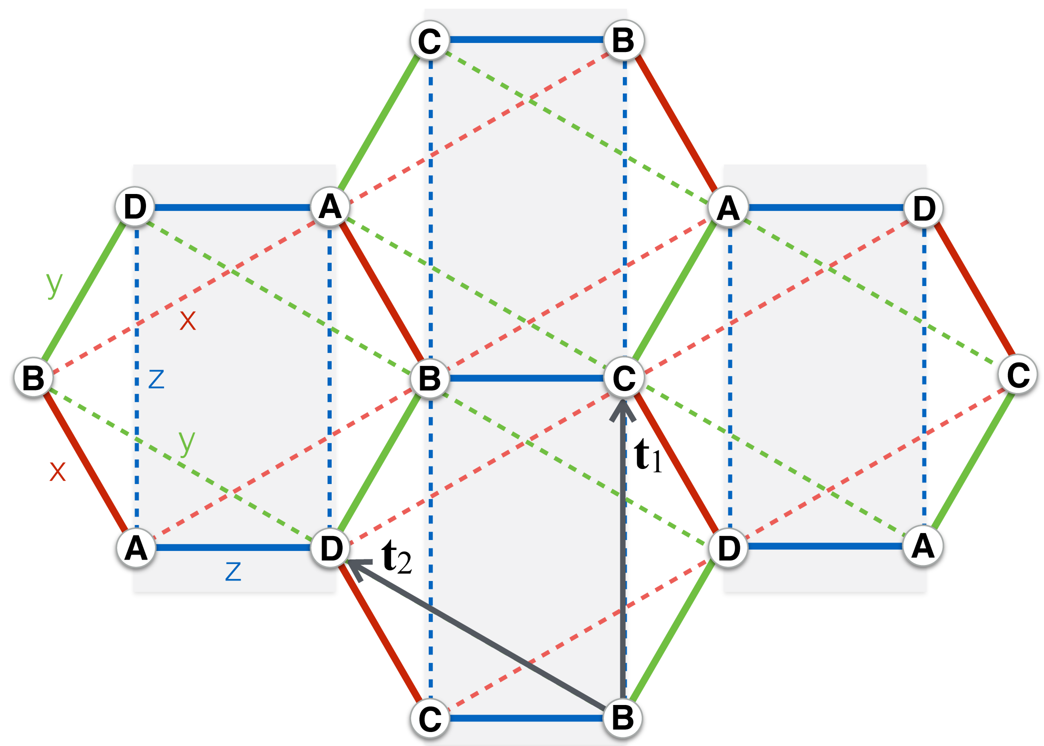
University of Minnesota
Driven to Discover

Phase diagram and quantum order-by-disorder in the Kitaev K_1-K_2 honeycomb magnet



Ioannis Rousochatzakis, University of Minnesota

Spin Orbit Summer School, Quantum Matter Institute, UBC Vancouver, Canada
24 October 2015

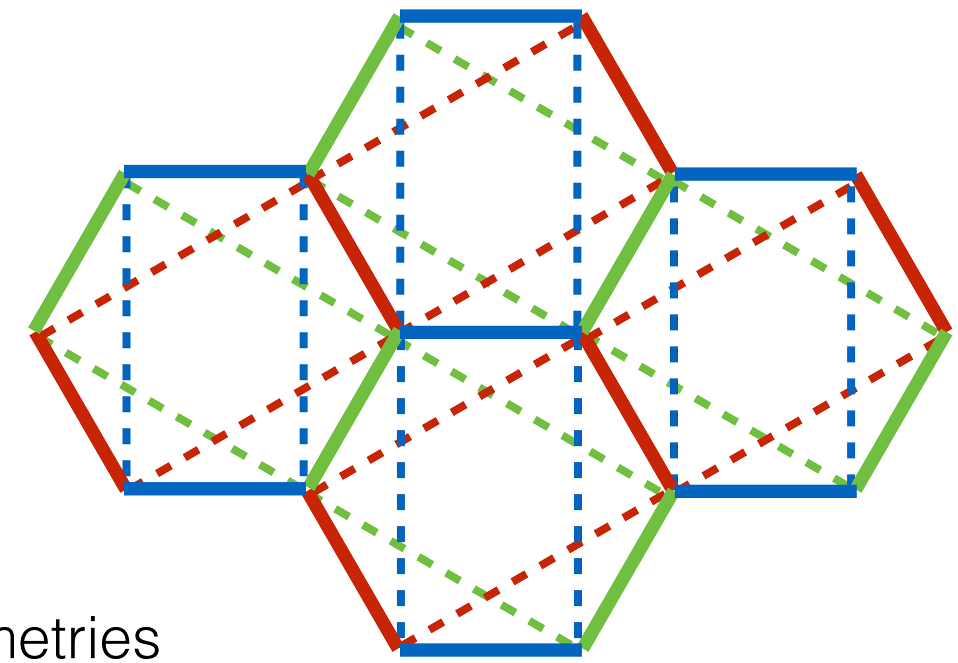
Collaborators

- Natalia Perkins (UMN) 
- Stephan Rachel (TU Dresden) 
- Johannes Reuter (TU Berlin) 
- Ronny Thomale (Wuerzburg University) 

I.R., J. Reuther, R. Thomale, S. Rachel, N. B. Perkins, PRX (in press) [arXiv: 1506.09185]

Outline

- Motivation: Kitaev spin liquid in real materials

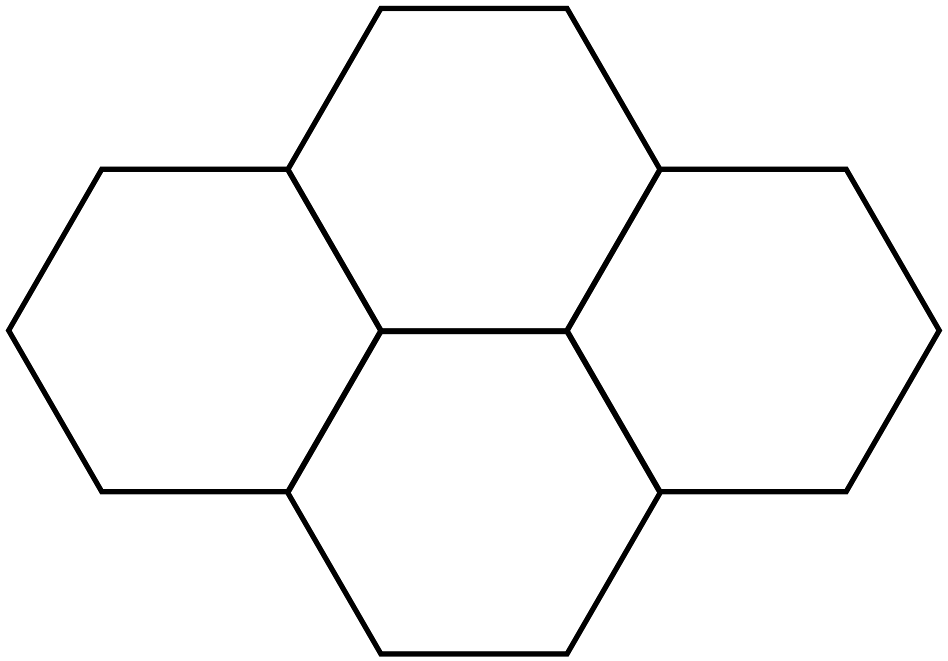


- Na_2IrO_3: importance of second-neighbor interaction K_2
- Quantum K_1-K_2 model: *hidden*, gauge-like dualities & symmetries
- Phase diagram from Exact Diagonalizations
- Physical mechanism of instability: Classical vs Quantum spins
- Outlook: back to materials

Main motivation: Kitaev spin liquid

A. Kitaev (2006)

Kitaev's honeycomb model (S=1/2):

