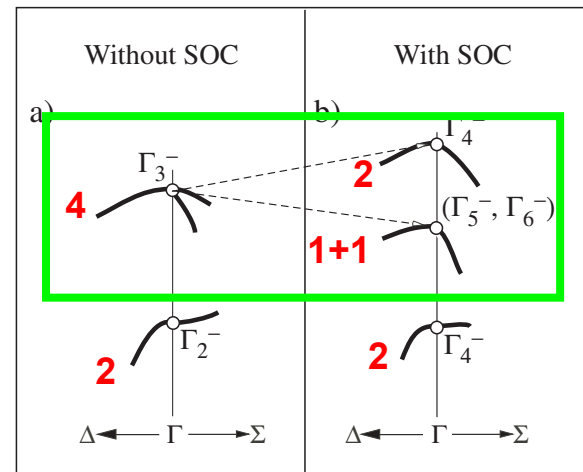
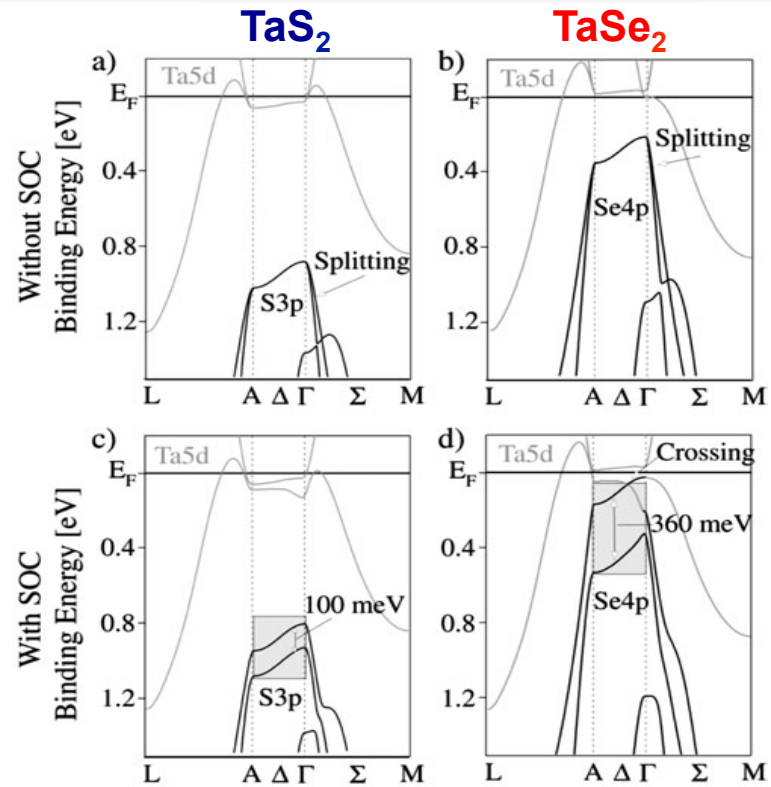
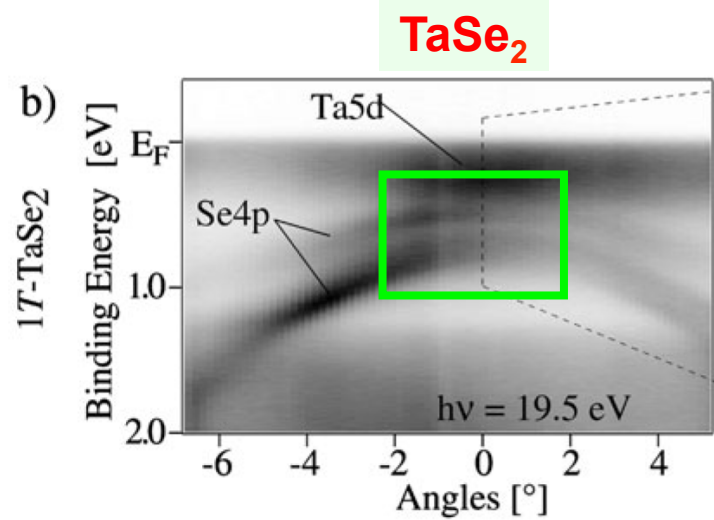
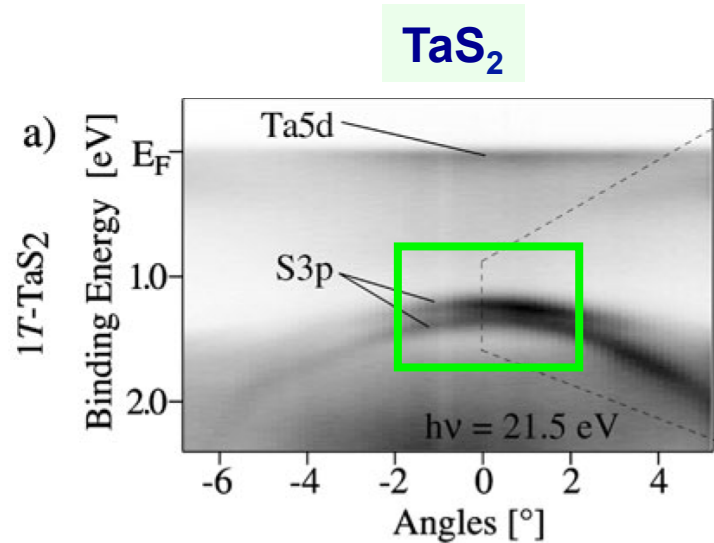


Figure 2. Spin-orbit projected densities of states  $N_{l,j}$  for gold,  $l = 2$ : (a),  $j = 3/2$  and (b),  $j = 5/2$  (curves shown with a full line). The broken curves represent the integrated partial DOS functions.

