Maiorana Ettore …the genius and the mystery

"There are several categories of scientists in the world; those of second or third rank do their best but never get very far. Then there is the first rank, those who make important discoveries, fundamental to scientific progress. But then there are the geniuses, like Galilei and Newton.

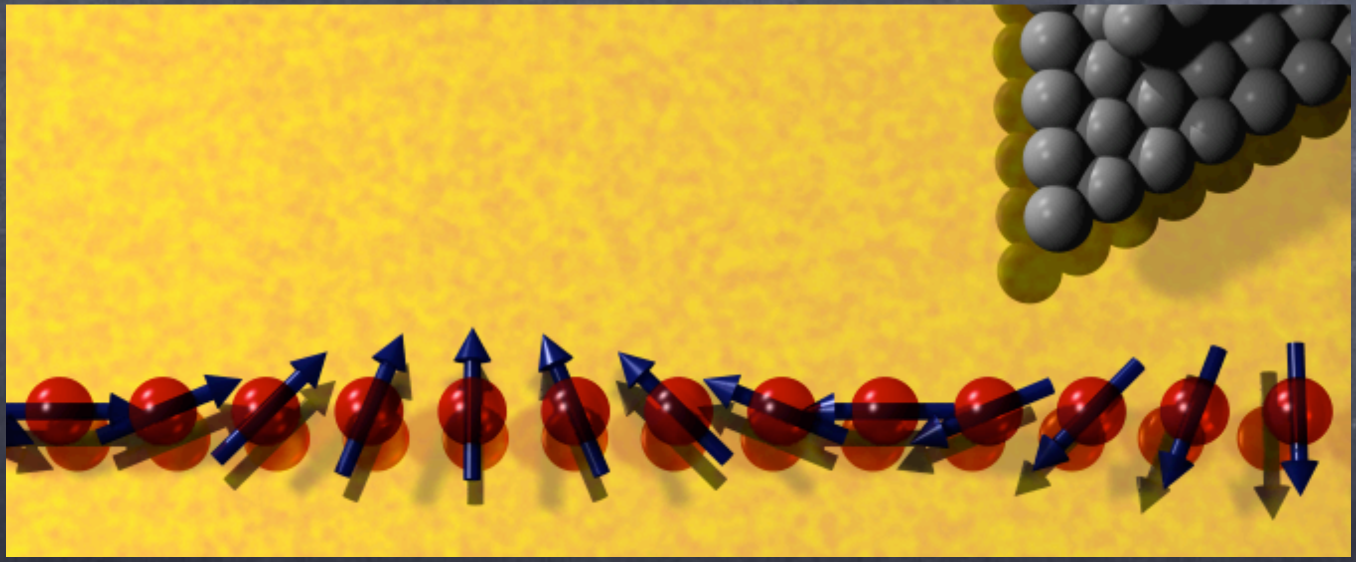
Majorana was one of these."

Enrico Fermi

Topological superconductors and Majorana fermions

M. Franz

University of British Columbia



Dirac fermions

$$i\dot{\psi} = (\boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m)\psi$$

Matter
(particles)

ψ

electrons, protons, neutrons

Paul Dirac (1902-84)

Antimatter
(antiparticles)

ψ^*

positrons, antiprotons,
antineutrons

particle $\neq$ antiparticle

Majorana fermions

$$i\dot{\psi} = (\tilde{\boldsymbol{\alpha}} \cdot \boldsymbol{p} + \tilde{\beta} m)\psi$$

particle = antiparticle

ψ

Ettore Majorana (1906-38)

ψ

REVIEWS OF MODERN PHYSICS, VOLUME 87, JANUARY–MARCH 2015

Colloquium: Majorana fermions in nuclear, particle, and solid-state physics

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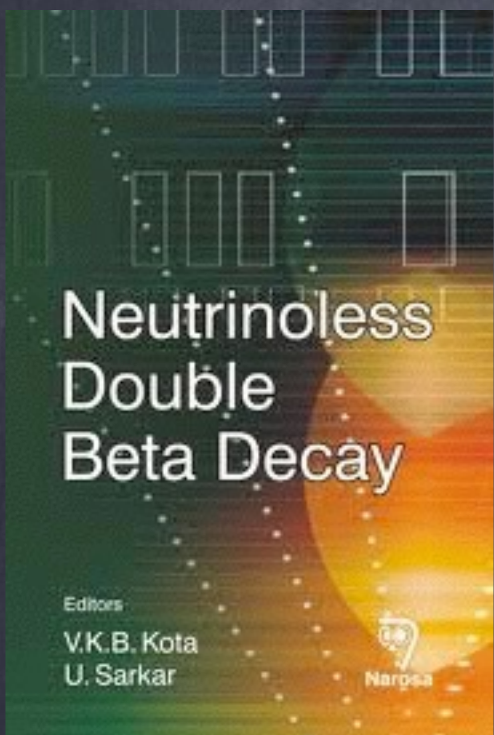
(published 11 February 2015)

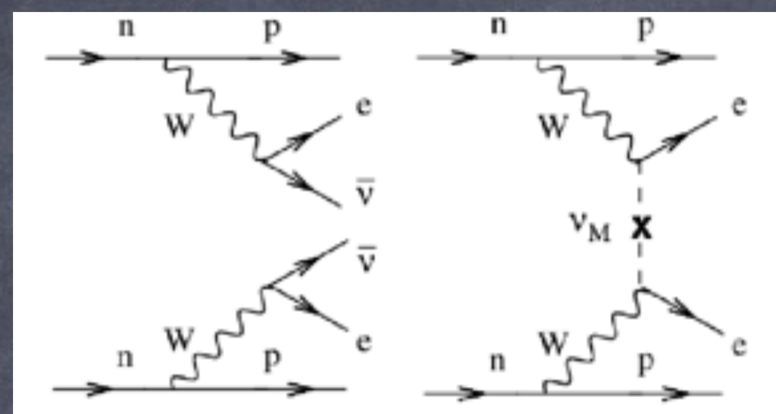
Ettore Majorana (1906–1938) disappeared while traveling by ship from Palermo to Naples in 1938. His fate has never been fully resolved and several articles have been written that explore the mystery itself. His demise intrigues us still today because of his seminal work, published the previous year, that established symmetric solutions to the Dirac equation that describe a fermionic particle that is its own antiparticle. This work has long had a significant impact in neutrino physics, where this fundamental question regarding the particle remains unanswered. But the formalism he developed has found many uses as there are now a number of candidate spin-$1/2$ neutral particles that may be truly neutral with no quantum number to distinguish them from their antiparticles. If such particles exist, they will influence many areas of nuclear and particle physics. Most notably the process of neutrinoless double beta decay can exist only if neutrinos are massive Majorana particles. Hence, many efforts to search for this process are underway. Majorana's influence does not stop with particle physics, however, even though that was his original consideration. The equations he derived also arise in solid-state physics where they describe electronic states in materials with superconducting order. Of special interest here is the class of solutions of the Majorana equation in one and two spatial dimensions at exactly zero energy. These Majorana zero modes are endowed with some remarkable physical properties that may lead to advances in quantum computing and, in fact, there is evidence that they have been experimentally observed. This Colloquium first summarizes the basics of Majorana's theory and its implications. It then provides an overview of the rich experimental programs trying to find a fermion that is its own antiparticle in nuclear, particle, and solid-state physics.

DOI: 10.1103/RevModPhys.87.137 PACS numbers: 71.10.Pm, 14.80.Va

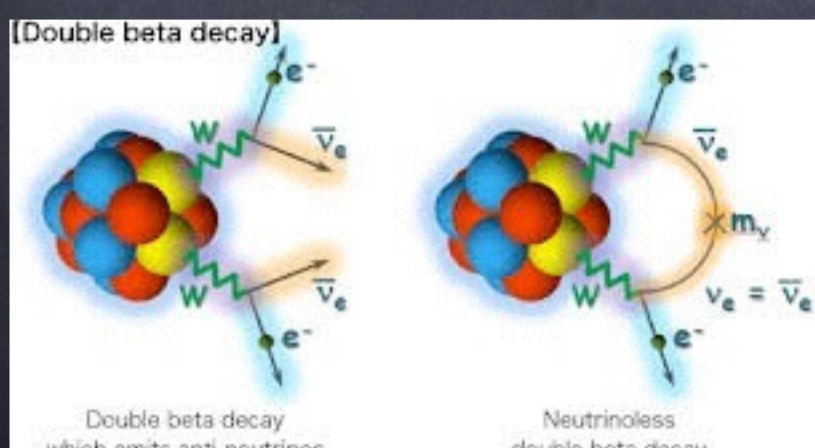
For a more technical discussion see RMP review article arXiv:1403.4976

Principal high-energy physics candidate for Majorana fermion: neutrino





This is expected to occur in certain isotopes, e.g. ^{76}Ge, ^{130}Te, ^{138}Xe but half-life is extremely long ~10^{25} years!



NATURE PHYSICS | VOL 5 | SEPTEMBER 2009 | www.nature.com/naturephysics

perspective

Majorana returns

Frank Wilczek

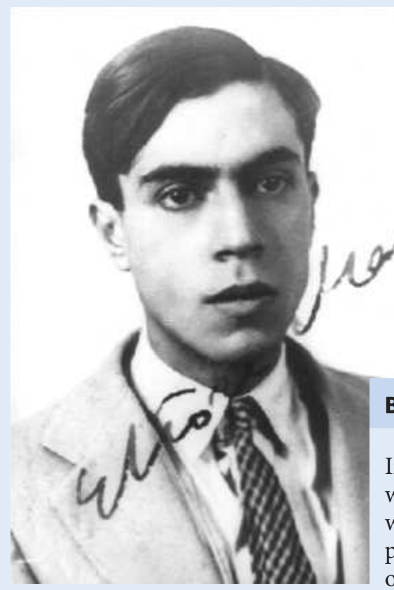
In his short care
of 'Majorana fer
modern physics

Enrico Fermi had
Ettore Majorana
his big idea: a m
Dirac equation that v
ramifications for par
afterwards, in 1938, I
disappeared, and for
equation remained a
footnote in theoretic
Now suddenly, it seer
concept is ubiquitous
is central to recent w
neutrino physics, sup
matter, but also on sc
ordinary matter.

Box 1 | The romance of Ettore Majorana

"There are many categories of scientists: people of second and third rank, who do their best, but do not go very far; there are also people of first-class rank, who make great discoveries, fundamental to the development of science. But then there are the geniuses, like Galileo and Newton. Well Ettore Majorana was one of them." Enrico Fermi, not known for flightiness or overstatement, is the source of these much-quoted lines.

The bare facts of Majorana's life are briefly told. Born in Catania, Italy, on 5 August 1906, into an accomplished family, he rose rapidly through the academic ranks, became a friend and scientific collaborator of Fermi, Werner Heisenberg and other luminaries, and produced a stream of high-quality papers. Then, beginning in 1933, things started to go terribly wrong. He complained of gastritis, became reclusive, with no official position, and published nothing for several years. In 1937, he allowed Fermi to write-up and submit, under his (Majorana's) name, his last and most profound paper — the point of departure of this article — containing results he had derived some years before. At Fermi's urging, Majorana applied for professorships and was awarded the Chair in Theoretical Physics at Naples,



RECAMI AND E. MAJORANA.

which he took up in January 1938. Tv
months later, he embarked on a myste
trip to Palermo, arrived, then boardec
straight back to Naples and disappear
without a trace.

Majorana published only nine pap
in his lifetime, none very lengthy. Th
are collected, with commentaries, all
both Italian and English versions, in
volume[30]. Each is a substantial contri
to quantum physics. At least two are

masterpieces: the last, as mentioned, and another on the quantum theory of spins in magnetic fields, which anticipates the later brilliant development of molecular-beam and magnetic resonance techniques.

In recent years, a small industry has developed, bringing Majorana's unpublished notebooks into print (see for example ref. 31). They are impressive documents, full of original calculations and expositions covering a wide range of physical problems. They leave an overwhelming impression of gathering

Box 2 | The Majorana equation

In 1928, Dirac proposed his relativistic wave equation for electrons[33]. This was a watershed event in theoretical physics, leading to a new understanding of spin, predicting the existence of antimatter, and impelling — for its adequate interpretation — the creation of quantum field theory. It also inaugurated a new method in theoretical physics, emphasizing mathematical aesthetics as a source of inspiration. Majorana's most influential work is especially poetic, in that it applies Dirac's method to Dirac's equation itself, to distill from it an equation both elegant and new. For many years, Majorana's idea seemed to be an ingenious but unfulfilled speculation. Recently, however, it has come into its own, and now occupies a central place in several of the most vibrant frontiers of modern physics.