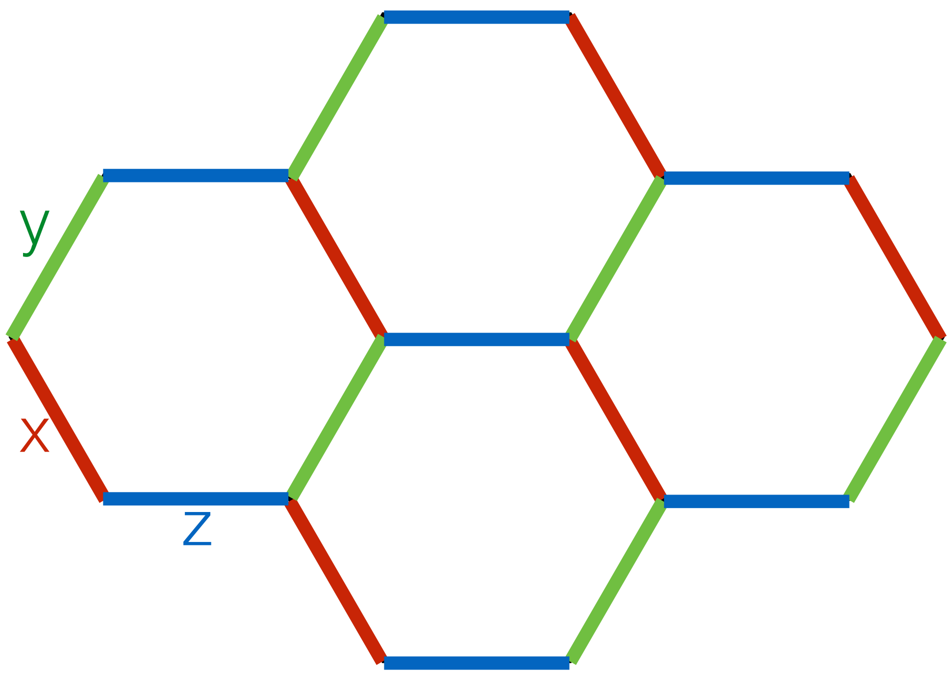


Main motivation: Kitaev spin liquid

A. Kitaev (2006)

Kitaev's honeycomb model (S=1/2):

$$\mathcal{H} = \sum_{\langle ij \rangle \in \mathrm{red}} S_i^x S_j^x + \sum_{\langle ij \rangle \in \mathrm{green}} S_i^y S_j^y + \sum_{\langle ij \rangle \in \mathrm{blue}} S_i^z S_j^z$$

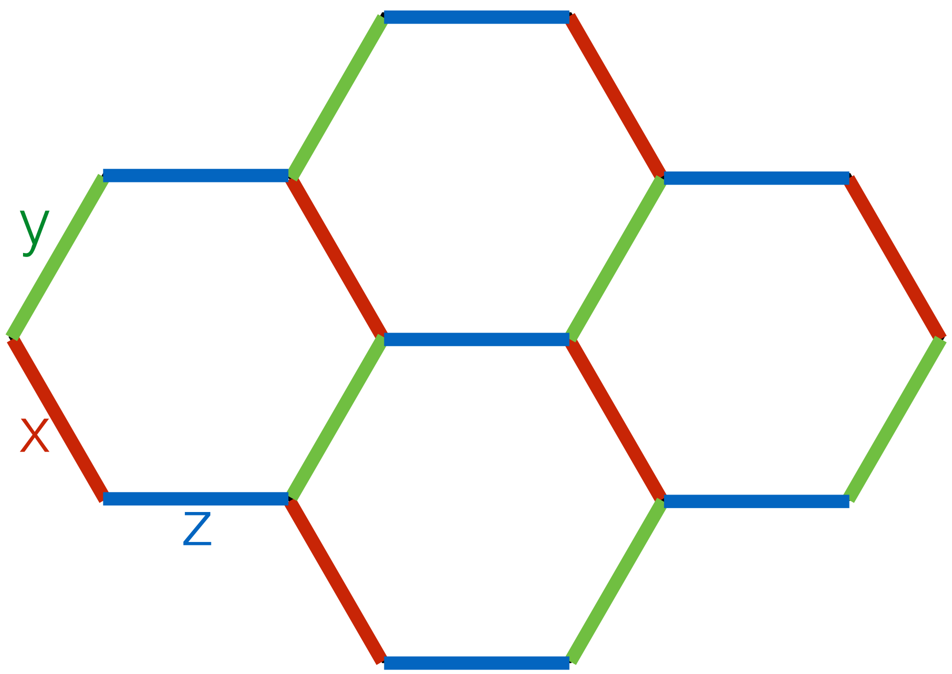


Main motivation: Kitaev spin liquid

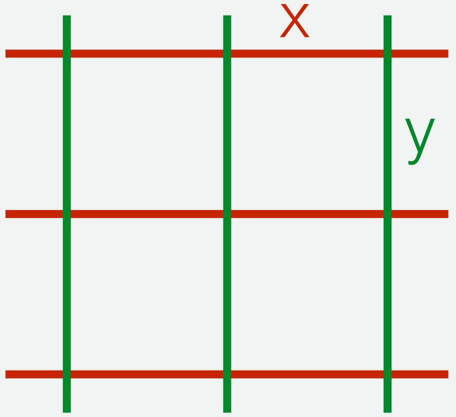
A. Kitaev (2006)

Kitaev's honeycomb model (S=1/2):

$$\mathcal{H} = \sum_{\langle ij\rangle \in \mathrm{red}} S_i^x S_j^x + \sum_{\langle ij\rangle \in \mathrm{green}} S_i^y S_j^y + \sum_{\langle ij\rangle \in \mathrm{blue}} S_i^z S_j^z$$



= Compass model   Kugel & Khomskii (1982)



- -no magnetic LRO
- -no spin liquid
- -**nematic** order

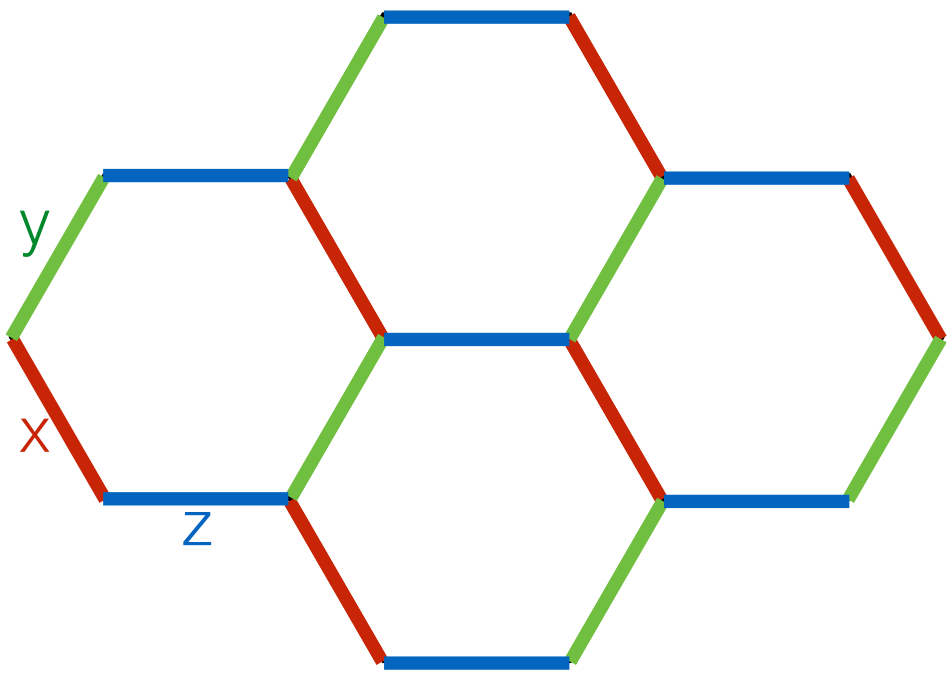
Nussinov & van den Brink ('15)