# Solids: valence bands - ARPES

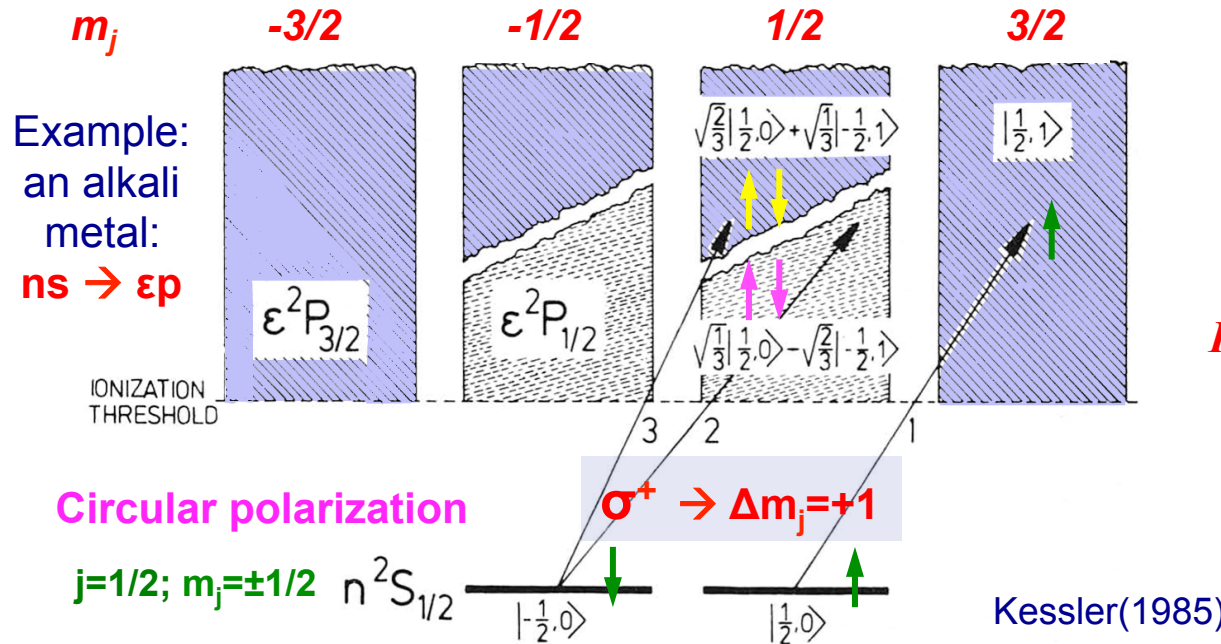


# The Fano effect: polarized electrons from unpolarized atoms

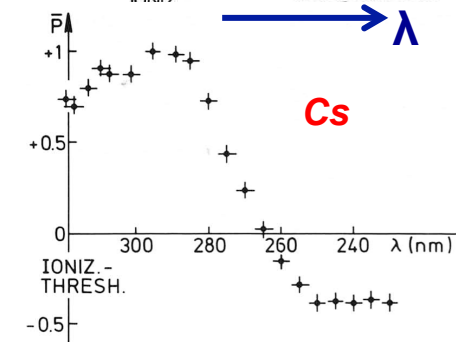
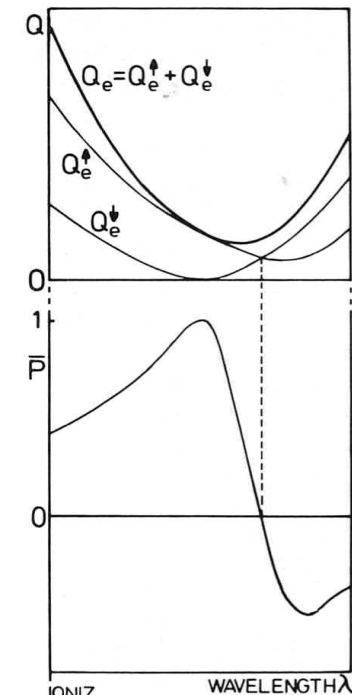
photon angular momentum  $\rightarrow$  dipole selection rules

$\rightarrow$  electron orbital momentum  $\rightarrow$  spin-orbit interaction  $\rightarrow$  electron spin

No SO  $\rightarrow$  no spin-flip!!



$$P = \frac{Q_e^\uparrow - Q_e^\downarrow}{Q_e^\uparrow + Q_e^\downarrow}$$



Angle-integrated:

$$\frac{Q_e^\uparrow}{Q_e^\downarrow} = \frac{9R_3^2 + 2(R_3 - R_1)^2}{(2R_1 + R_3)^2}; \quad R_{1,3} = \langle R_{n,0} | r | R_{\epsilon,1,m_j=1/2;3/2} \rangle$$

No spin-orbit  $\rightarrow$   $R_1 = R_3$ ;  $\frac{Q_e^\uparrow}{Q_e^\downarrow} = 1$ ;  $P = 0$



# Exploiting the spin polarization: circular dichroism in XAS/PES

Goal(s): to determine separately the spin- and orbital moments (and the expectation values of other observables)

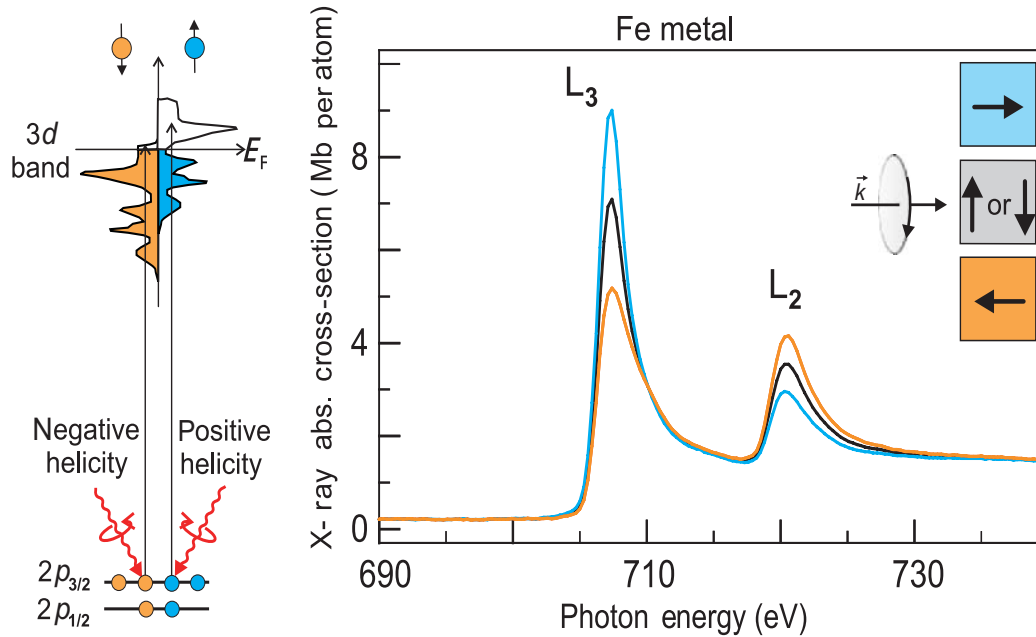
**XAS:** Transitions into low-lying (localized) correlated final states, polarized by an external field or because of magnetic order