Dirac's equation connects the four components of a field ψ. In modern (covariant) notation it reads

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

The γ matrices are required to obey the rules of Clifford algebra, that is

$$\{\gamma^\mu \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$$

where $\eta^{\mu\nu}$ is the metric tensor of flat space. Spelling it out, we have

$$(\gamma^0)^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = -(\gamma^3)^2 = 1$$
$$\gamma^j\gamma^k = -\gamma^k\gamma^j \text{ for } i \neq j$$

(in which I have adopted units such that $\hbar = c = 1$). Furthermore, we require that γ^0 be Hermitian, and the remaining marices anti-Hermitian. These conditions ensure that the equation properly describes the wavefunction of a spin-½ particle with mass m.

Dirac found a suitable set of 4 × 4 γ matrices, whose entries contain both real and imaginary numbers. For the equation to make sense, ψ must then be a complex field. Dirac and most other physicists regarded this consequence as a good feature, because electrons are electrically charged, and the description of charged particles requires complex fields, even at the level of the Schrödinger equation. This is also true in the language of quantum field theory. In quantum field theory, if a given field φ creates the particle A (and destroys its antiparticle $\bar{A}$), the complex conjugate φ^* will create $\bar{A}$ and destroy A. Particles that are their own antiparticles must be associated with fields obeying $\varphi = \varphi^*$, that is, real fields. Because electrons and positrons are distinct, the associated fields ψ and ψ^* and must therefore be different; this feature appeared naturally in Dirac's equation.

Majorana inquired whether it might be possible for a spin-½ particle to be its own antiparticle, by attempting to find the equation that such an object would satisfy. To get an equation of Dirac's type (that is, suitable for spin-½) but capable of governing a real field, requires γ matrices that satisfy the Clifford algebra and are purely imaginary. Majorana found such matrices. Written as tensor products of the usual Pauli matrices σ, they take the form:

$$\tilde{\gamma}^0 = \sigma_2 \otimes \sigma_1$$
$$\tilde{\gamma}^1 = i\sigma_1 \otimes 1$$
$$\tilde{\gamma}^2 = i\sigma_3 \otimes 1$$
$$\tilde{\gamma}^3 = i\sigma_2 \otimes \sigma_2$$

or alternatively, as ordinary matrices:

$$\tilde{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^2 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$$\tilde{\gamma}^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

Majorana's equation, then, is simply

$$(i\tilde{\gamma}^\mu \partial_\mu - m)\psi = 0)$$

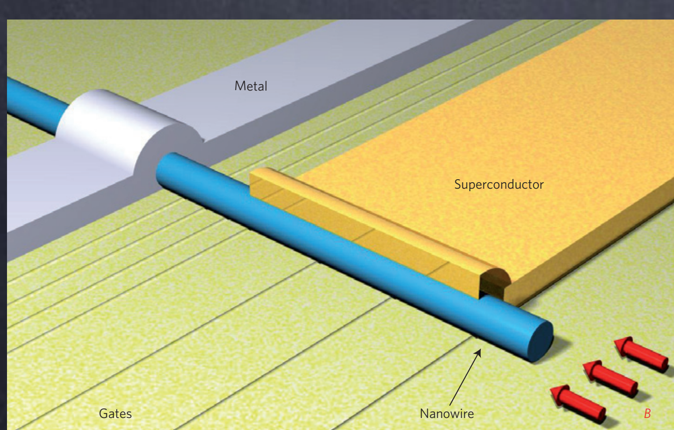
Because the $\tilde{\gamma}^\mu$ matrices are purely imaginary, the matrices $i\tilde{\gamma}^\mu$ are real, and consequently this equation can govern a real field ψ.

NATURE|Vol 464|11 March 2010|doi:10.1038/nature08915 REVIEW INSIGHT

Non-Abelian states of matter

Ady Stern[1]

Quantum mechanics classifies all elementary particles as either fermions or bosons, and this classification is crucial to the understanding of a variety of physical systems, such as lasers, metals and superconductors. In certain two-dimensional systems, interactions between electrons or atoms lead to the formation of quasiparticles that break the fermion–boson dichotomy. A particularly interesting alternative is offered by 'non-Abelian' states of matter, in which the presence of quasiparticles makes the ground state degenerate, and interchanges of identical quasiparticles shift the system between different ground states. Present experimental studies attempt to identify non-Abelian states in systems that manifest the fractional quantum Hall effect. If such states can be identified, they may become useful for quantum computation.



Physics

Physics **3**, 24 (2010)

Viewpoint

Race for Majorana fermions

Marcel Franz
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Published March 15, 2010

The race for realizing Majorana fermions—elusive particles that act as their own antiparticles—heats up, but we still await ideal materials to work with.

Subject Areas: **Semiconductor Physics, Mesoscopics, Particles and Fields**

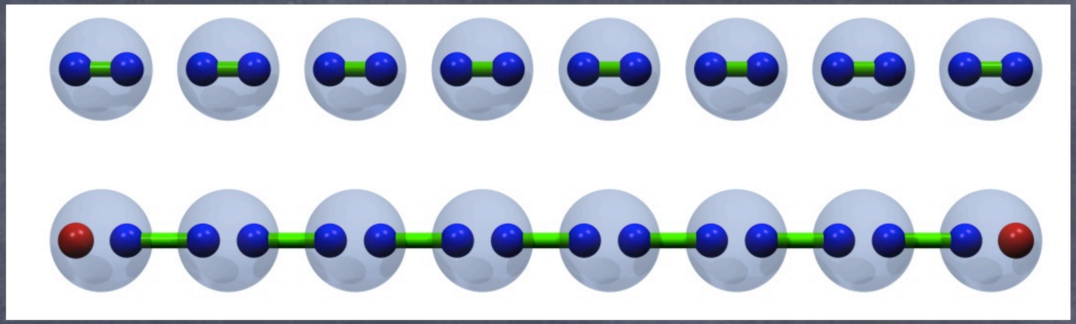
Majorana fermions vs. Majorana zero modes

(for experts)

- neutrinos in particle physics (?)
- electrons in superconductors

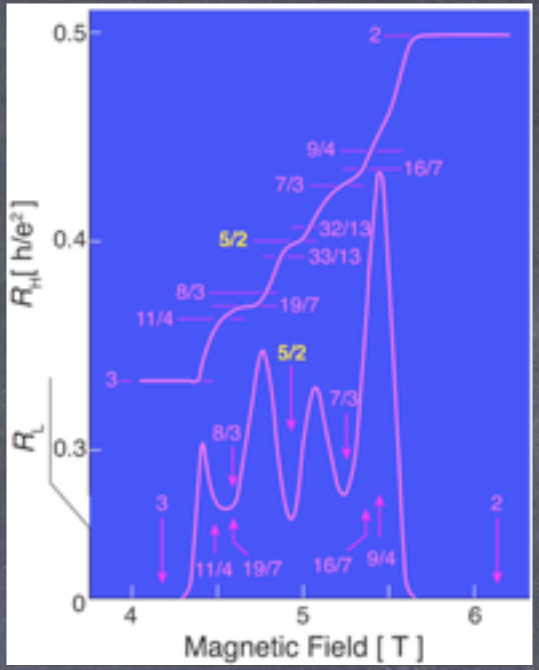
- special case of Majorana fermion of interest in CM physics
- occur as zero-energy excitations in 1D and 2D topological SC
- obey non-Abelian exchange statistics

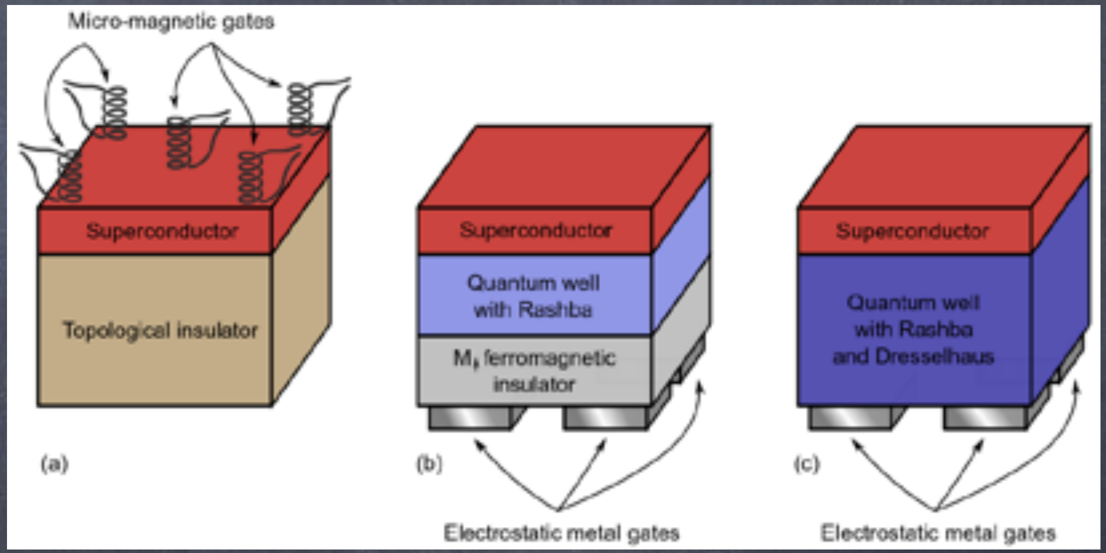
Majorana fermions in solid-state systems

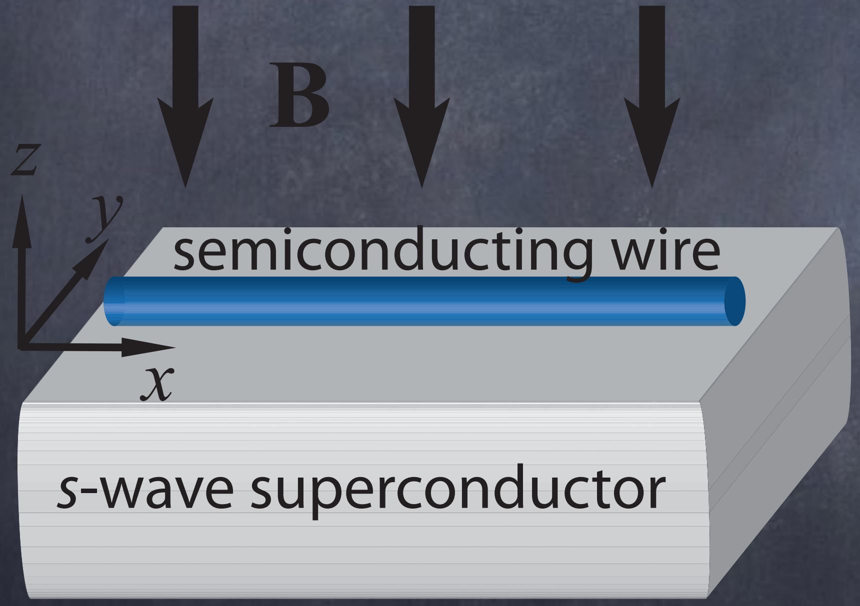


- Majorana fermions - particles that are identical to their antiparticles
- Can occur as collective excitations in solids with unconventional superconducting order.
- Majorana zero modes: Obey non-abelian exchange statistics and can serve as a platform for fault-tolerant quantum computation.