Main motivation: Kitaev spin liquid

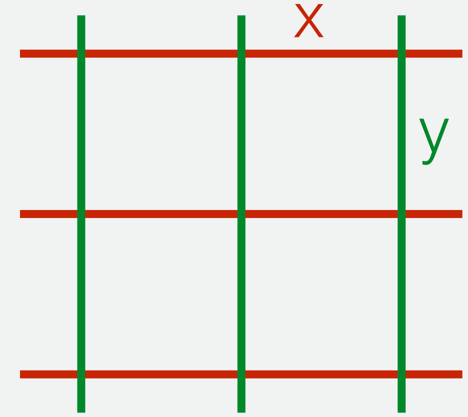
A. Kitaev (2006)

Kitaev's honeycomb model (S=1/2):

$$\mathcal{H} = \sum_{\langle ij\rangle \in \text{red}} S_i^x S_j^x + \sum_{\langle ij\rangle \in \text{green}} S_i^y S_j^y + \sum_{\langle ij\rangle \in \text{blue}} S_i^z S_j^z$$



= Compass model   Kugel & Khomskii (1982)



- -no magnetic LRO
- -no spin liquid
- -**nematic** order

Nussinov & van den Brink ('15)

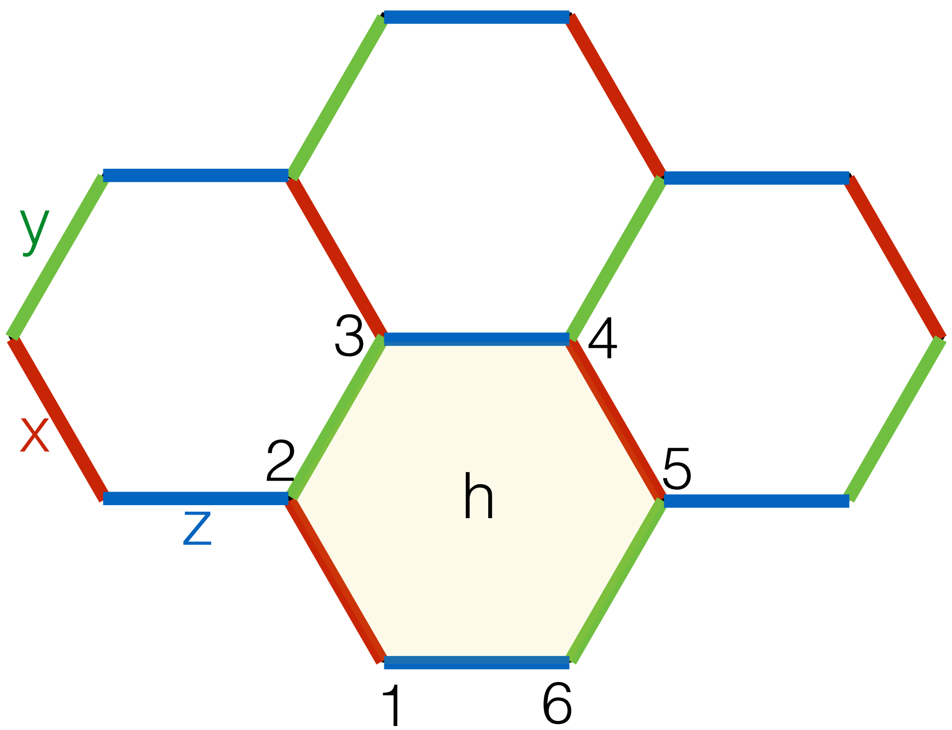
- what's special about honeycomb?

Main motivation: Kitaev spin liquid

A. Kitaev (2006)

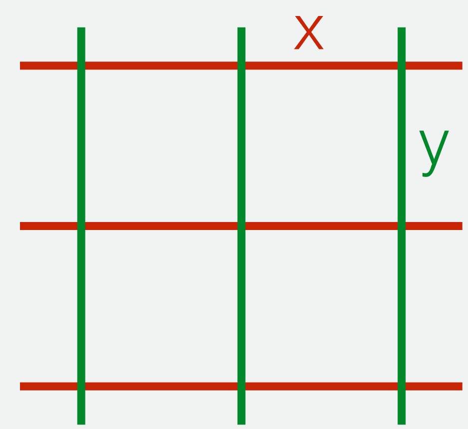
Kitaev's honeycomb model (S=1/2):

$$\mathcal{H} = \sum_{\langle ij\rangle \in \mathrm{red}} S_i^x S_j^x + \sum_{\langle ij\rangle \in \mathrm{green}} S_i^y S_j^y + \sum_{\langle ij\rangle \in \mathrm{blue}} S_i^z S_j^z$$



= Compass model

Kugel & Khomskii (1982)



-no magnetic LRO

-no spin liquid

-**nematic** order

Nussinov & van den Brink ('15)

- what's special about honeycomb?

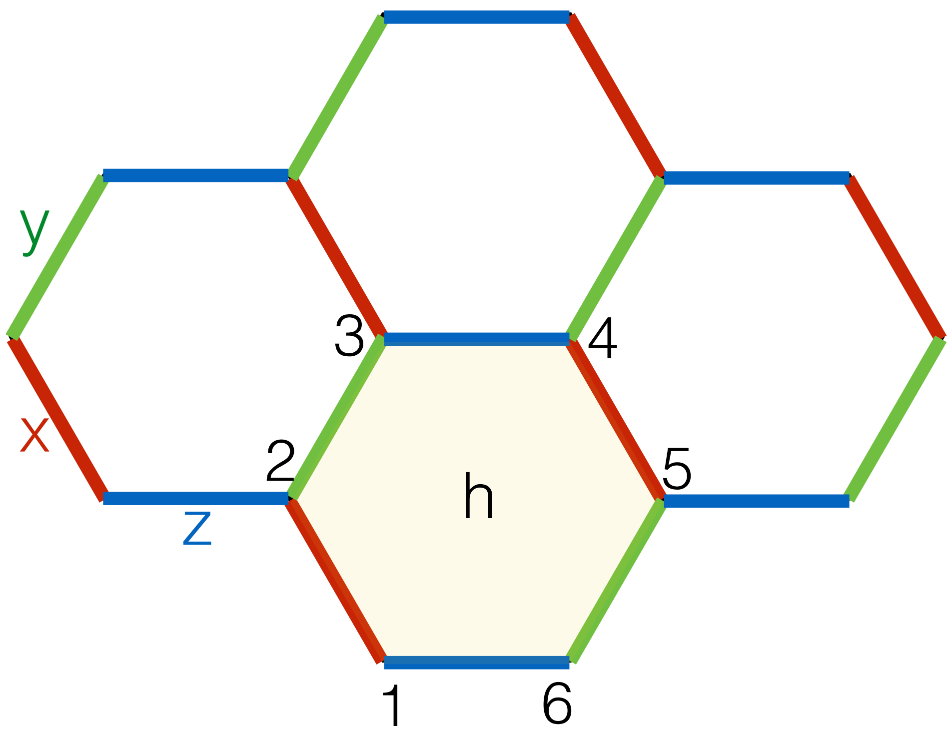
→Extensive number of conserved fluxes: $W_h = 2^6\ S_1^y\ S_2^z\ S_3^x\ S_4^y\ S_5^z\ S_6^x$

Main motivation: Kitaev spin liquid

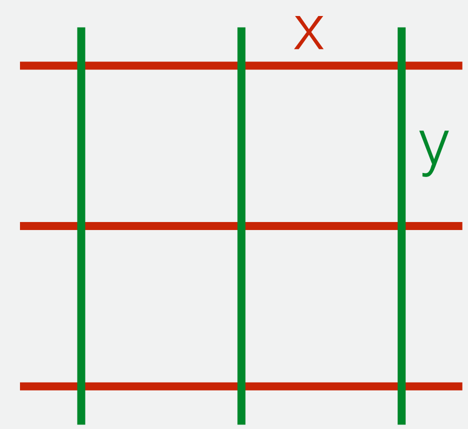
A. Kitaev (2006)

Kitaev's honeycomb model (S=1/2):

$$\mathcal{H} = \sum_{\langle ij\rangle \in \mathrm{red}} S_i^x S_j^x + \sum_{\langle ij\rangle \in \mathrm{green}} S_i^y S_j^y + \sum_{\langle ij\rangle \in \mathrm{blue}} S_i^z S_j^z$$



= Compass model Kugel & Khomskii (1982)



- -no magnetic LRO
- -no spin liquid
- -**nematic** order

Nussinov & van den Brink ('15)

- what's special about honeycomb?