- The Pauli principle enforces spin anisotropy

**PES:** Transitions to free-electron final states, weakly correlated and non-polarized

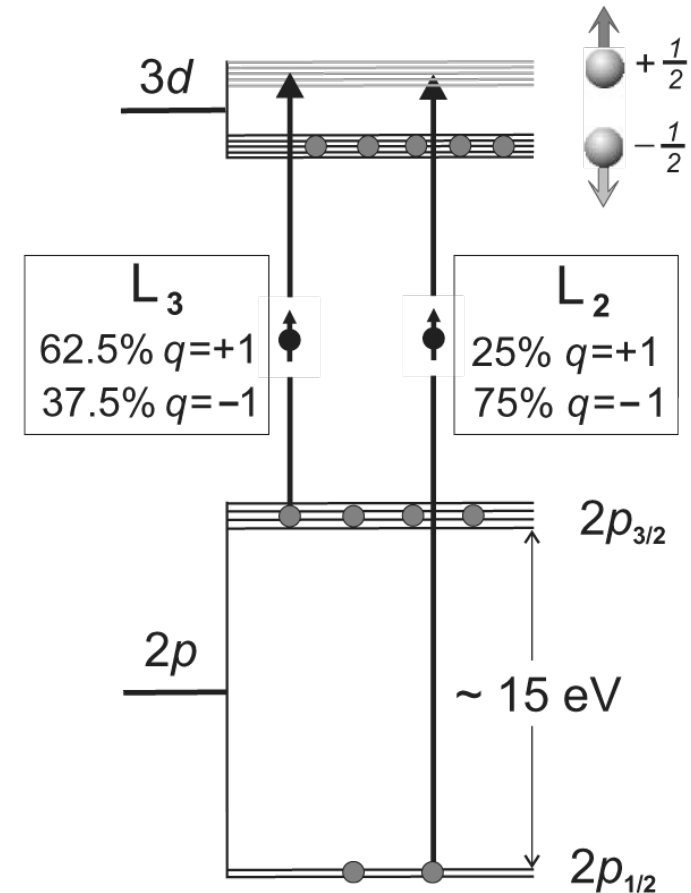
- No need for macroscopic magnetization
- typically needs spin detection

# Dichroism in XAS



The absorption depends on the **photon polarization** and on the **spin orientation**

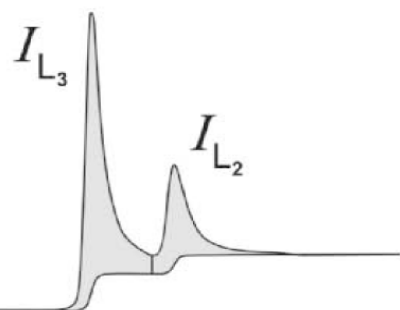
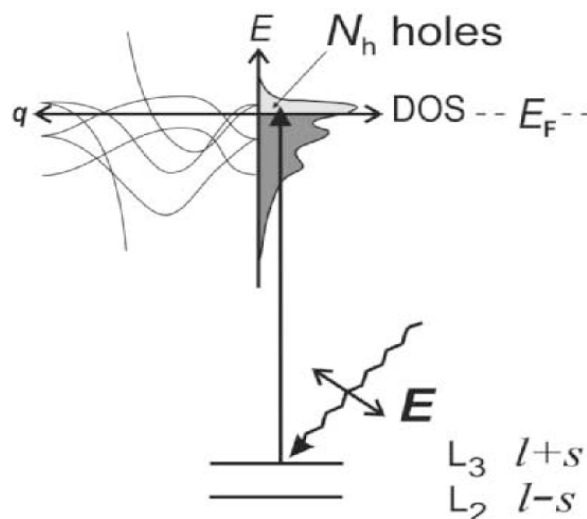
**Simple case: fully polarized 3d band**



**An atomic model**

# XMCD sum rules: quantitative determination of $m_{orb}$ e $m_{spin}$

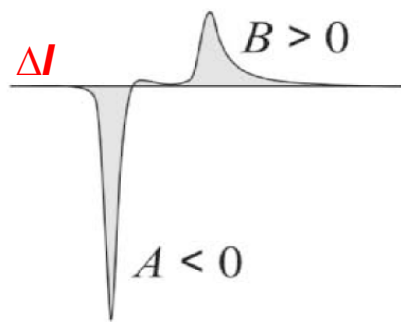
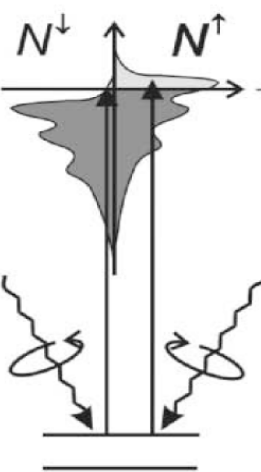
## Orbital occupation



$$N_h = \langle I_{L_3} + I_{L_2} \rangle / C$$

## Spin moment

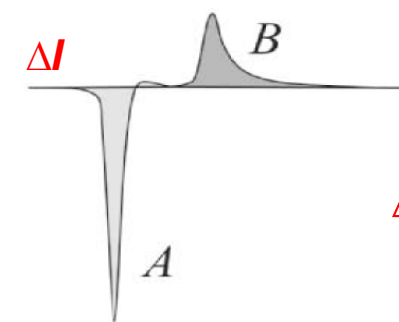
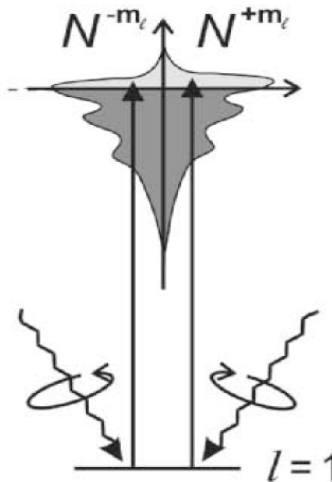
$$-\frac{1}{2} \downarrow \quad \uparrow + \frac{1}{2}$$



$$m_s = \mu_B \langle -A + 2B \rangle / C$$

## Orbital moment

$$-m_l \quad \uparrow + m_l$$



$$m_o = -2\mu_B \langle A + B \rangle / 3C$$

$$\Delta I = | \downarrow \uparrow - | \uparrow \uparrow$$

# Photoemission – Polarization and spin detection

TABLE I. The six fundamental photoemission spectra  $I^{xy}$ , which are linear combinations of the primitive spin-polarized spectra.  $x=0,1,2$  denotes isotropically, circularly, and linearly polarized radiation, respectively.  $y=0,1$  denotes without and with spin-polarization measurement, respectively. In the column labeled  $z$ , a value 1, 2, or 3 denotes that  $\langle M \rangle$ ,  $\langle M^2 - \frac{1}{3} J(J+1) \rangle$ , or  $\langle M^3 - \frac{3}{5} M[J(J+1) - \frac{1}{3}] \rangle$  in the ground state has to be nonzero to obtain the spectrum;  $z=0$  denotes the value of the monopole, which is always unity.