- Proposed realizations:
  a. Moore-Read FQHE
  b. Spin-polarized p+ip superconductor
  c. TI/SC interface
  d. Rashba-coupled semicond. + SC + magnetic insulator
  e. 1D quantum wires







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Majorana fermions
in the solid state context

Ordinary fermions $\{c_i^\dagger, c_j\} = \delta_{ij}$

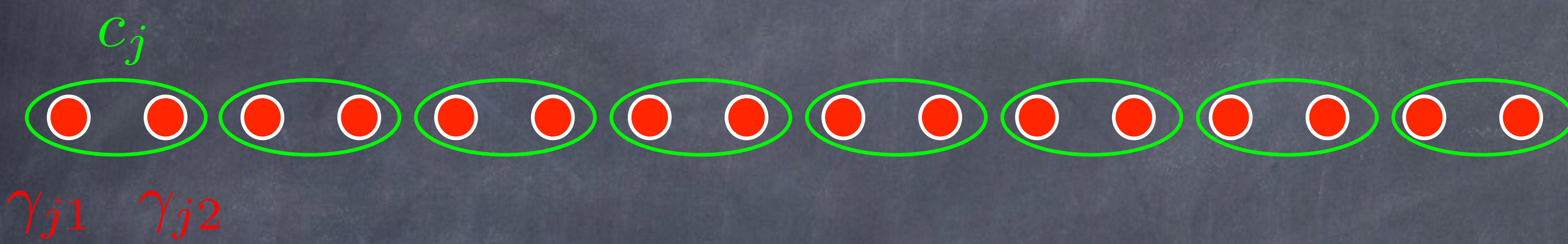
Write in terms of Majorana fermions: $$\boxed{c_j = (\gamma_{j1} + i\gamma_{j2})/2}$$

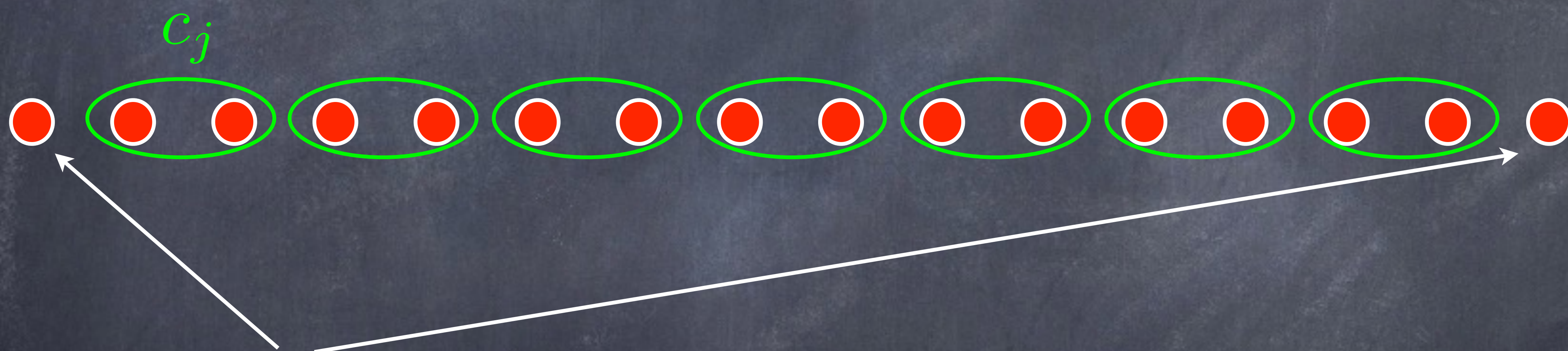
$$\{\gamma_{i\alpha}, \gamma_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \gamma_{i\alpha}^\dagger = \gamma_{i\alpha}$$

Canonical transformation: can be used to recast ANY fermionic Hamiltonian in terms of Majorana operators

Certain Hamiltonians can support solutions with isolated localized Majorana fermions

Example: `Kitaev 1D model' [Phys. Usp. 44, 131 (2001)]





isolated, unpaired Majoranas

These also encode one complex fermion but in a way that is robust to any local perturbation --> ideal quantum bit.

Kitaev model details:

$$\mathcal{H} = \sum_j \left[-t(c_j^\dagger c_{j+1} + \text{h.c.}) + (\Delta c_j^\dagger c_{j+1}^\dagger + \text{h.c.}) \right]$$

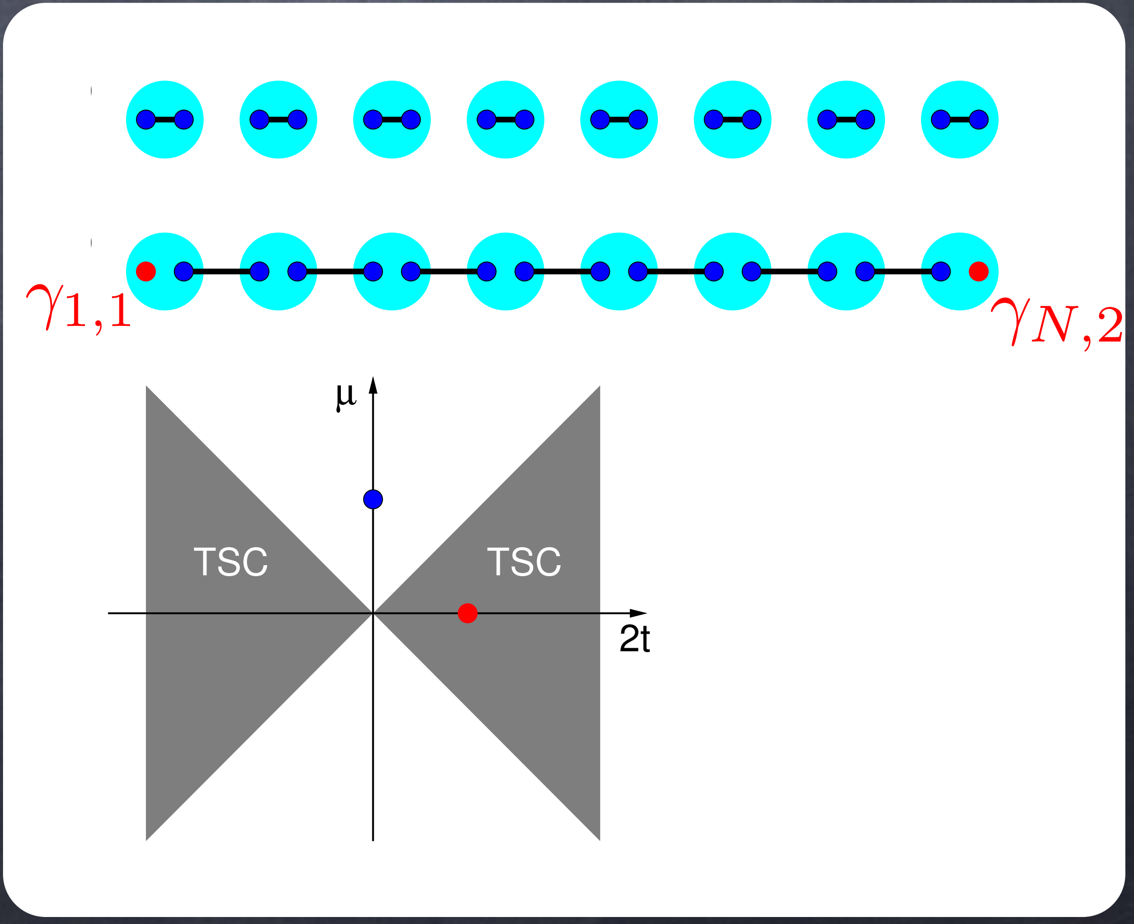
Transform to Majorana basis $c_j = (\gamma_{j1} + i\gamma_{j2})/2$

$$\mathcal{H} = \frac{i}{2} \sum_j [(t+\Delta)\gamma_{j2}\gamma_{j+1,1} + (-t+\Delta)\gamma_{j1}\gamma_{j+1,2}]$$

Focus on a special point $t = \Delta$

$$\mathcal{H} = it \sum_{j=1}^{N-1} \gamma_{j2}\gamma_{j+1,1}$$

$\gamma_{1,1}$ and $\gamma_{N,2}$ missing!

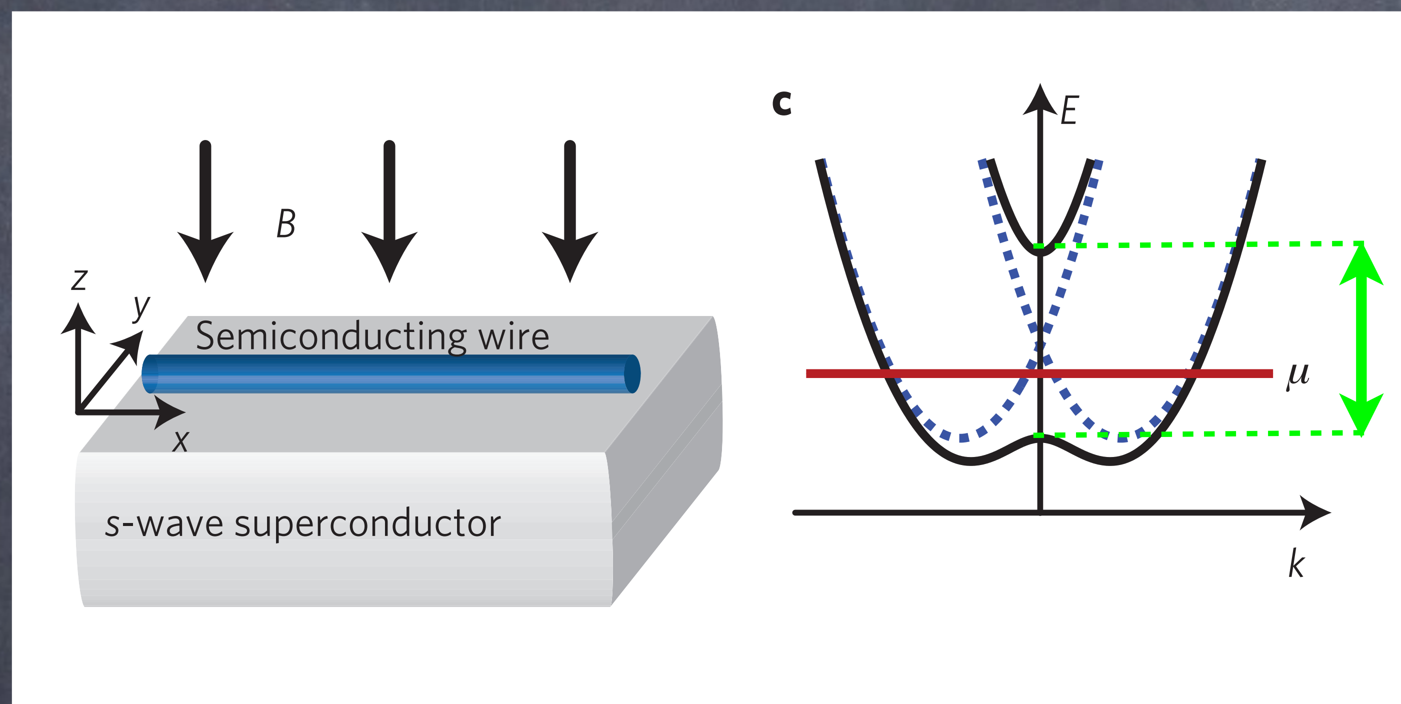


Rashba-coupled semiconductor quantum wire

(a physical realization of the Kitaev chain)

Lutchyn et al. PRL 2010, Oreg et al. PRL 2010

$$H_0 = \int_{-\infty}^{\infty} dx\, \psi_\sigma^\dagger(x) \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'}(x),$$



1-10K

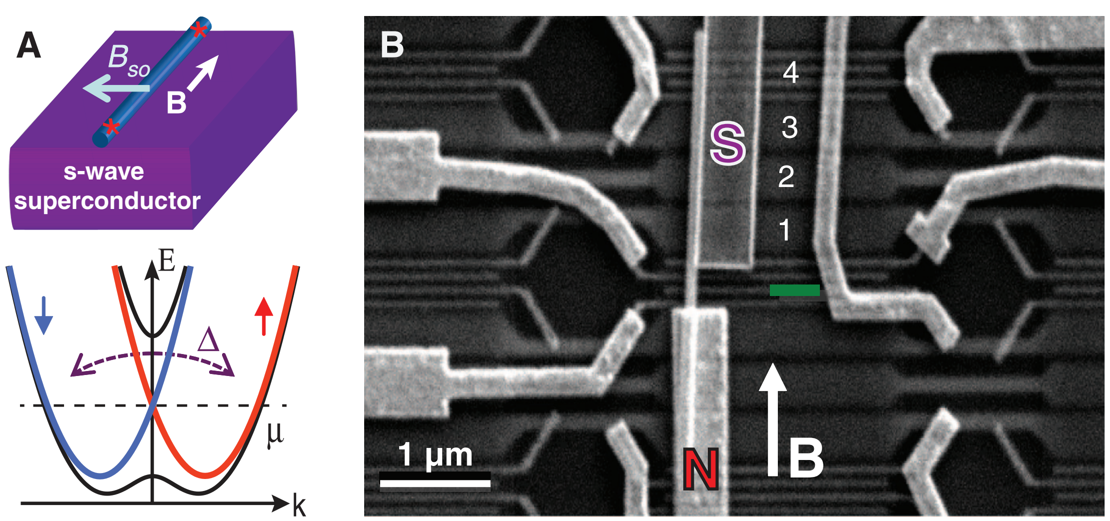
Potential issues:

- Chemical potential tuning
- Effects of disorder
- Detection

Experimental realizations

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

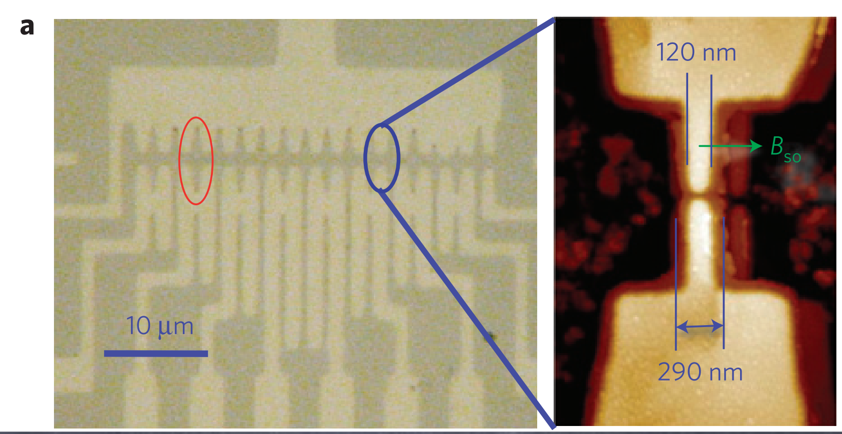
V. Mourik,[1*] K. Zuo,[1*] S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1†]