→Extensive number of conserved fluxes: $W_h = 2^6\ S_1^y\ S_2^z\ S_3^x\ S_4^y\ S_5^z\ S_6^x$

→Exact solvability: free Majorana fermions in the background of static fluxes

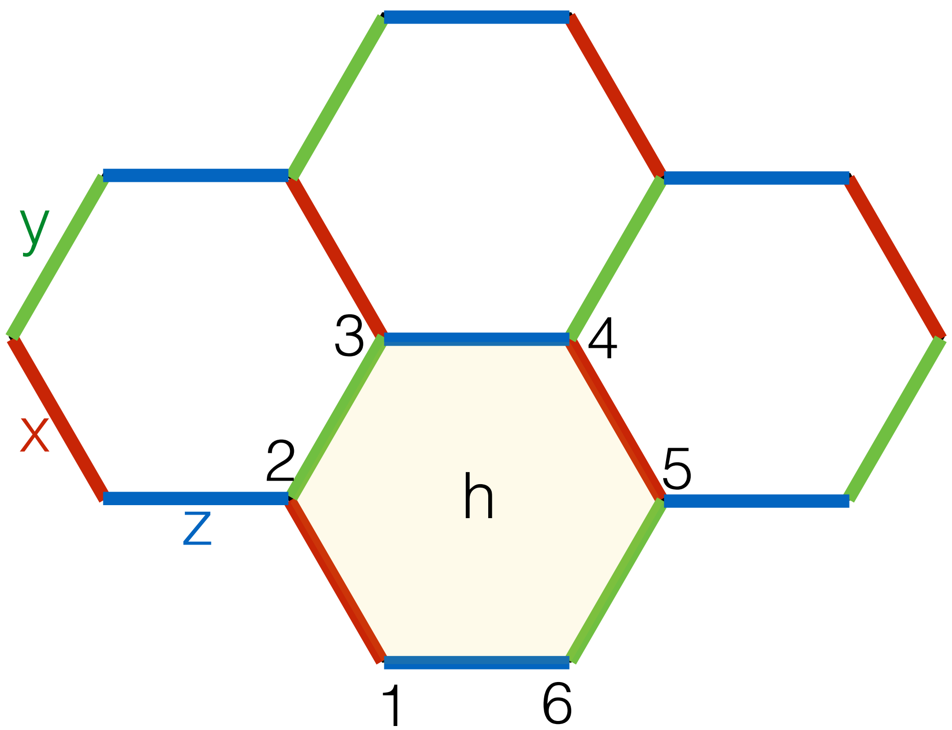
Ground state sector: W_h=+1, for all h

Main motivation: Kitaev spin liquid

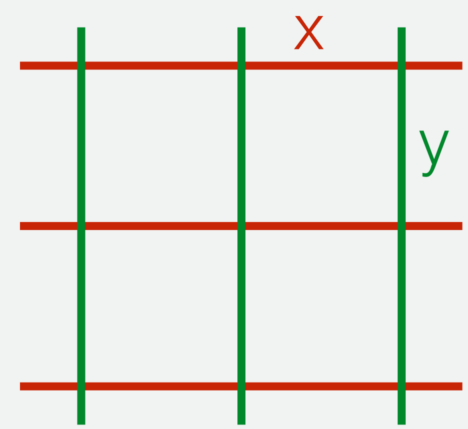
A. Kitaev (2006)

Kitaev's honeycomb model (S=1/2):

$$\mathcal{H} = \sum_{\langle ij\rangle \in \mathrm{red}} S_i^x S_j^x + \sum_{\langle ij\rangle \in \mathrm{green}} S_i^y S_j^y + \sum_{\langle ij\rangle \in \mathrm{blue}} S_i^z S_j^z$$



= Compass model Kugel & Khomskii (1982)



- -no magnetic LRO
- -no spin liquid
- -**nematic** order

Nussinov & van den Brink ('15)

- what's special about honeycomb?

  →Extensive number of conserved fluxes: $W_h = 2^6\ S_1^y\ S_2^z\ S_3^x\ S_4^y\ S_5^z\ S_6^x$

  →Exact solvability: free Majorana fermions in the background of static fluxes

  Ground state sector: $W_h=+1$, for all h

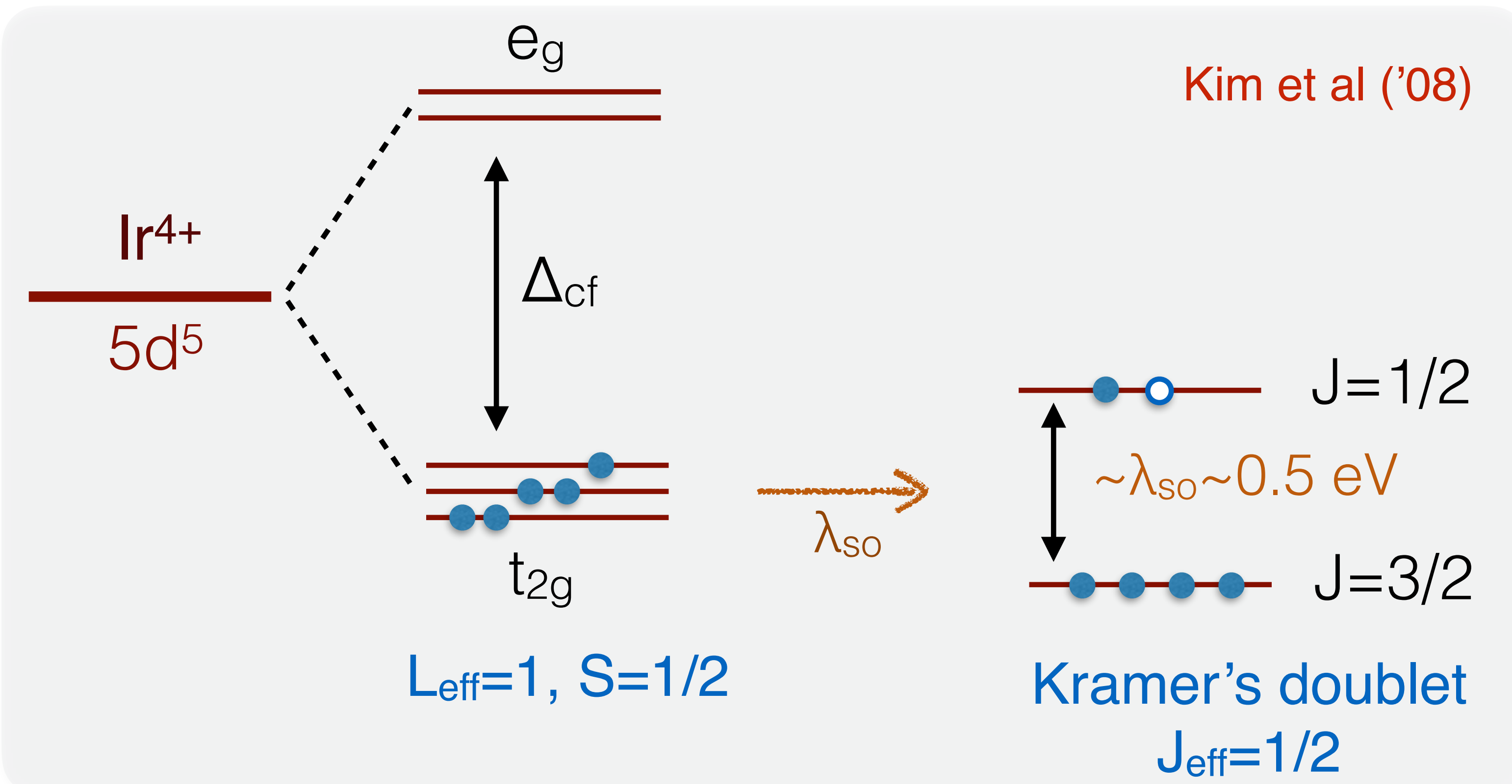
- excitations: gapless Majorana fermions and gapped $W_h=-1$ vortices