$I^{xy}$	Combination of primitive spectra	$z$	Significance
$I^{00}$	$I_{1\uparrow} + I_{0\uparrow} + I_{-1\uparrow} + I_{1\downarrow} + I_{0\downarrow} + I_{-1\downarrow}$	0	Isotropic spectrum
$I^{01}$	$I_{1\uparrow} + I_{0\uparrow} + I_{-1\uparrow} - I_{1\downarrow} - I_{0\downarrow} - I_{-1\downarrow}$	1	Spin spectrum
$I^{10}$	$I_{1\uparrow} - I_{-1\uparrow} + I_{1\downarrow} - I_{-1\downarrow}$	1	Orbit spectrum (MCD)
$I^{11}$	$I_{1\uparrow} - I_{-1\uparrow} - I_{1\downarrow} + I_{-1\downarrow}$	0,2	Spin-orbit spectrum
$I^{20}$	$I_{1\uparrow} - 2I_{0\uparrow} + I_{-1\uparrow} + I_{1\downarrow} - 2I_{0\downarrow} + I_{-1\downarrow}$	2	Anisotropic spectrum (MLD)
$I^{21}$	$I_{1\uparrow} - 2I_{0\uparrow} + I_{-1\uparrow} - I_{1\downarrow} + 2I_{0\downarrow} - I_{-1\downarrow}$	1,3	Anisotropic spin magnetic spectrum

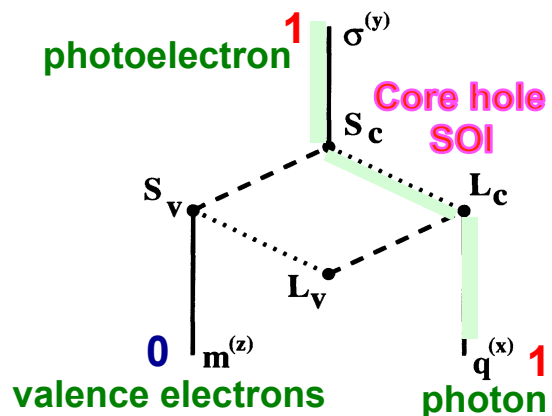
# Core level photoemission

## FUNDAMENTAL SPECTRA

Photoemission spectrum	$x$	$y$	$z$	s.o.	$U_{cv}$
Isotropic spectrum	0	0	0	0	0
Spin spectrum	0	1	1	0	1
Orbit spectrum (MCD)	1	0	1	1	1
Spin-orbit spectrum	1	1	0	1	0
Spin-orbit magnetic quadrupole spectrum	1	1	2	1	1
Anisotropic spectrum (MLD)	2	0	2	1	1
Anisotropic spin magnetic dipole spectrum	2	1	1	1	1
Anisotropic spin magnetic octupole spectrum	2	1	3	1	1

spin detection  
Pol. moment

$$I^{11} = I_{1\uparrow} + I_{-1\downarrow} - (I_{1\downarrow} + I_{-1\uparrow})$$

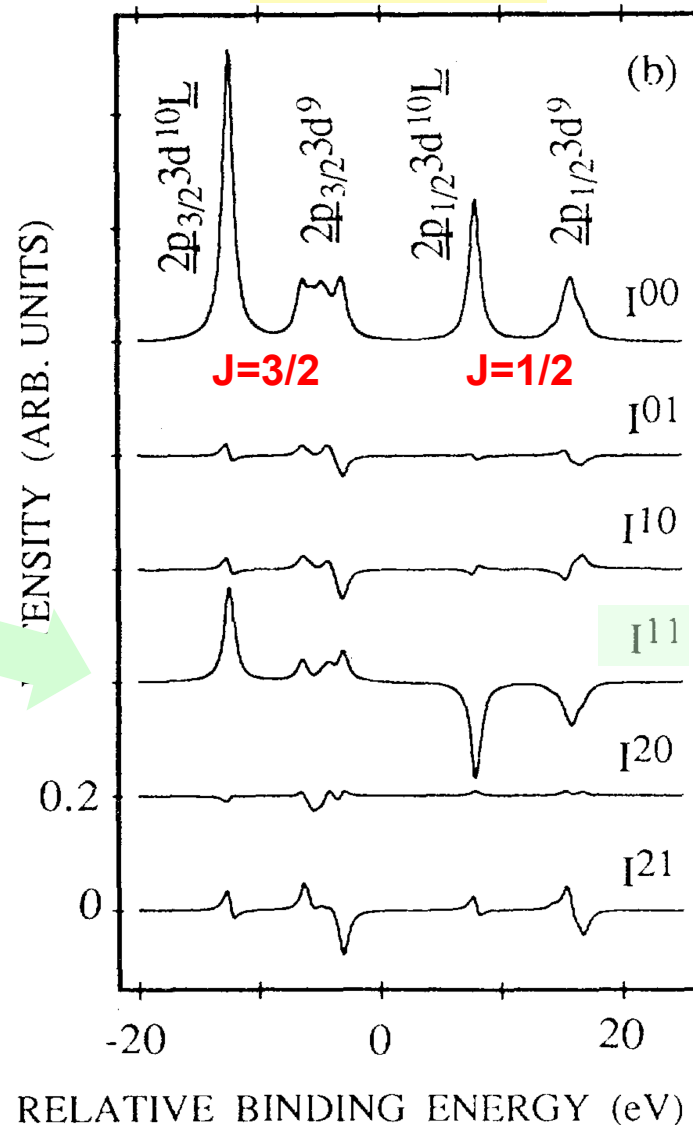


Integrated

$$\approx \langle L_z S_z \rangle = 0$$

(closed shell)