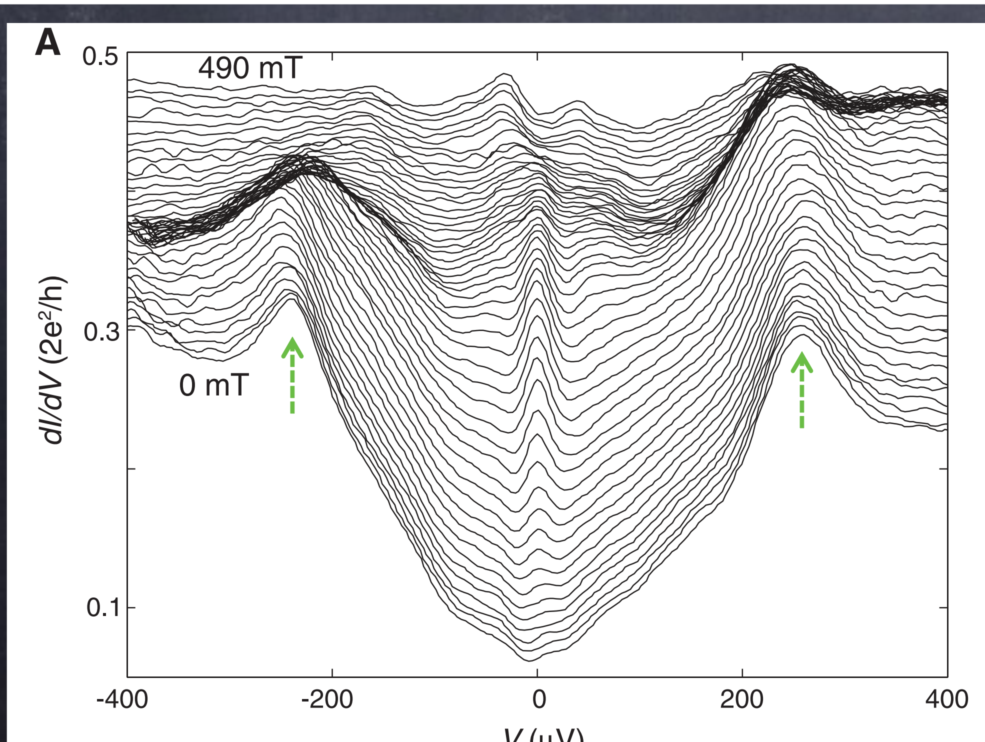


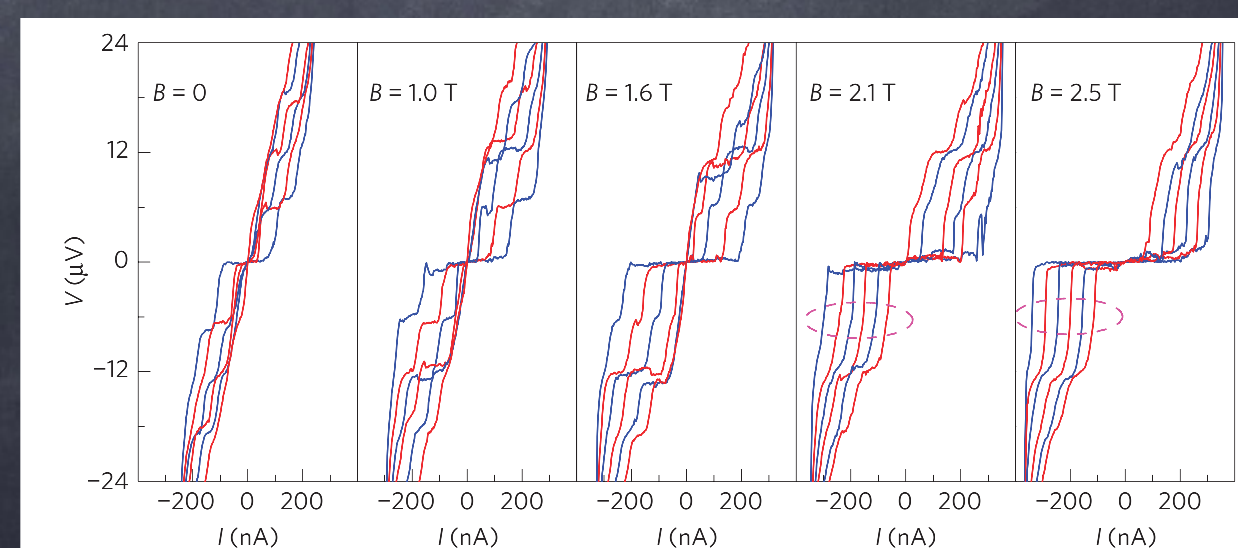
nature physics
LETTERS
PUBLISHED ONLINE: 23 SEPTEMBER 2012 | DOI: 10.1038/NPHYS2429

The fractional a.c. Josephson effect in a semiconductor–superconductor nanowire as a signature of Majorana particles

Leonid P. Rokhinson[1★], Xinyu Liu[2] and Jacek K. Furdyna[2]

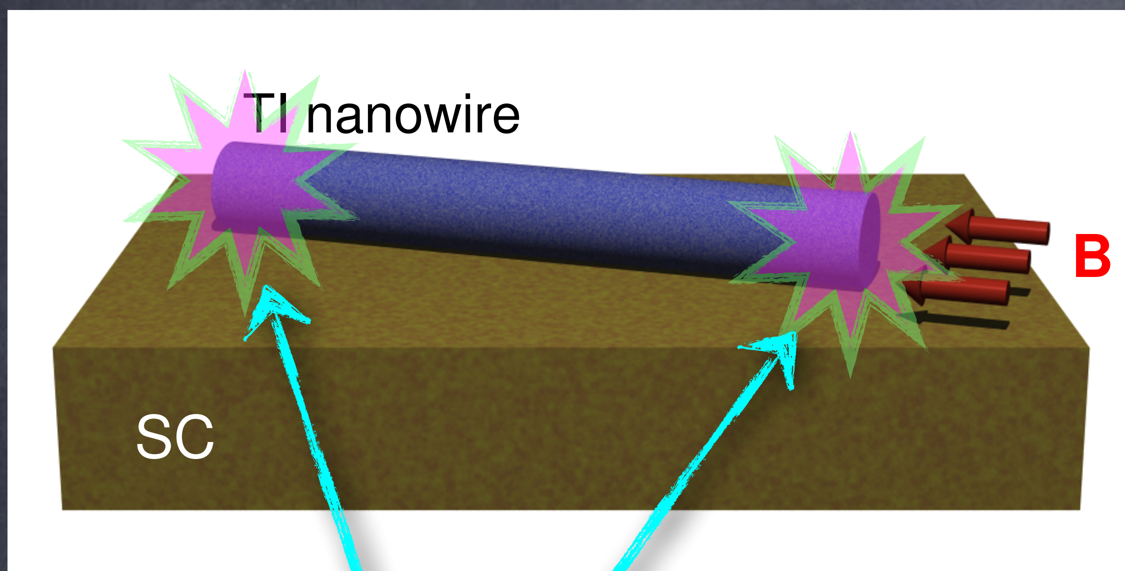






Other proposals: topological insulator nanowire

[A. Cook and M. Franz, Phys. Rev. B 84, 201105R (2011)]



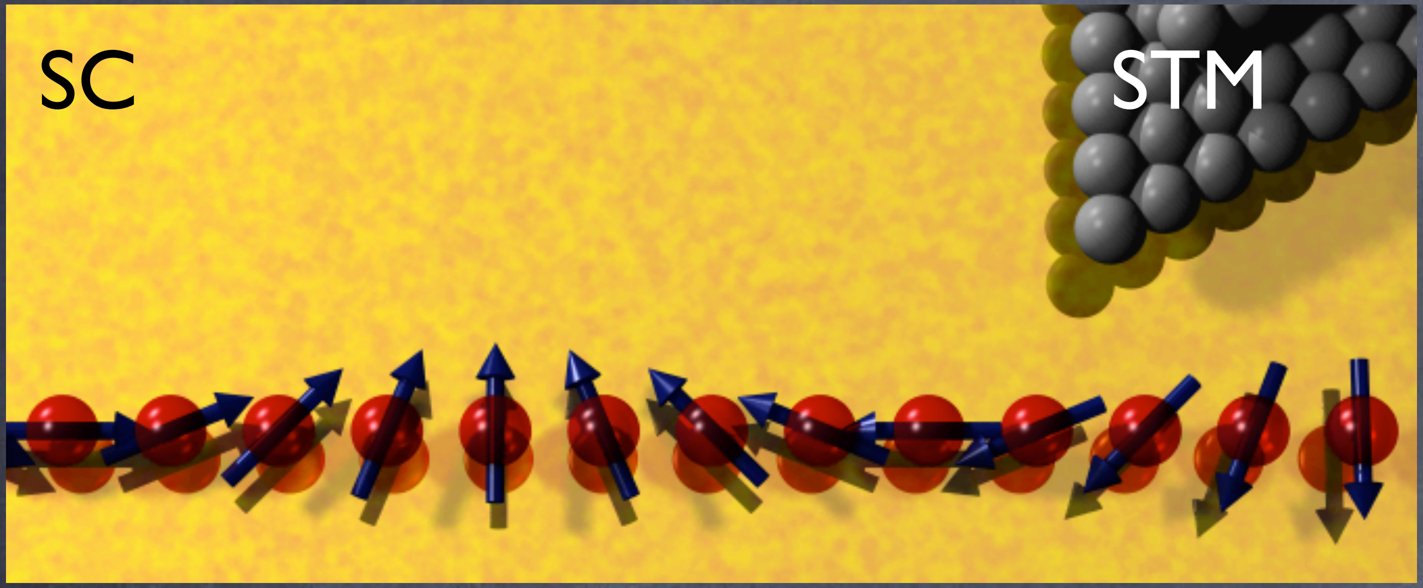
Topological insulator nanowire placed on top of an ordinary s-wave SC in longitudinal applied magnetic field.

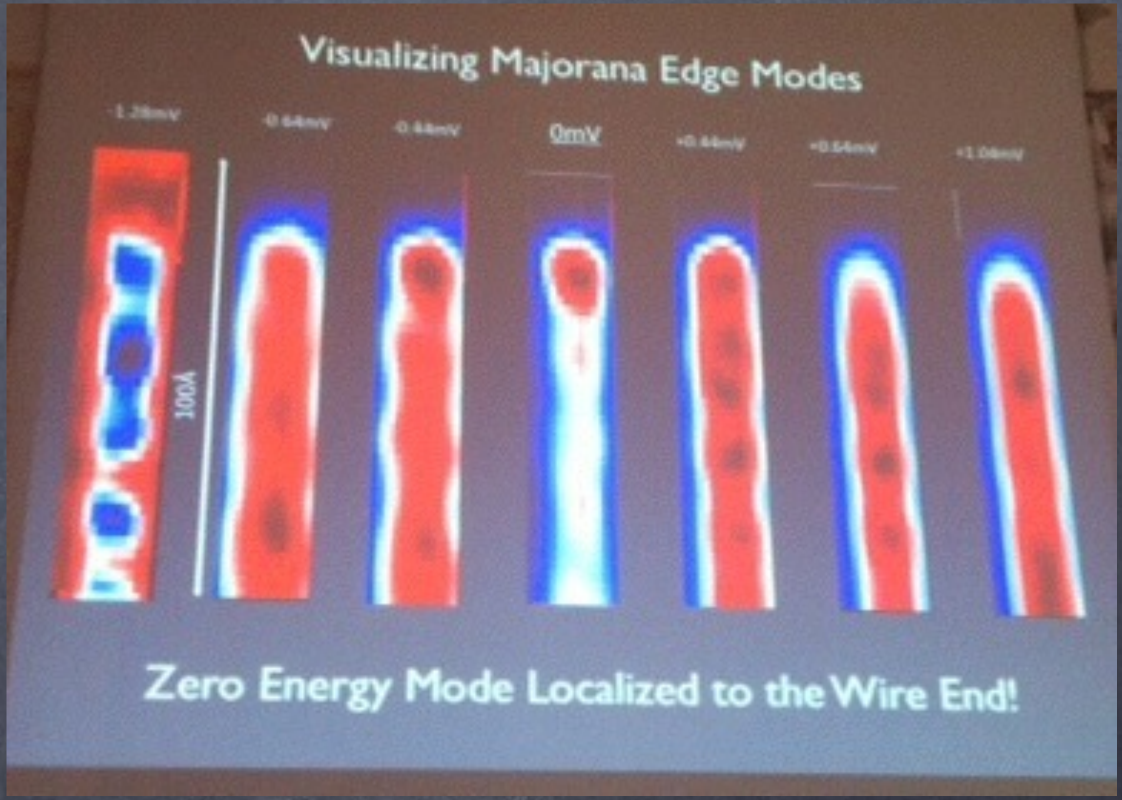
Majorana fermions

No fine tuning:

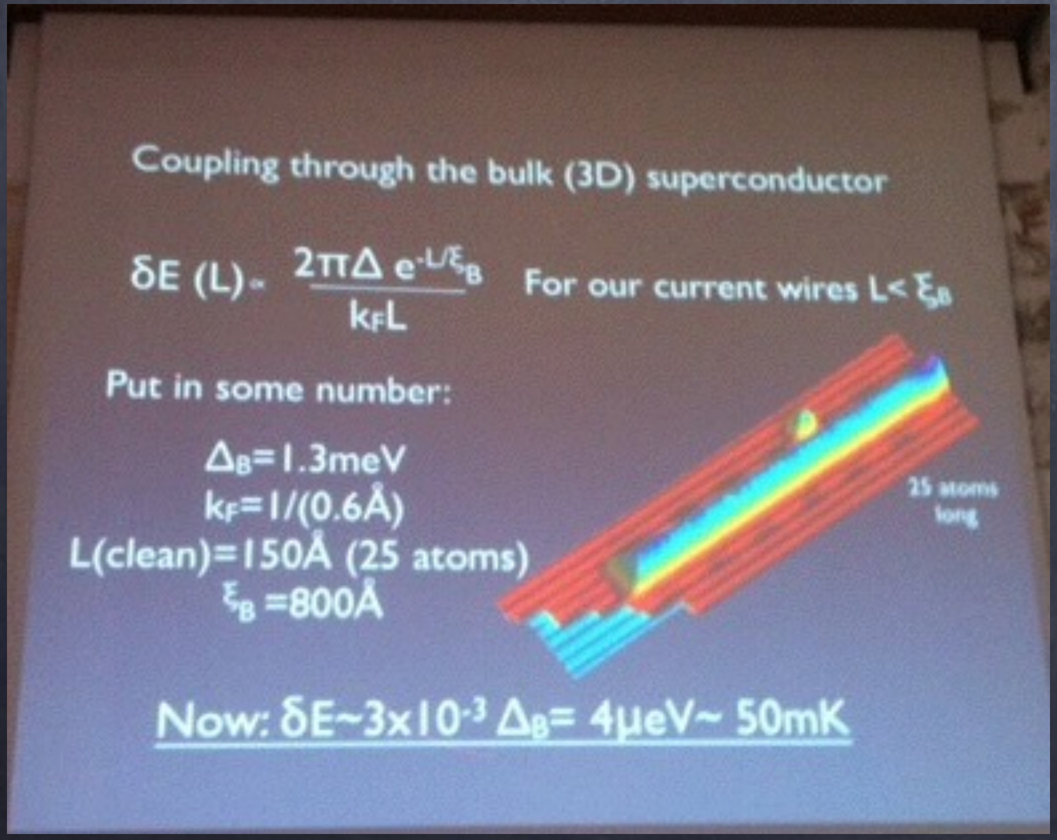
1. Chemical potential inside the bulk gap (~300meV in Bi_2Se_3).
2. Flux close to 1/2 flux quantum.
3. Robust against non-magnetic disorder.

A chain of magnetic atoms deposited on the surface of an ordinary s-wave superconductor [S. Nadj-Perge, I.K. Drozdov, B.A. Bernevig, and A. Yazdani, PRB 88, 020407 (2013); M.M. Vazifeh & M.F., PRL 111, 206802 (2013)]





A. Yazdani, Erice talk July 16, 2013



RESEARCH ARTICLES

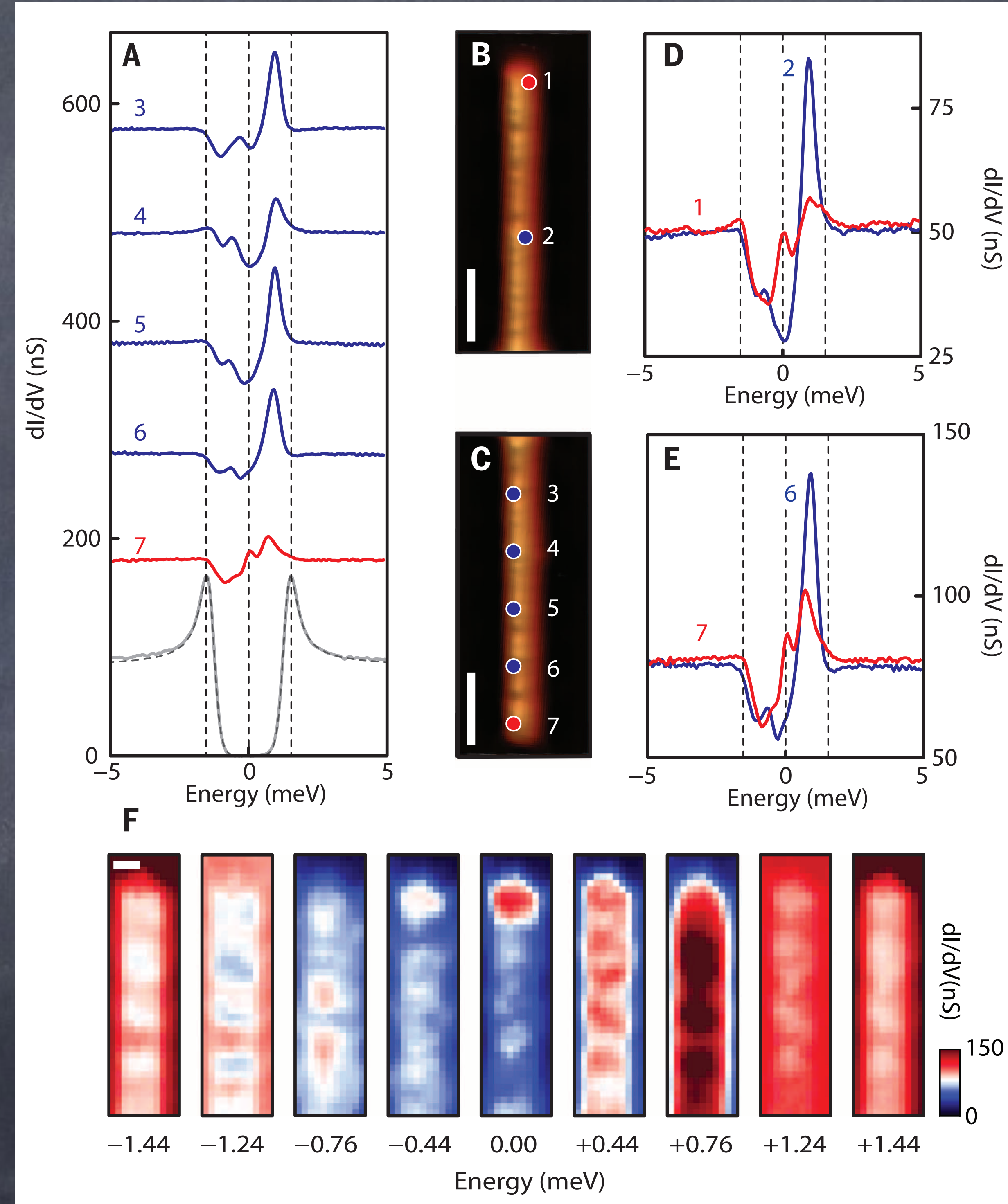
Science AAAS

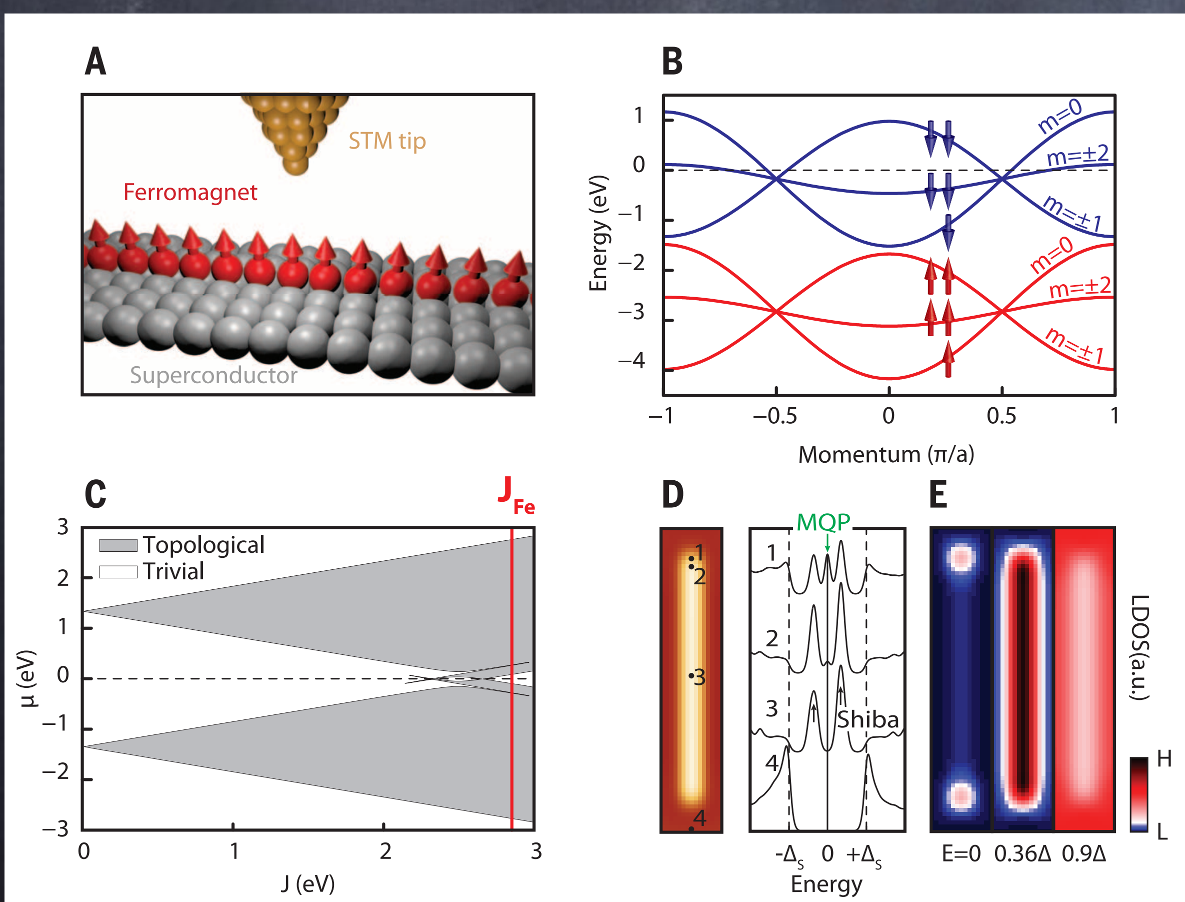
TOPOLOGICAL MATTER

Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor

Stevan Nadj-Perge,[1*] Ilya K. Drozdov,[1*] Jian Li,[1*] Hua Chen,[2*] Sangjun Jeon,[1] Jungpil Seo,[1] Allan H. MacDonald,[2] B. Andrei Bernevig,[1] Ali Yazdani[1]†