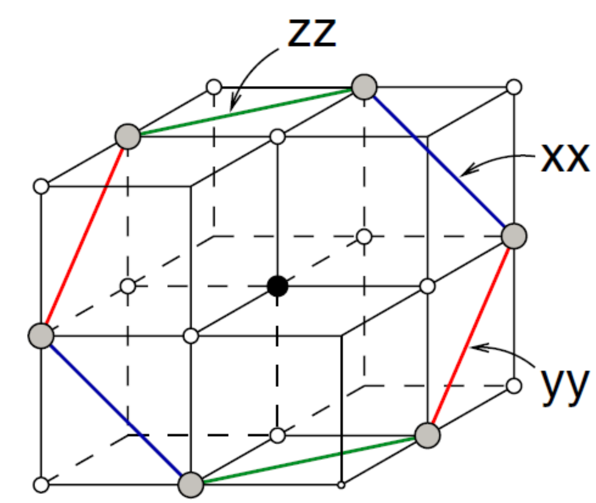
Material realizations: strong SOC with 90° oxygen bonding

Material realizations: strong SOC with 90˚ oxygen bonding

Jackeli & Khaliullin (2009)

- the honeycomb Iridates A_2IrO_3 (A=Li, Na):

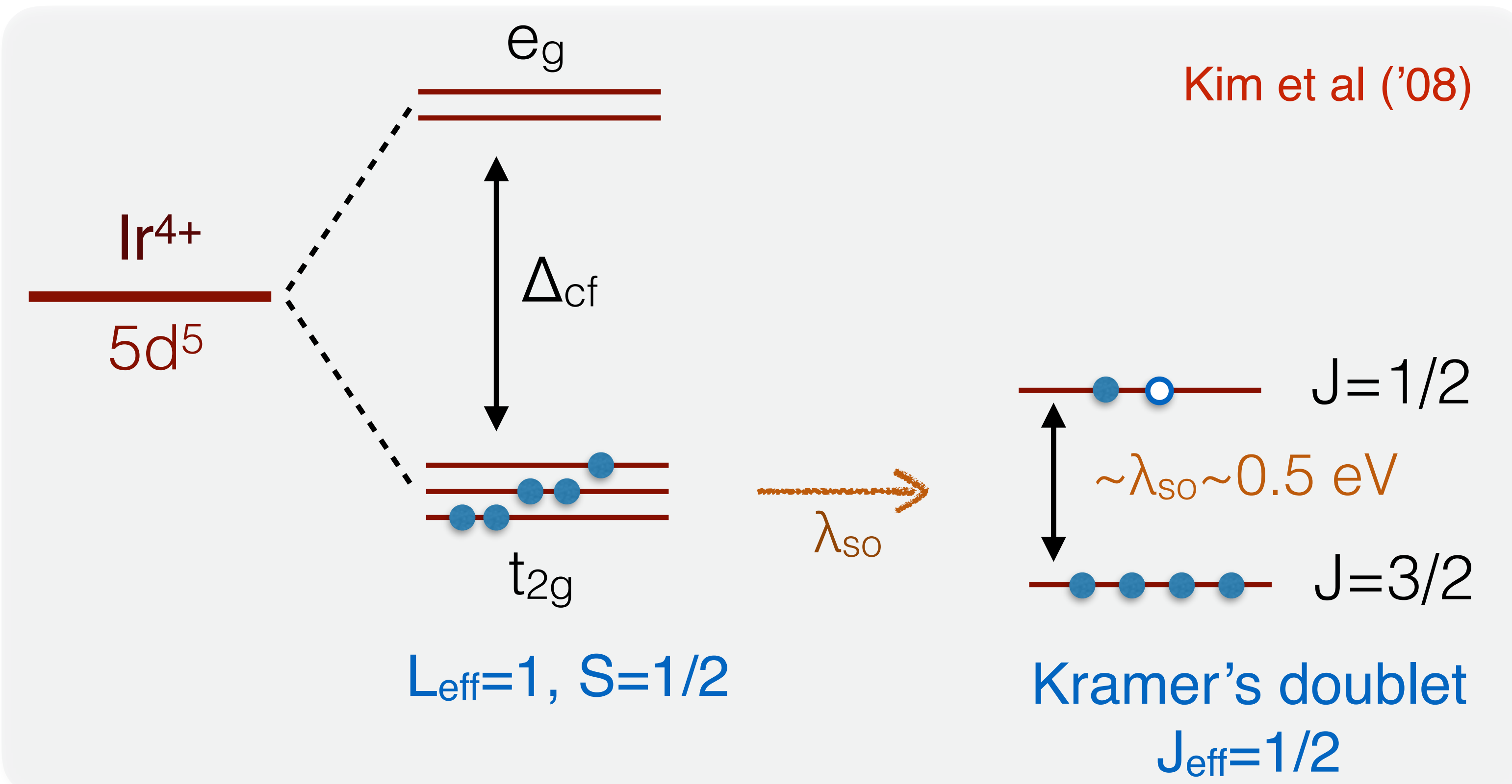


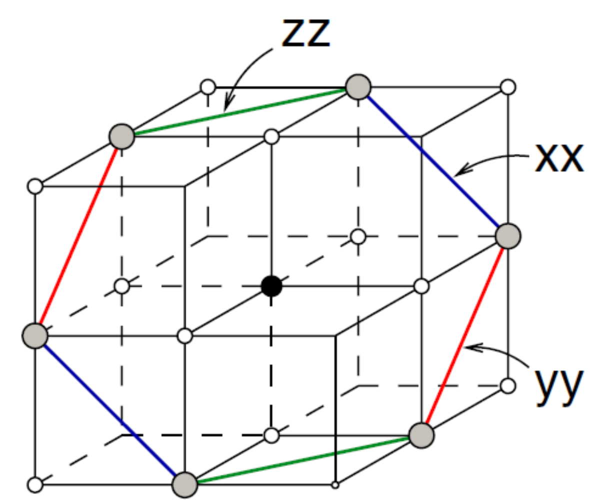


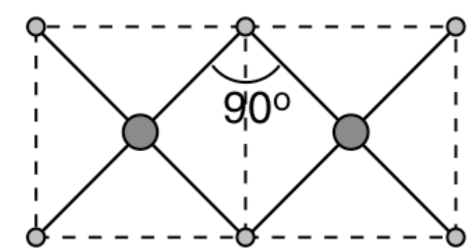
Material realizations: strong SOC with 90˚ oxygen bonding

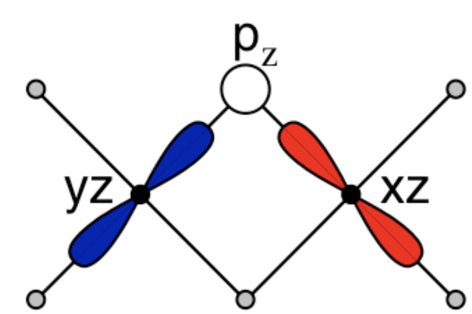
Jackeli & Khaliullin (2009)