Cu<sup>II</sup>, Cu 2p



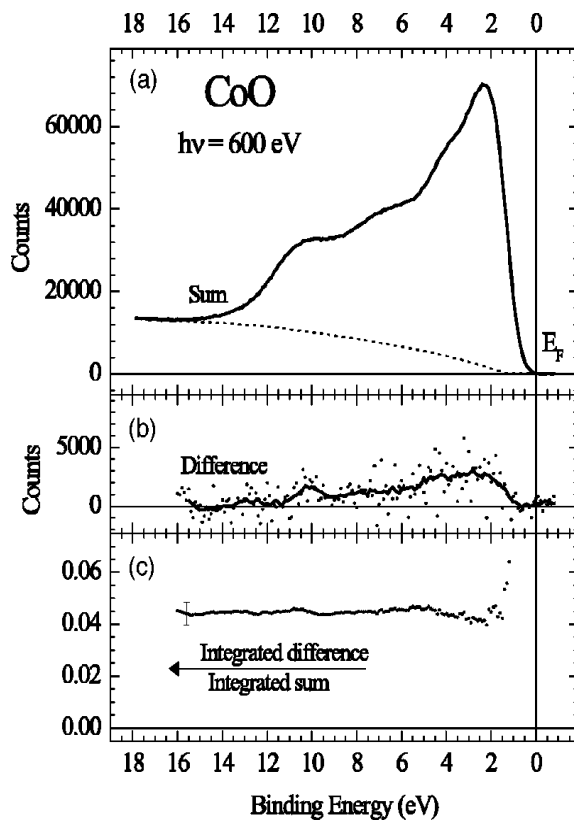


# Valence band: Integrated spin-orbit spectrum

$$\frac{\rho^{11}}{\rho^{00}} = \frac{2A_1}{A_0} \frac{\langle \sum_i l_{zi} s_{zi} \rangle}{\langle n \rangle} = \frac{2A_1}{A_0} \frac{\langle \sum_i l_i \cdot s_i \rangle}{3\langle n \rangle} \quad \text{if } \langle M \rangle = 0$$

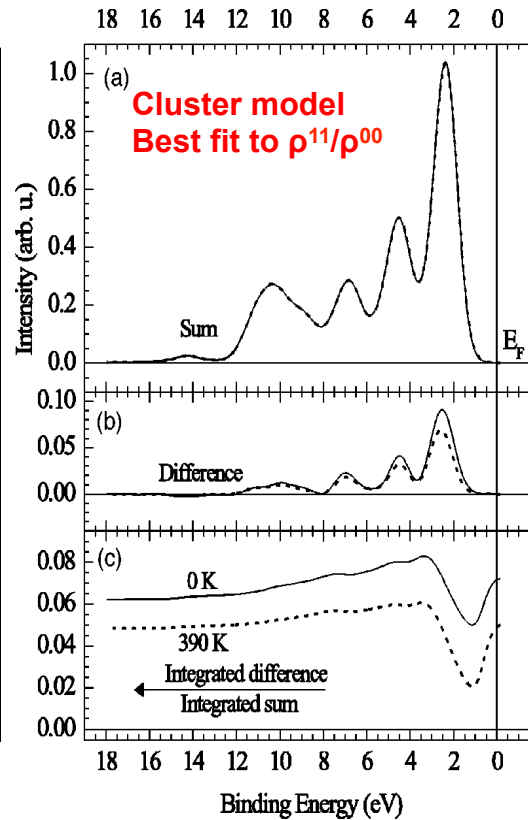
$\rho =$  integrated spectra  
 $z =$  spin quantization axis, //  $k_{\text{photon}}$

Unlike XMCD or XMLD SP-PES does not require FM or AFM order



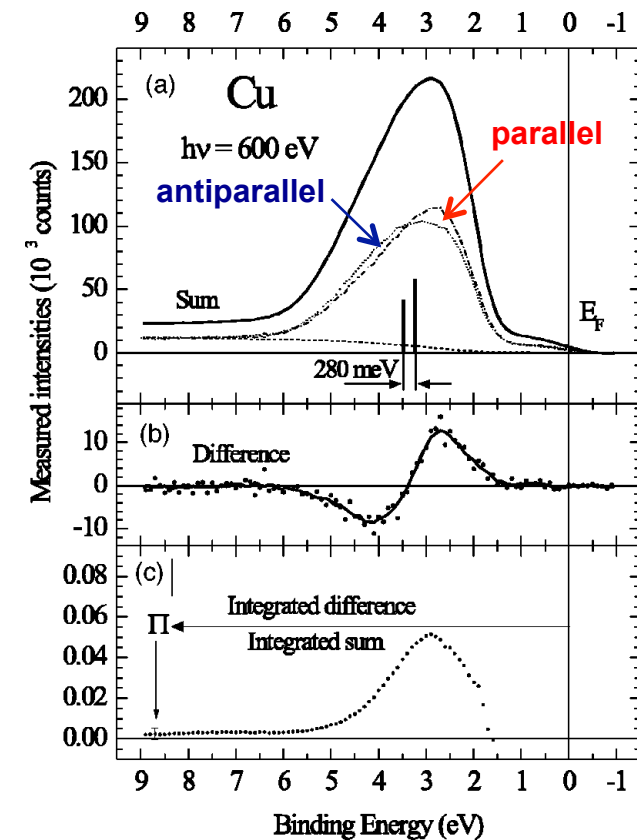
$$\langle \sum_i l_{zi} s_{zi} \rangle = -0.33 \pm 0.04 \hbar^2$$

G. Ghiringhelli (2002)



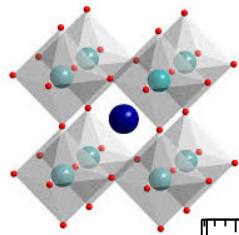
Theory:  $\langle L_z \rangle / 2\langle S_z \rangle = 0.57$

Exp.:  $m = 3.98 \mu_B \rightarrow \langle L_z \rangle = 1.36 \hbar ; \langle S_z \rangle = 1.31 \hbar$



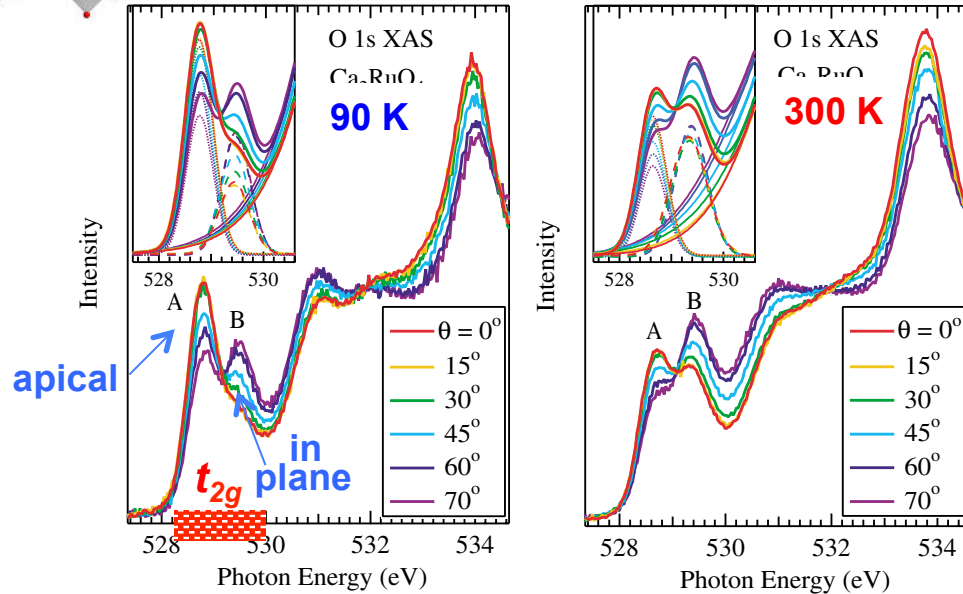
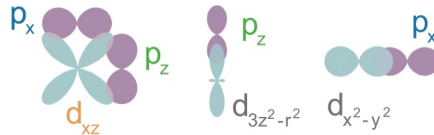
$$\langle \sum_i l_{zi} s_{zi} \rangle = 0$$