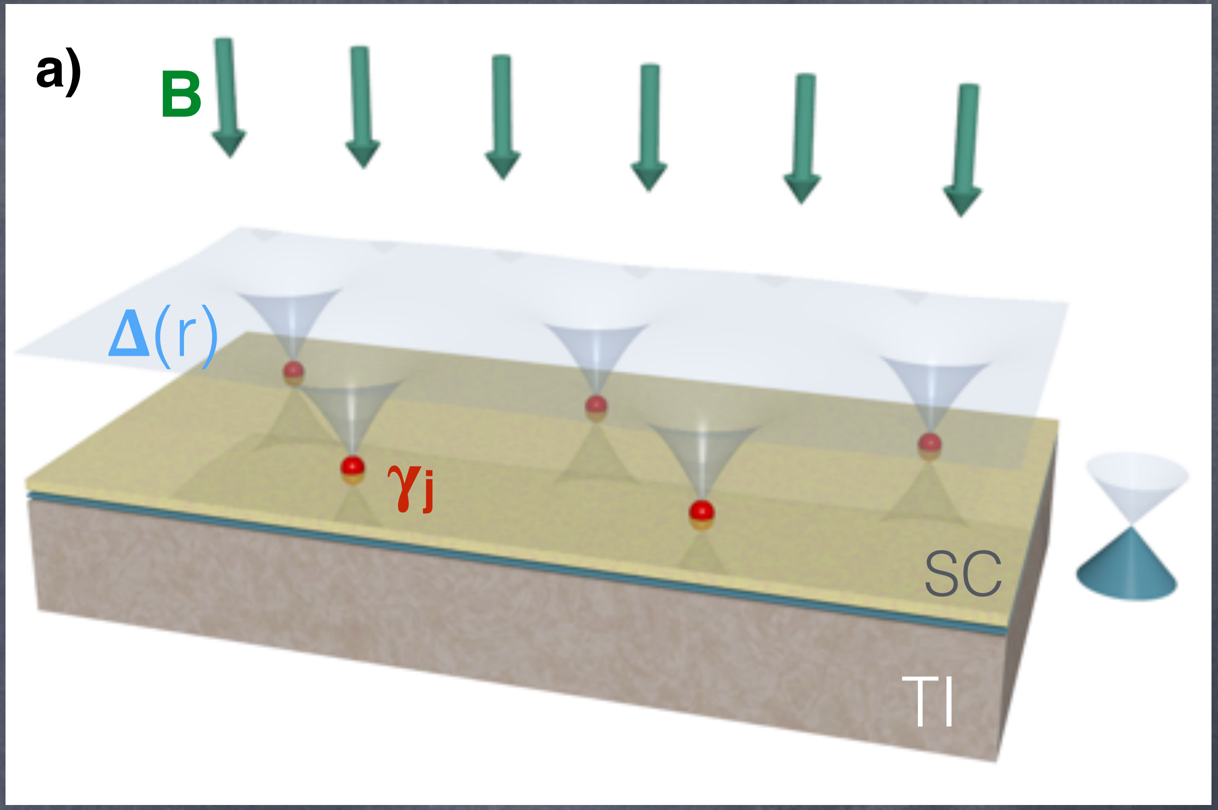
Majorana fermions are predicted to localize at the edge of a topological superconductor, a state of matter that can form when a ferromagnetic system is placed in proximity to a conventional superconductor with strong spin-orbit interaction. With the goal of realizing a one-dimensional topological superconductor, we have fabricated ferromagnetic iron (Fe) atomic chains on the surface of superconducting lead (Pb). Using high-resolution spectroscopic imaging techniques, we show that the onset of superconductivity, which gaps the electronic density of states in the bulk of the Fe chains, is accompanied by the appearance of zero-energy end-states. This spatially resolved signature provides strong evidence, corroborated by other observations, for the formation of a topological phase and edge-bound Majorana fermions in our atomic chains.





The Fu-Kane superconductor (2D)

Surface of a 3D TI interfaced with an ordinary s-wave superconductor.



For experts: this is similar (although not identical) to the spin-polarized p_x+ip_y superconductor as may be realized in $SrRu_2O_4$

PRL **114,** 017001 (2015) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2015

Experimental Detection of a Majorana Mode in the core of a Magnetic Vortex inside a Topological Insulator-Superconductor $Bi_2Te_3/NbSe_2$ Heterostructure

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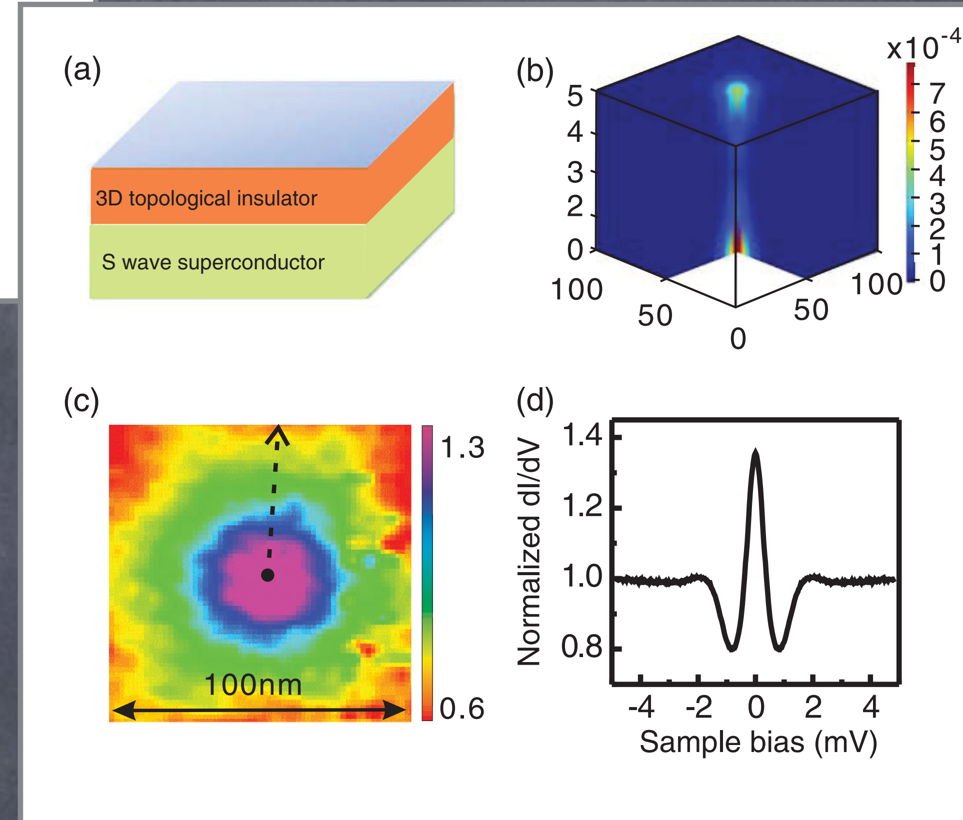
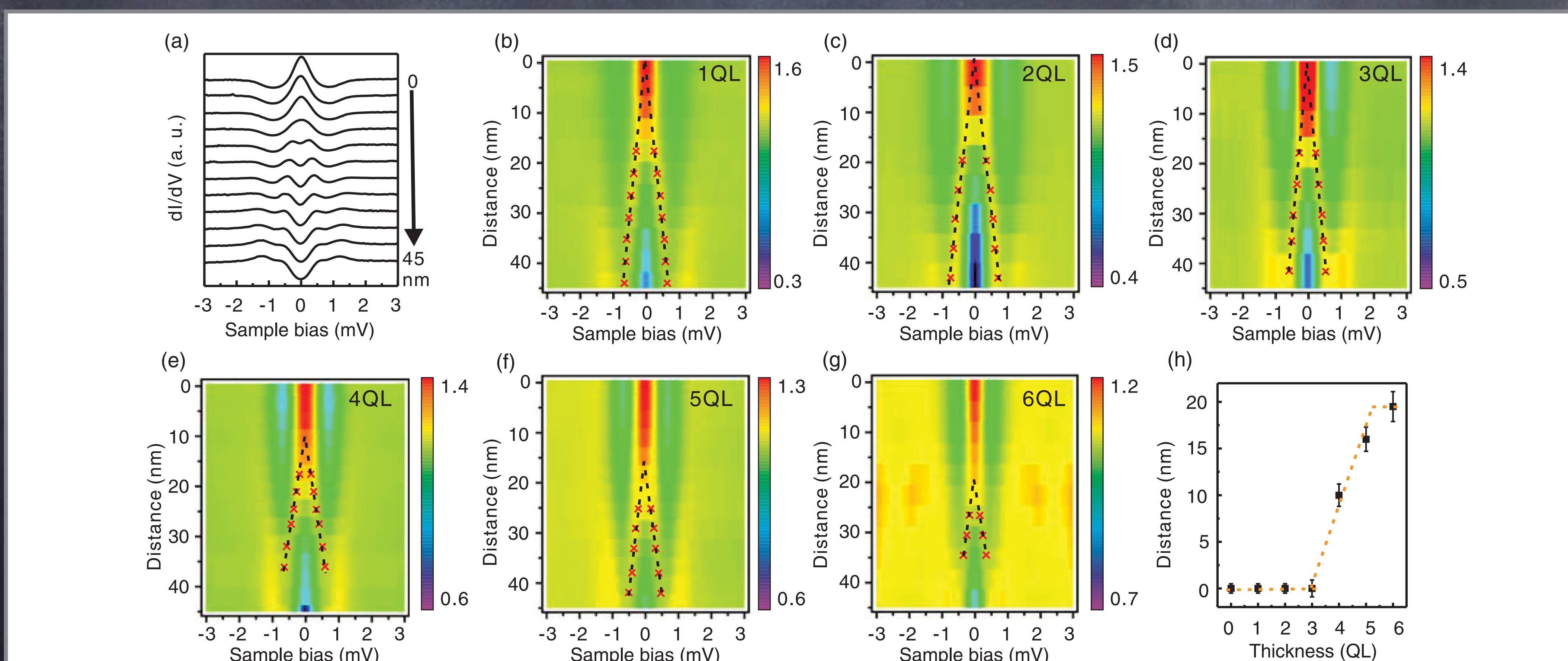
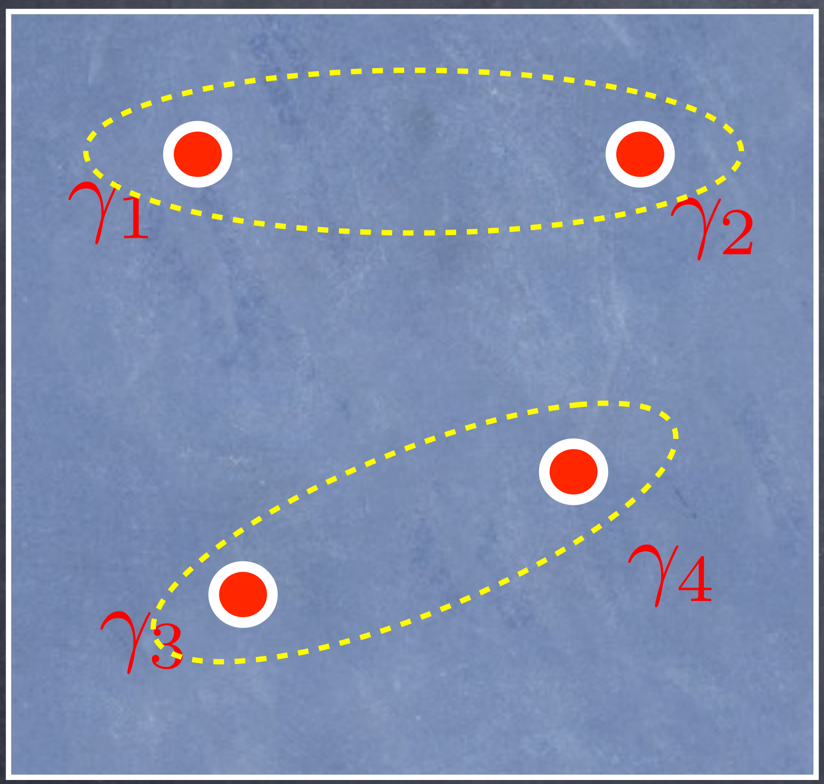




Majorana fermions:
Ground state degeneracy and non-Abelian exchange statistics



Define two fermions

$$c_a = \frac{1}{2}(\gamma_1 + i\gamma_2) \qquad c_b = \frac{1}{2}(\gamma_3 + i\gamma_4)$$

and label the resulting Hilbert space by eigenvalues $n_{a,b} = 0, 1$

$$\hat{n}_a = c_a^\dagger c_a = \frac{1}{2}(1 + i\gamma_1\gamma_2)$$

$$\hat{n}_b = c_b^\dagger c_b = \frac{1}{2}(1 + i\gamma_3\gamma_4)$$

$|n_a, n_b\rangle : \quad |0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle$ 4-fold degenerate ground state