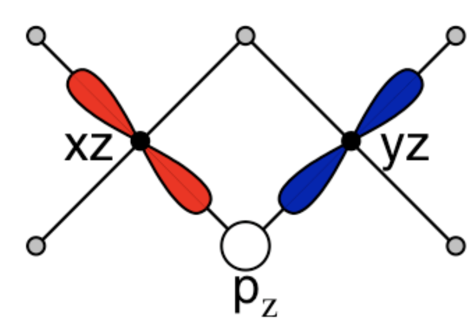
- the honeycomb Iridates A_2IrO_3 (A=Li, Na):









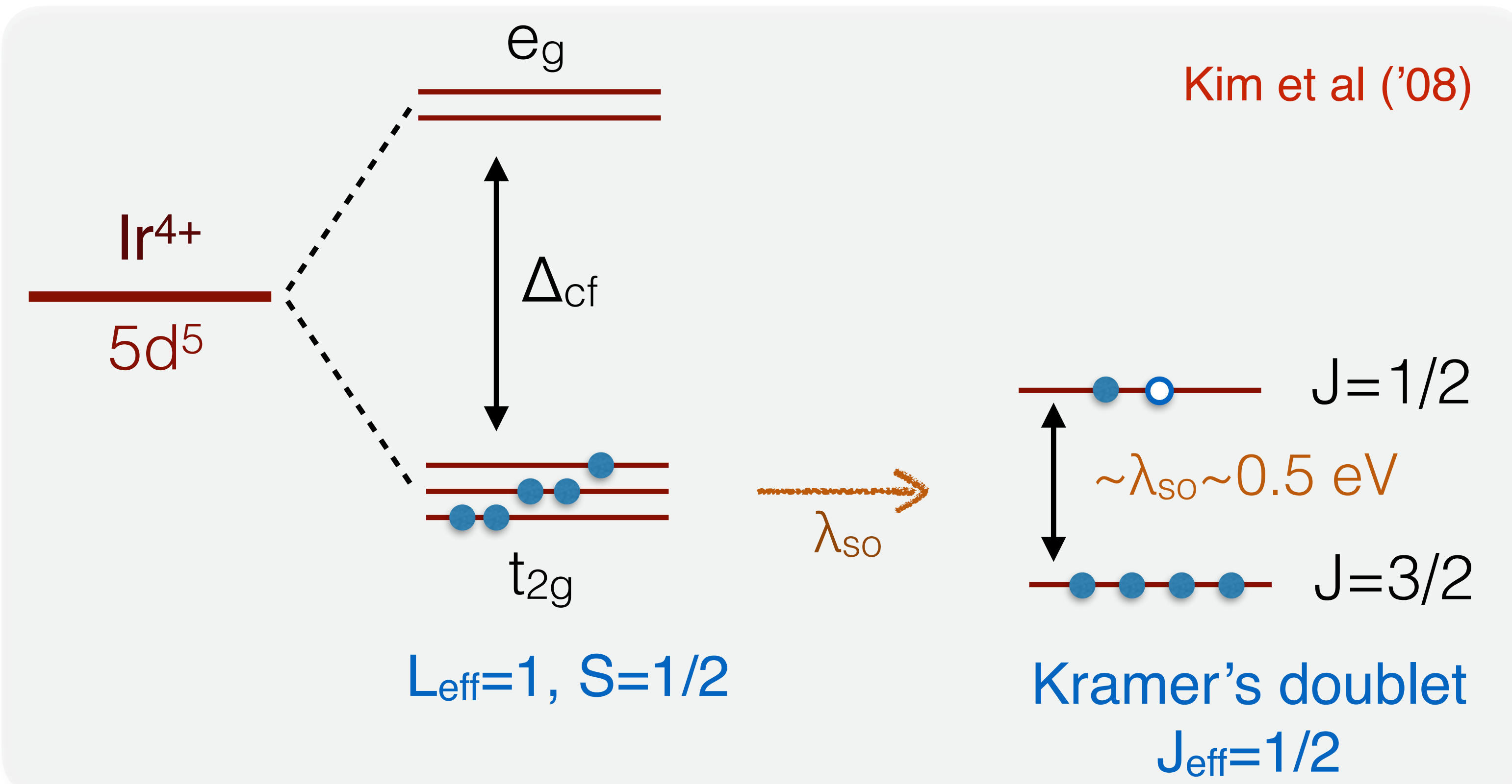


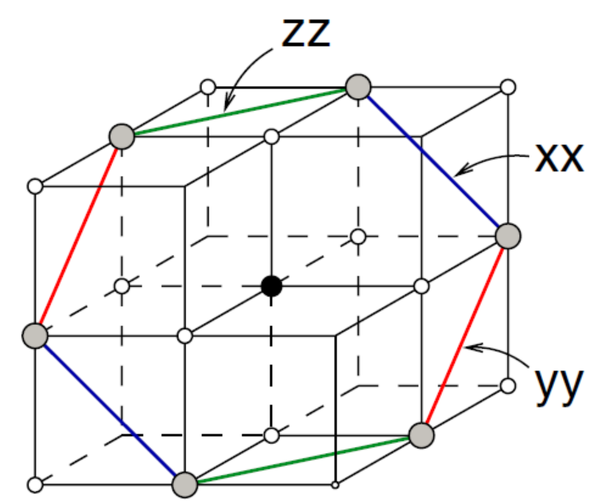
- 90˚ oxygen bonding: destructive interference
  → J_1=0; but K_1 is finite

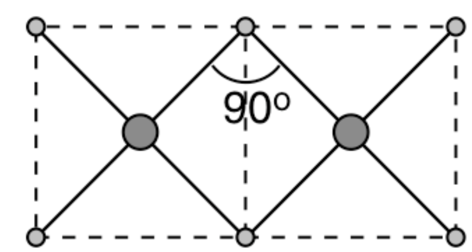
Material realizations: strong SOC with 90˚ oxygen bonding

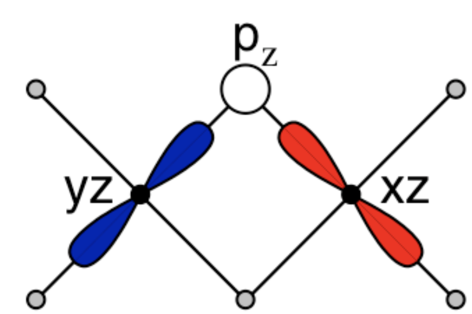
Jackeli & Khaliullin (2009)

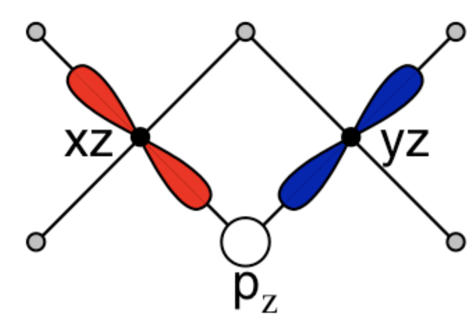
- the honeycomb Iridates A_2IrO_3 (A=Li, Na):











- 90˚ oxygen bonding: destructive interference
  → J_1=0; but K_1 is finite

- Quantum chemistry data for Na_2OIr_3: K_1=-17 meV, J_1=3 meV, ….
  Katukuri et al (2014)

Kitaev spin liquid is robust against Heisenberg exchange

Chaloupka, Jackeli & Khaliullin (2013)

$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Kitaev spin liquid is robust against Heisenberg exchange

Chaloupka, Jackeli & Khaliullin (2013)