# Ca<sub>2</sub>RuO<sub>4</sub>: an AFM (T<sub>N</sub>=110K) Mott insulator



Ru<sup>4+</sup>(4d<sup>4</sup>; t<sub>2g</sub><sup>4</sup>)

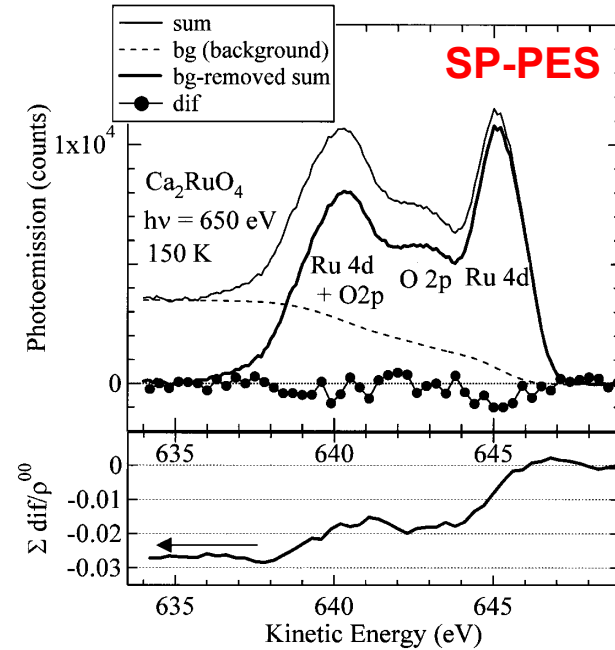
O K-edge XAS



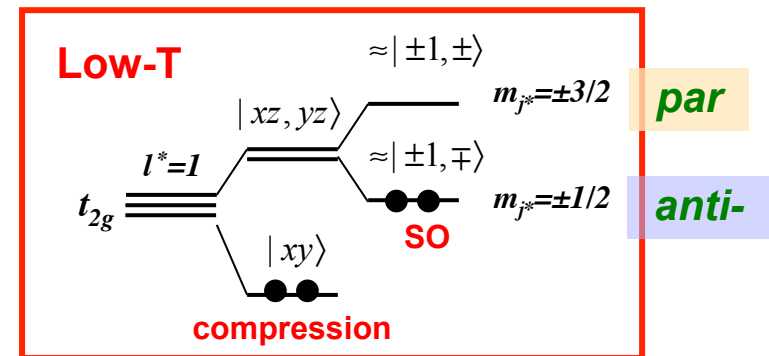
t<sub>2g</sub>: strong angular dependence  
(→ O p<sub>xyz</sub> orbitals / in-plane, out-of-plane)

strong temperature dependence  
(Tetragonal distortion → variable orbital occupancy)

300 K: n<sub>xy</sub>:n<sub>yz/xz</sub> = 1:1 ; 90 K: n<sub>xy</sub>:n<sub>yz/xz</sub> = 1/2:3/2



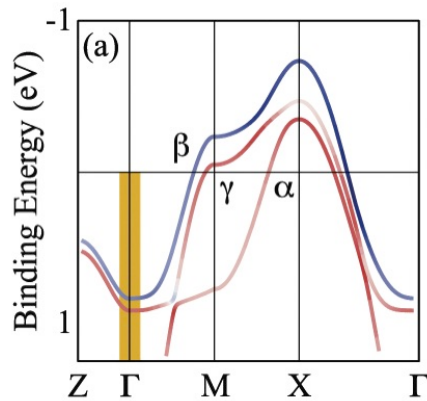
$$\left\langle \sum_i l_{zi} s_{zi} \right\rangle = -0.28 \pm 0.07 \hbar^2 \quad \text{L not quenched}$$



T. Mizokawa et al., PRL (2001)

SO mixes and splits d<sub>xz</sub> and d<sub>yz</sub> : d<sub>xz</sub> + id<sub>yz</sub> ≈ |-1> ; d<sub>xz</sub> - id<sub>yz</sub> ≈ |1>