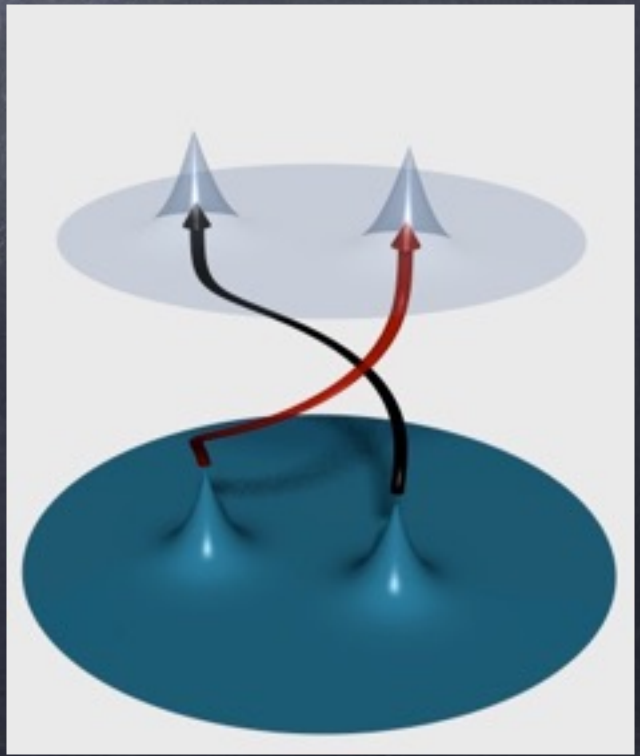
More generally, in the presence of 2N Majorana zero modes the ground state exhibits 2^{N-1}-fold degeneracy:

$$|\Phi_{\{n_j\}}\rangle = |n_1, n_2, \ldots n_N\rangle$$

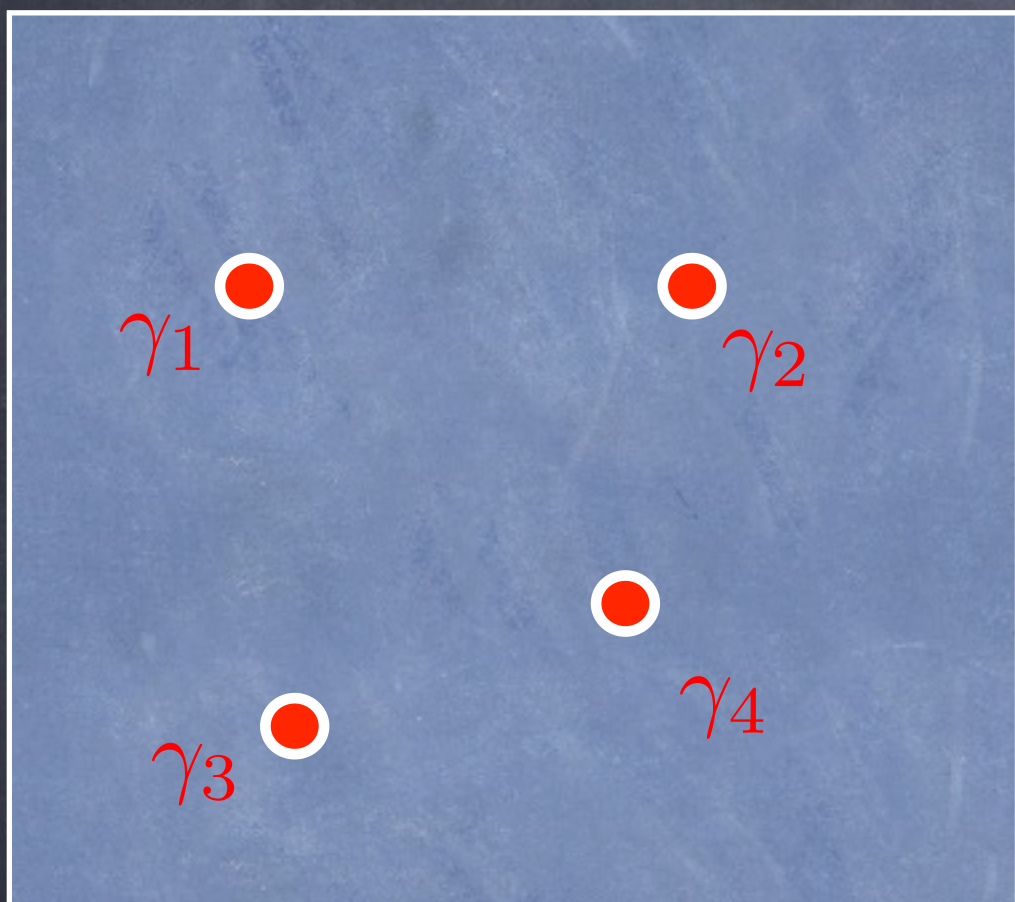
- One can imagine encoding quantum information into the ground-state wavefunction in a way that is protected against environmental decoherence

$$|\Psi\rangle = \sum C_{\{n_j\}} |\Phi_{\{n_j\}}\rangle$$

- One can furthermore manipulate this quantum information by braiding the Majorana fermions



Exchange and braiding



Upon exchange the wavefunction transforms as

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \mapsto \Psi(\mathbf{r}_2, \mathbf{r}_1) \quad \text{(bosons)}$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \mapsto -\Psi(\mathbf{r}_2, \mathbf{r}_1) \quad \text{(fermions)}$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \mapsto e^{i\alpha}\Psi(\mathbf{r}_2, \mathbf{r}_1) \quad \text{(anyons)}$$

Majorana fermions are "non-Abelian anyons" and upon exchange transform as:

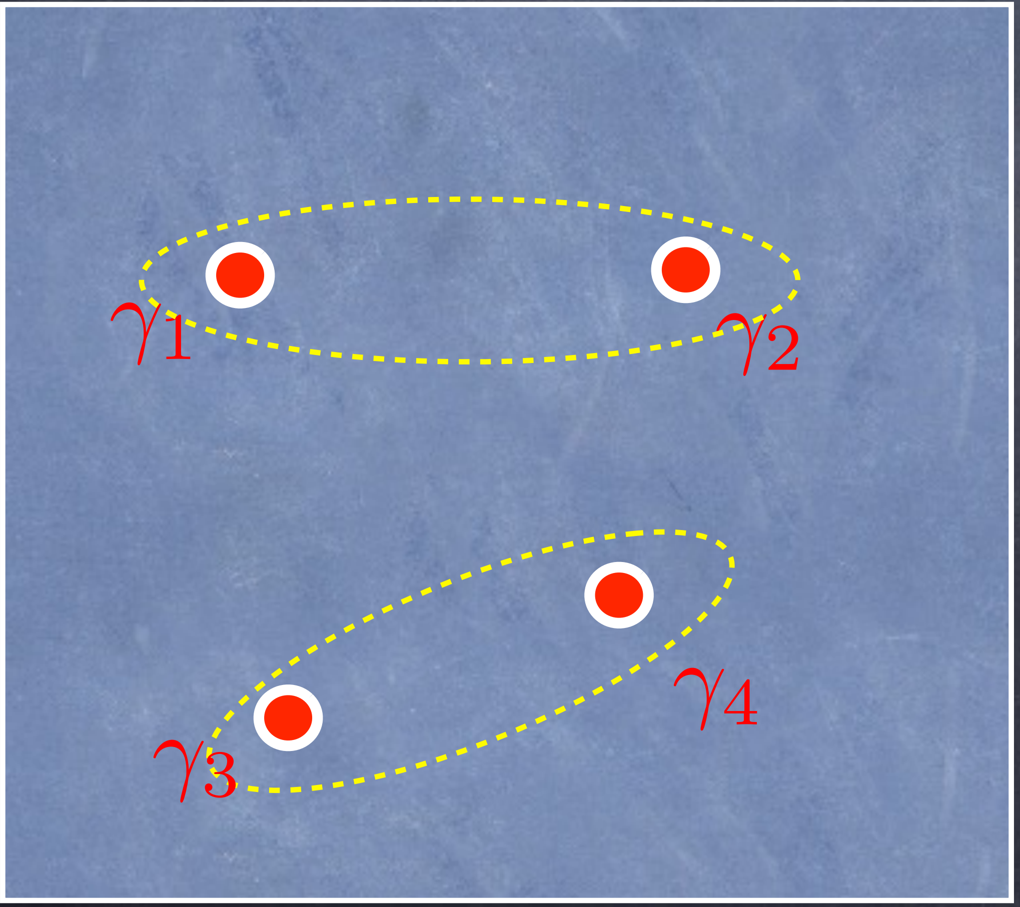
$$\gamma_1 \mapsto -\gamma_2$$

$$\gamma_2 \mapsto \gamma_1$$

For 2N Majoranas a pairwise exchange can be implemented in terms of a unitary transformation

$$\gamma_k \mapsto T_{ij}\gamma_k T_{ij}^\dagger, \qquad T_{ij} = \frac{1}{\sqrt{2}}(1+\gamma_i\gamma_j)$$

From this we can read off the effect of the exchange on the quantum state



$$T_{12}|n_a, n_b\rangle = e^{i\frac{\pi}{4}(1-2n_a)}|n_a, n_b\rangle$$

$$T_{12}T_{12}|n_a, n_b\rangle = i(-1)^{n_a}|n_a, n_b\rangle$$

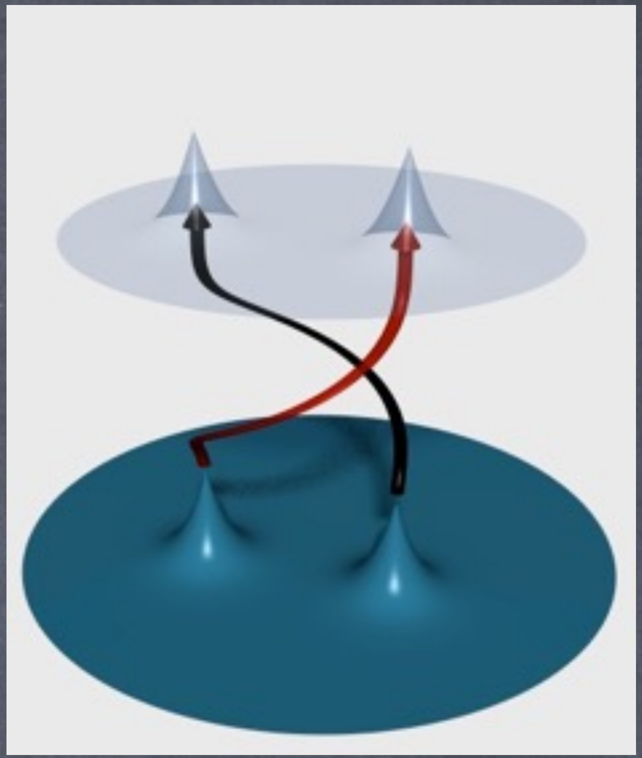
$$T_{13}|n_a, n_b\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(|n_a, n_b\rangle + (-1)^{n_a}|\bar{n}_a, \bar{n}_b\rangle\right)$$

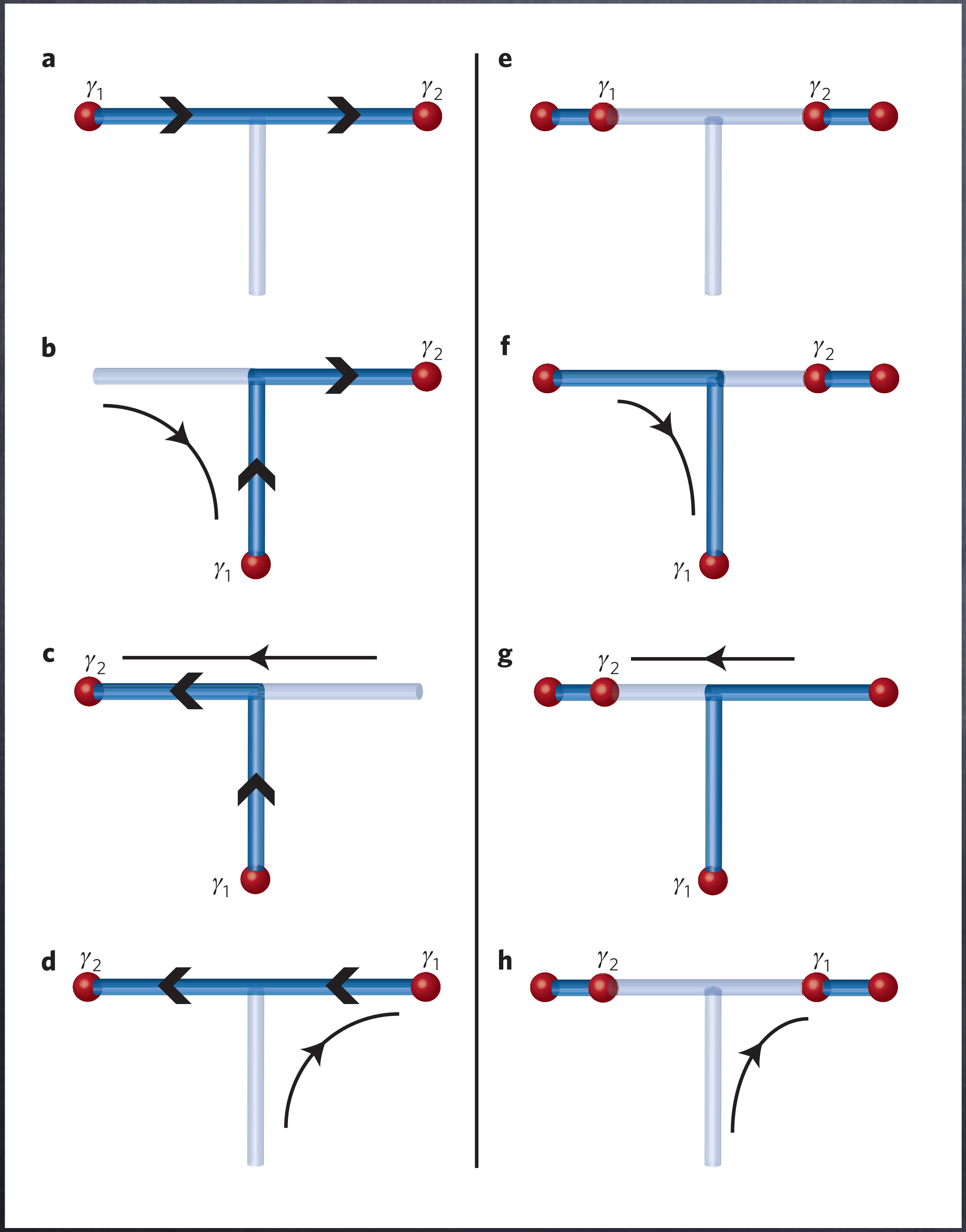
$$T_{13}T_{13}|n_a, n_b\rangle = (-1)^{n_a}|\bar{n}_a, \bar{n}_b\rangle$$

Majorana exchange summary:

- The Hilbert space spanned by Majorana zero modes can be used to encode and manipulate quantum information.
- This is effected by pairwise exchanges which implement unitary transformations on the state vector $|\Psi\rangle$
- The operations form a non-Abelian group, e.g. $T_{12}T_{23} \neq T_{23}T_{12}$.
- Unfortunately, the group structure is not sufficiently rich to implement a universal quantum computer.