$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

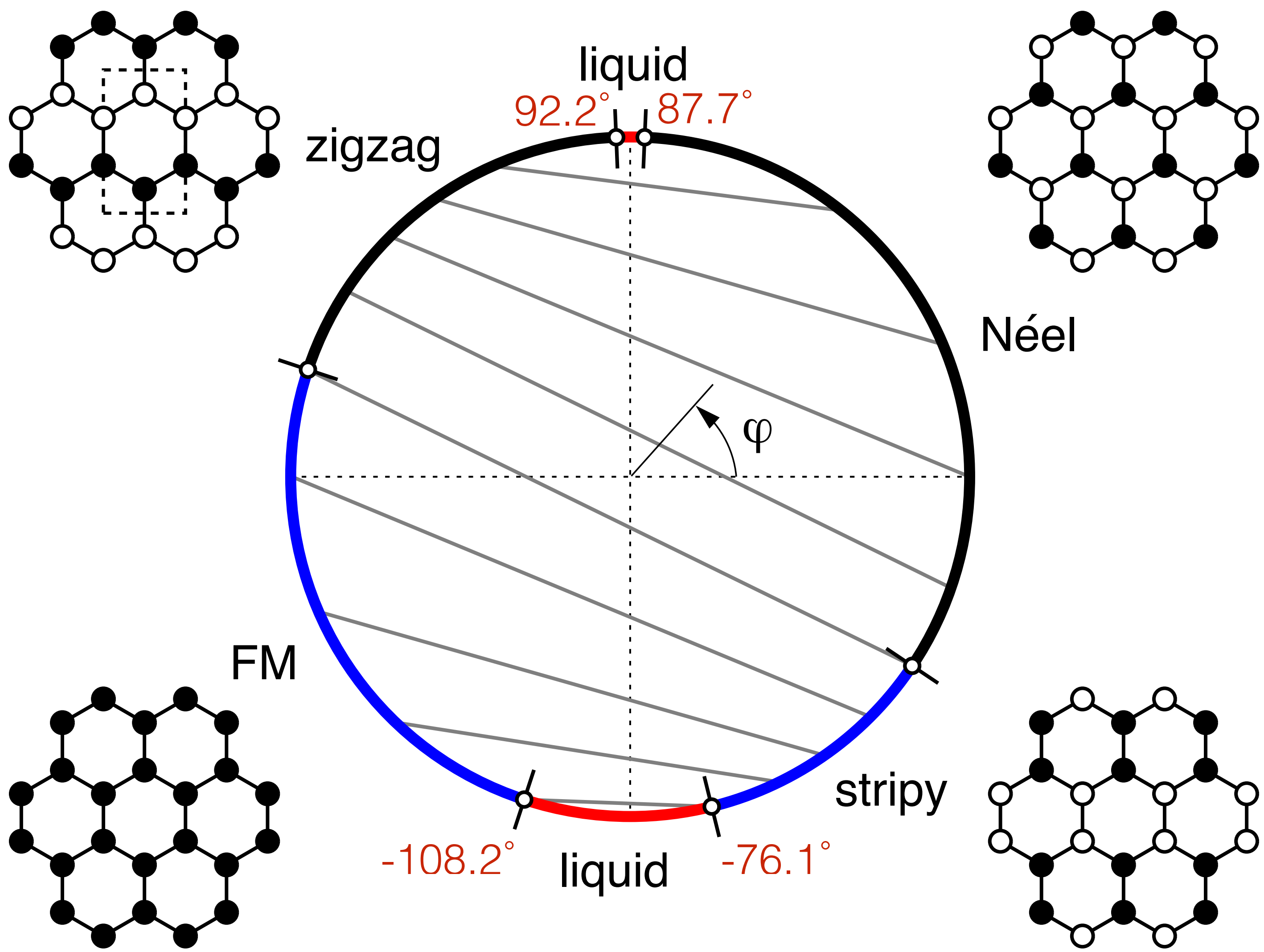
$$J_1 = \cos\phi$$
$$K_1 = 2\sin\phi$$

Kitaev spin liquid is robust against Heisenberg exchange

Chaloupka, Jackeli & Khaliullin (2013)

$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$J_1 = \cos\phi$$
$$K_1 = 2\sin\phi$$

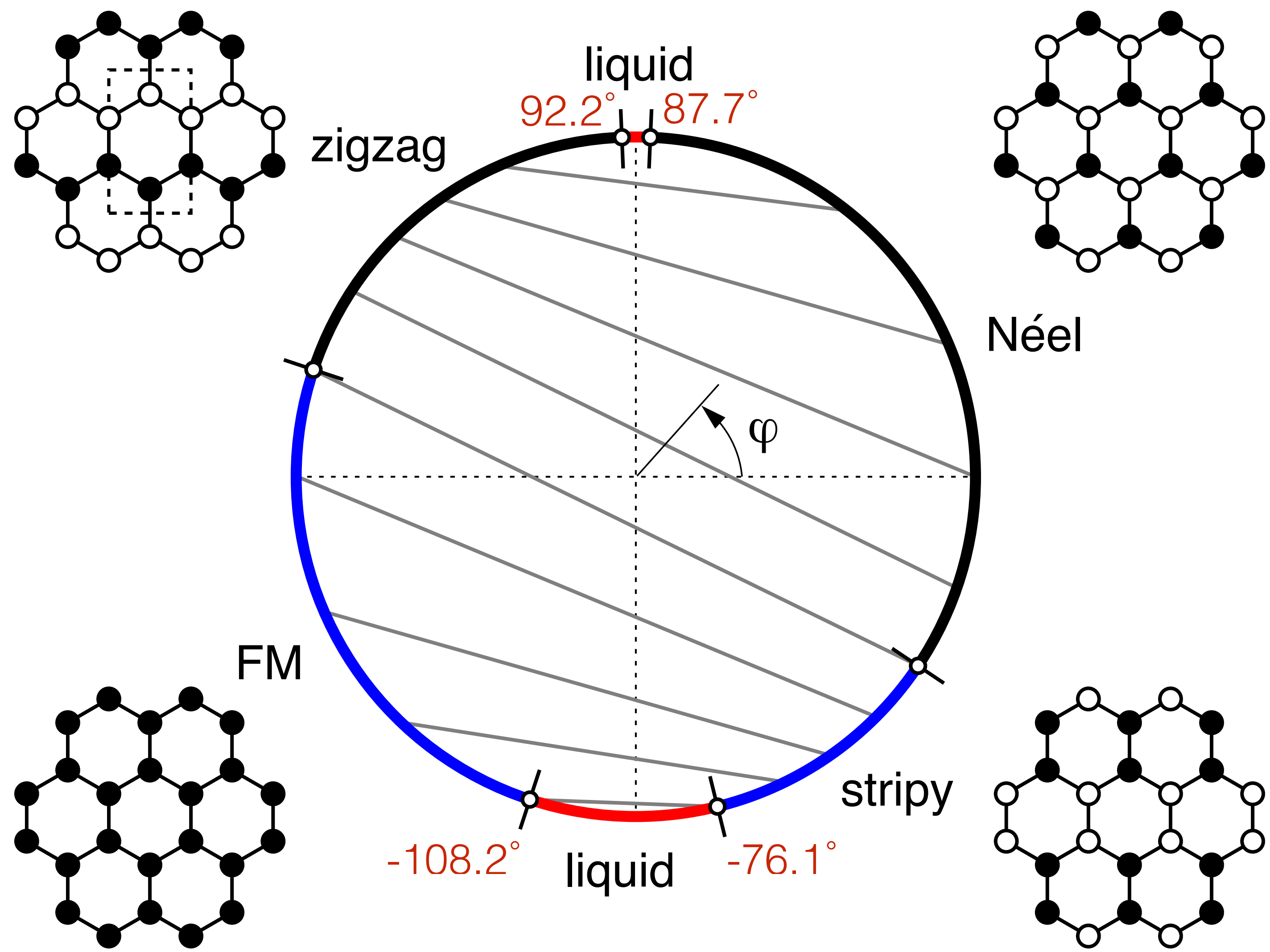


Kitaev spin liquid is robust against Heisenberg exchange

Chaloupka, Jackeli & Khaliullin (2013)

$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$J_1 = \cos\phi$$
$$K_1 = 2\sin\phi$$