# SP-ARPES - Sr<sub>2</sub>RuO<sub>4</sub> : a spin-triplet superconductor?

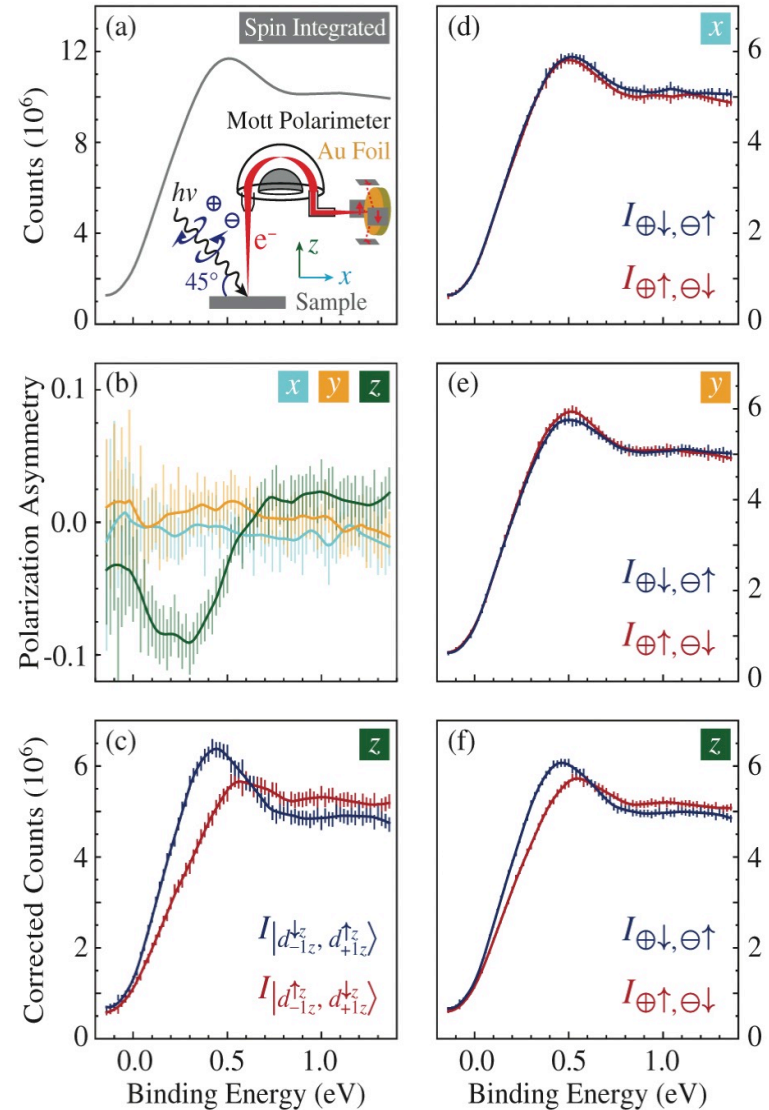
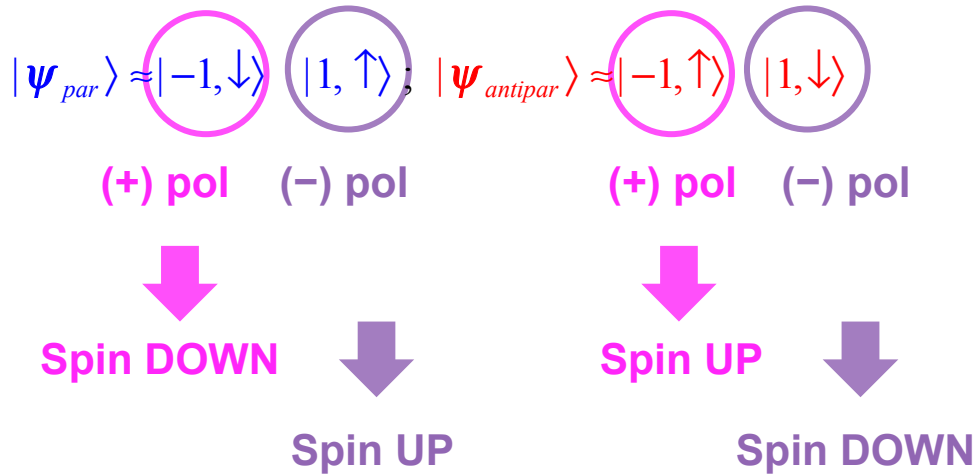


SO mixes and splits  $d_{xz}$  and  $d_{yz}$  :  
 $\rightarrow d_{xz} + id_{yz} \approx |-1\rangle$ ;  $d_{xz} - id_{yz} \approx |1\rangle$

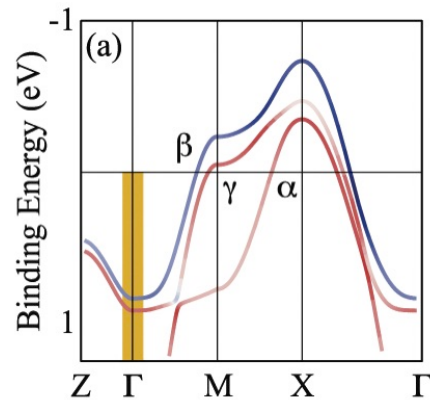
$$|\psi_{par}\rangle \approx |-1, \downarrow\rangle; |1, \uparrow\rangle \quad m_{j^*} = \pm 3/2$$

$$|\psi_{antipar}\rangle \approx |-1, \uparrow\rangle; |1, \downarrow\rangle \quad m_{j^*} = \pm 1/2$$

Selection rules: (mainly)  $\Delta l = -1$  ( $d \rightarrow p$ ) ;  $\Delta m_l = \pm 1$   
 (but  $|m_l| < 2$  for p states!)



# SP-ARPES - $\text{Sr}_2\text{RuO}_4$ : a spin-triplet superconductor?



**SO** is large ( $\sim 130$  meV);  $|m_l, m_s\rangle$  and  $| -m_l, -m_s\rangle$  states are degenerate and mixed

$$|\psi_{par}\rangle \approx a|-1, \downarrow\rangle + b|1, \uparrow\rangle$$

$$|\psi_{antipar}\rangle \approx c|-1, \uparrow\rangle + d|1, \downarrow\rangle$$

Local description:  
orbital and spin moments  
are entangled

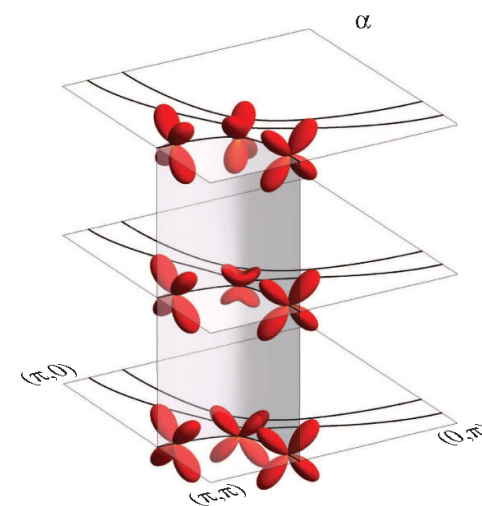
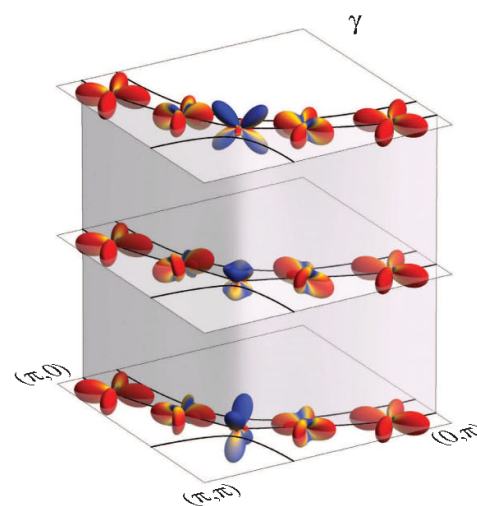
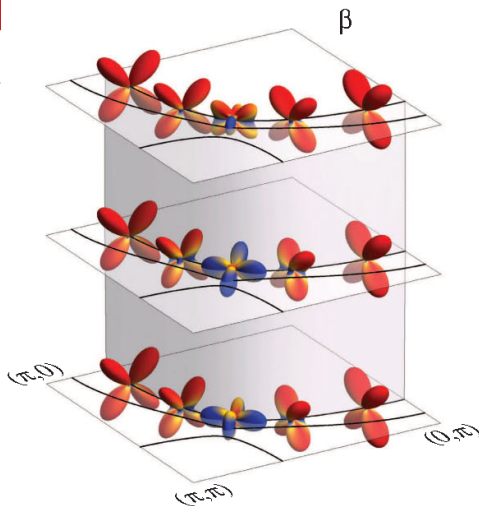
In the solid (one Kramers' partner):