How about exchange in 1D structures?



Braiding of Majoranas in T-junctions shows non-Abelian exchange statistics.
[Alicea et al. Nat Phys 2010]

Universal topological quantum computer?

PHYSICAL REVIEW X **4**, 011036 (2014)

Universal Topological Quantum Computation from a Superconductor-Abelian Quantum Hall Heterostructure

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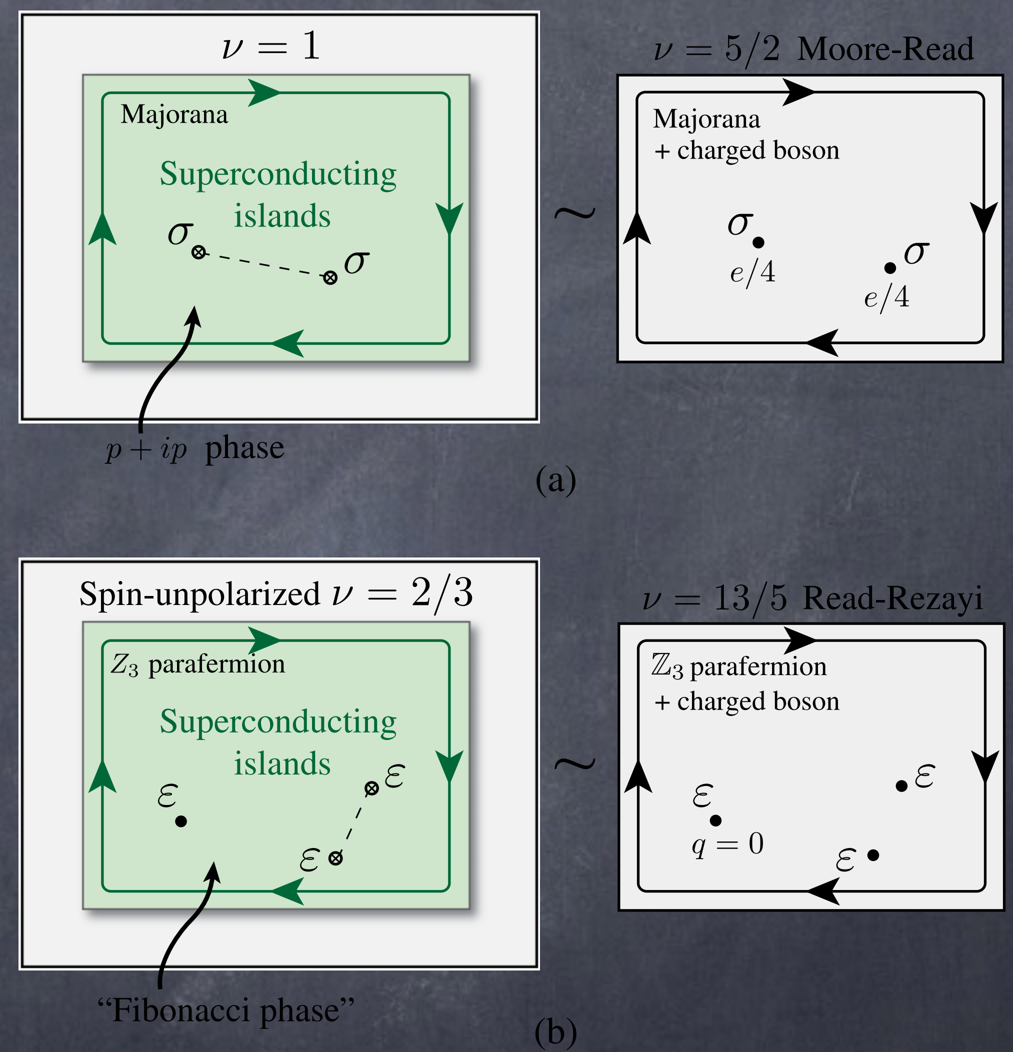
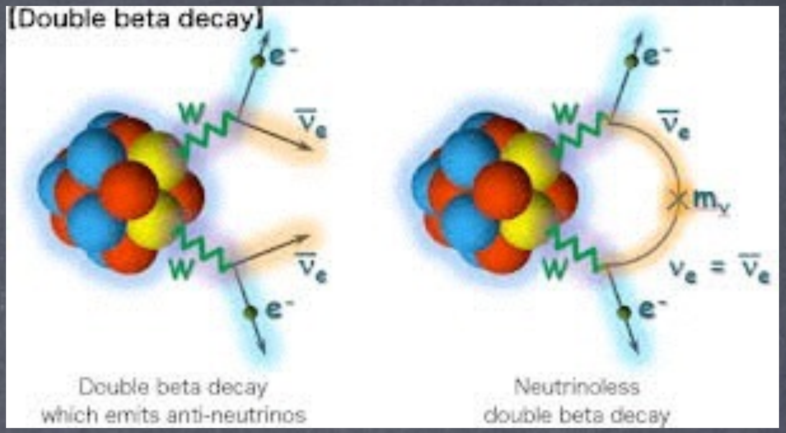
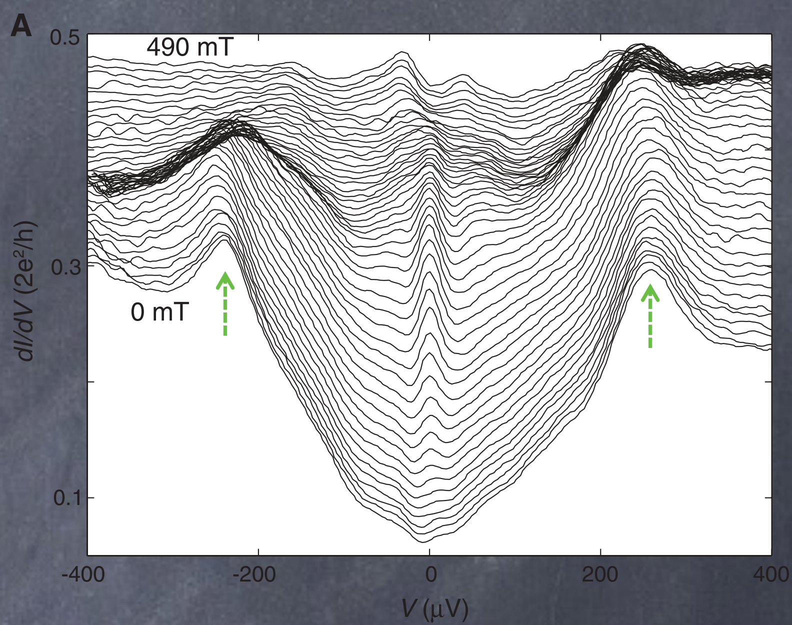
Non-Abelian anyons promise to reveal spectacular features of quantum mechanics that could ultimately provide the foundation for a decoherence-free quantum computer. A key breakthrough in the pursuit of these exotic particles originated from Read and Green's observation that the Moore-Read quantum Hall state and a (relatively simple) two-dimensional $p + ip$ superconductor both support so-called Ising non-Abelian anyons. Here, we establish a similar correspondence between the $\mathbb{Z}_3$ Read-Rezayi quantum Hall state and a novel two-dimensional superconductor in which charge-$2e$ Cooper pairs are built from fractionalized quasiparticles. In particular, both phases harbor Fibonacci anyons that—unlike Ising anyons—allow for universal topological quantum computation solely through braiding. Using a variant of Teo and Kane's construction of non-Abelian phases from weakly coupled chains, we provide a blueprint for such a superconductor using Abelian quantum Hall states interlaced with an array of superconducting islands. Fibonacci anyons appear as neutral deconfined particles that lead to a twofold ground-state degeneracy on a torus. In contrast to a $p + ip$ superconductor, vortices do not yield additional particle types, yet depending on nonuniversal energetics can serve as a trap for Fibonacci anyons. These results imply that one can, in principle, combine well-understood and widely available phases of matter to realize non-Abelian anyons with universal braid statistics. Numerous future directions are discussed, including speculations on alternative realizations with fewer experimental requirements.



Conclusions



- Pursuing an ideal of simplicity and mathematical elegance Majorana predicted in 1938 a new type of a particle identical to its antiparticle.

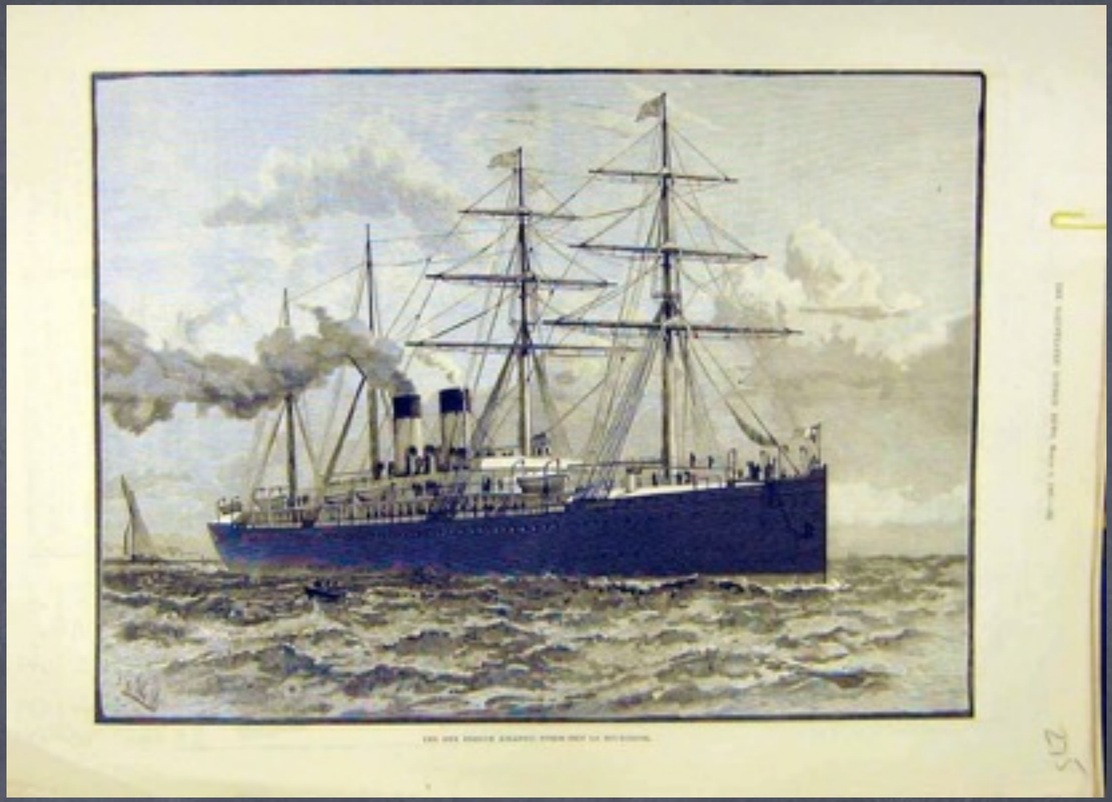


- After 75 years it remains unclear if neutrino realizes such Majorana fermion; experiments are ongoing.

- It appears likely that Majorana fermions have been realized in solid state devices.

- They obey non-Abelian exchange statistics and could represent platforms for a future quantum computer.

On March 27, 1938 Majorana boarded a ship from Palermo to Naples and was never seen again. To this date, his disappearance at the age of 32 remains a mystery. His legacy endures.



"Majorana had greater gifts than anyone else in the world. Unfortunately he lacked one quality which other men generally have: plain common sense."

Enrico Fermi