$$k_z = 3\pi/4$$

$$k_z = 3\pi/8$$

$$k_z = 0$$

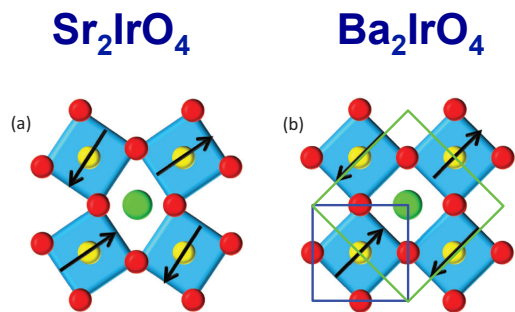


**It is not possible to associate a well-defined spin value to a state at the Fermi surface.**

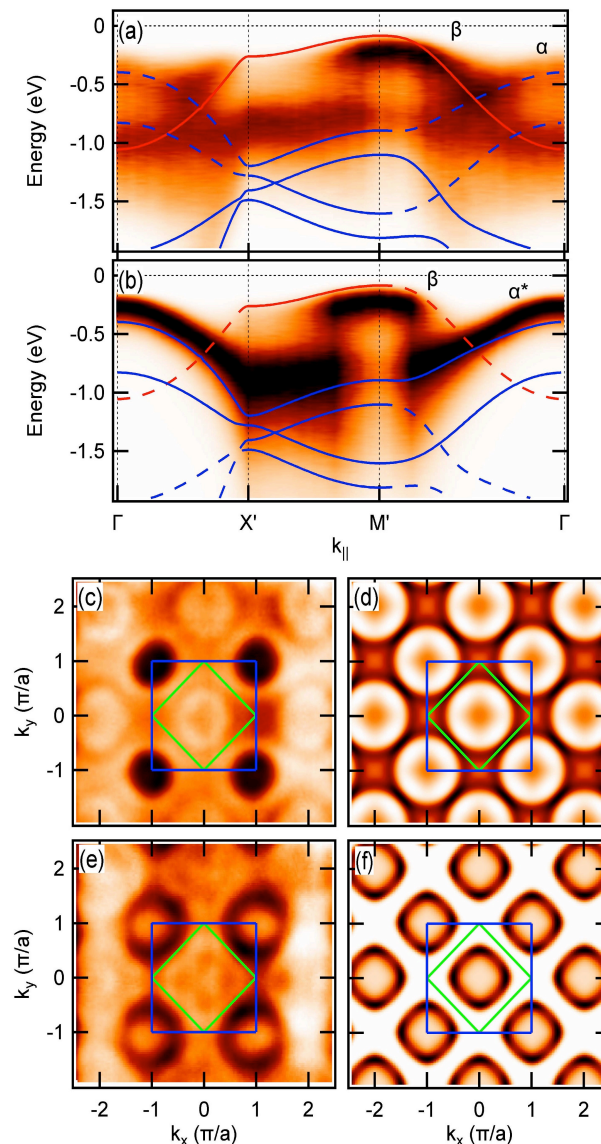
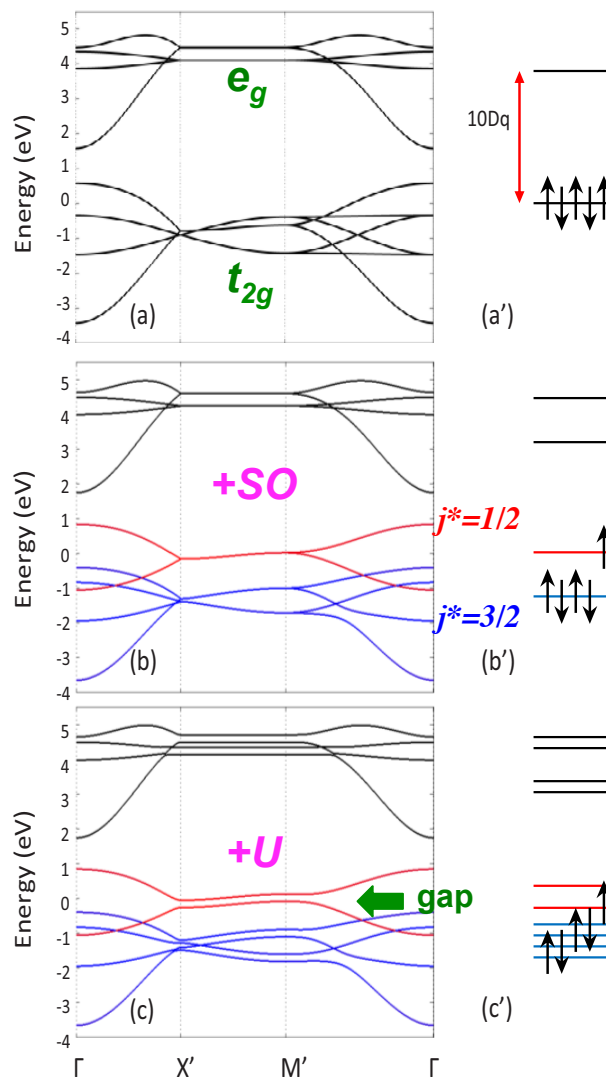
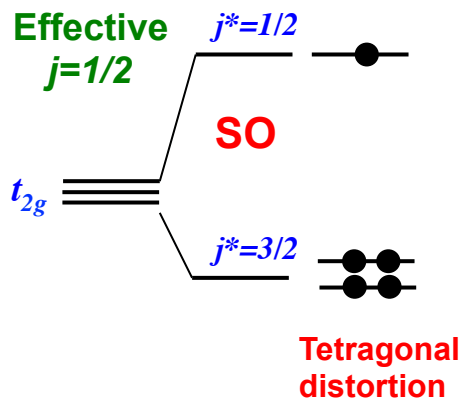
**Cooper pairs cannot be simply described as “singlets” or “triplets”**

C.N. Veenstra et al., PRL (2014)

# Strong SOI in 5d TM compounds: the iridates



$\text{Ir}^{4+}; t_{2g}^5$



Constant energy maps

Moser et al., NJP (2014)