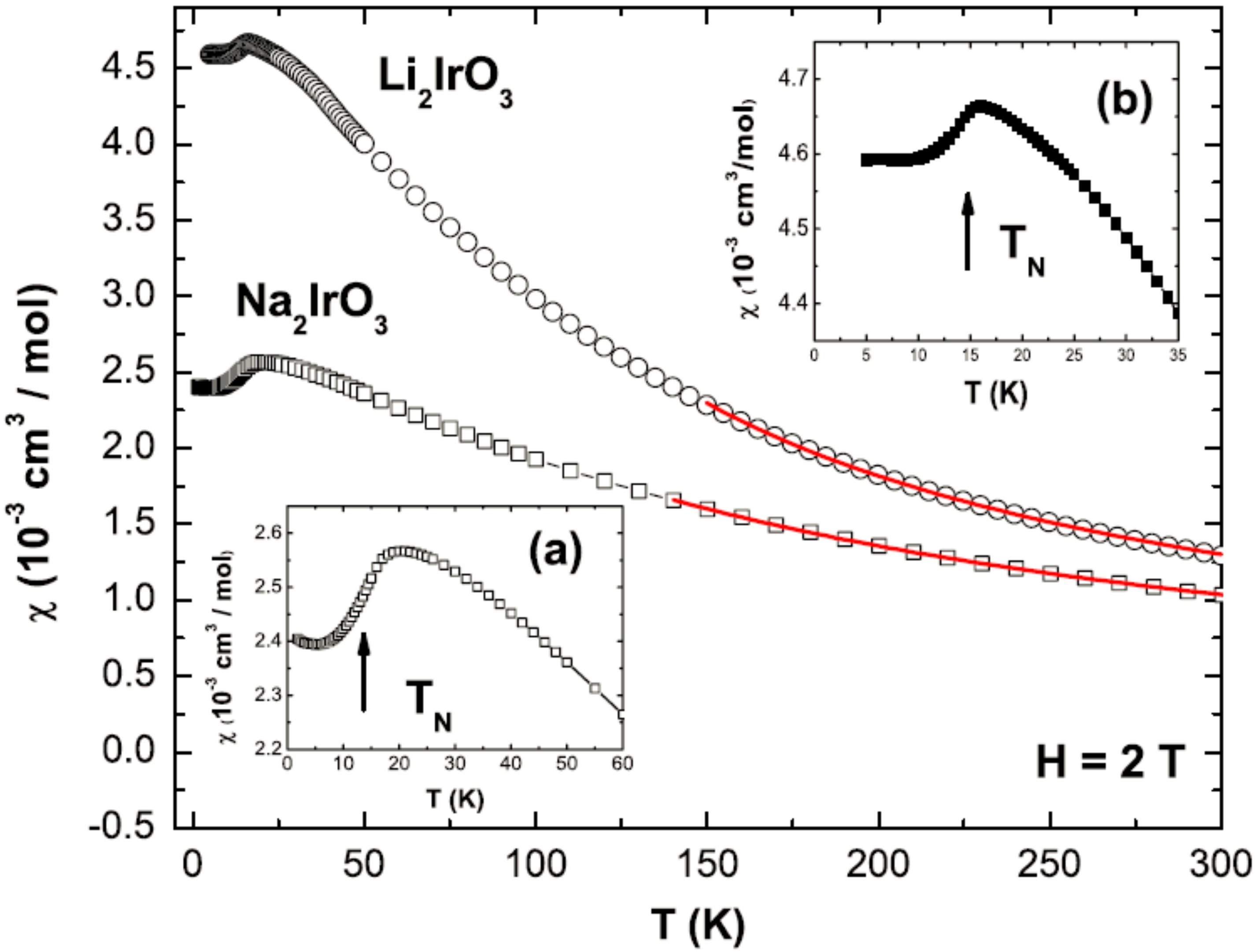


*lines represent a hidden duality

However: Kitaev spin liquid has not been observed so far

- Na213, Li213 & a-RuCl3: all show magnetic LRO at low T

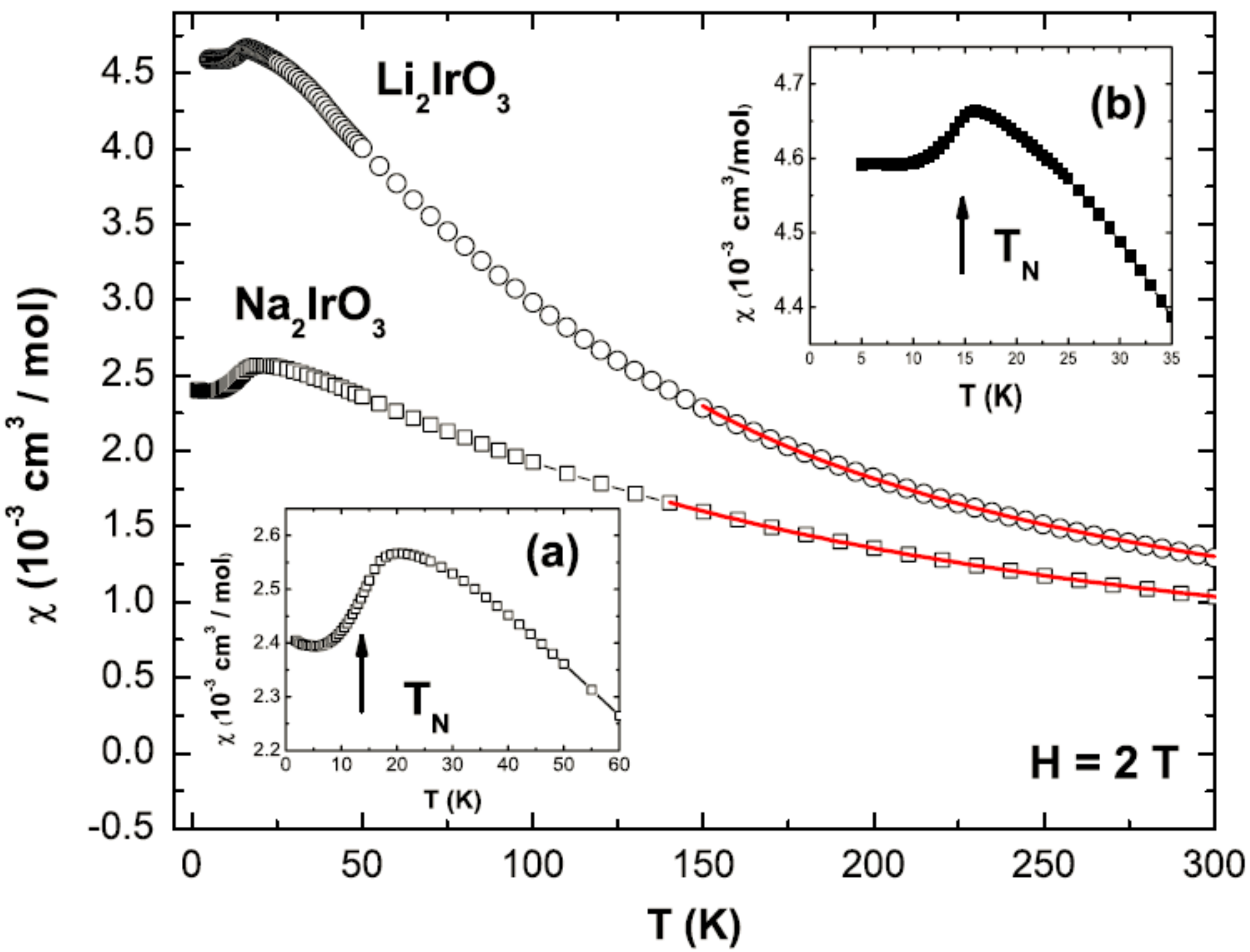
Singh & Gegenwart ('10), Singh et al ('12)



However: Kitaev spin liquid has not been observed so far

- Na213, Li213 & a-RuCl3: all show magnetic LRO at low T

Singh & Gegenwart ('10), Singh et al ('12)

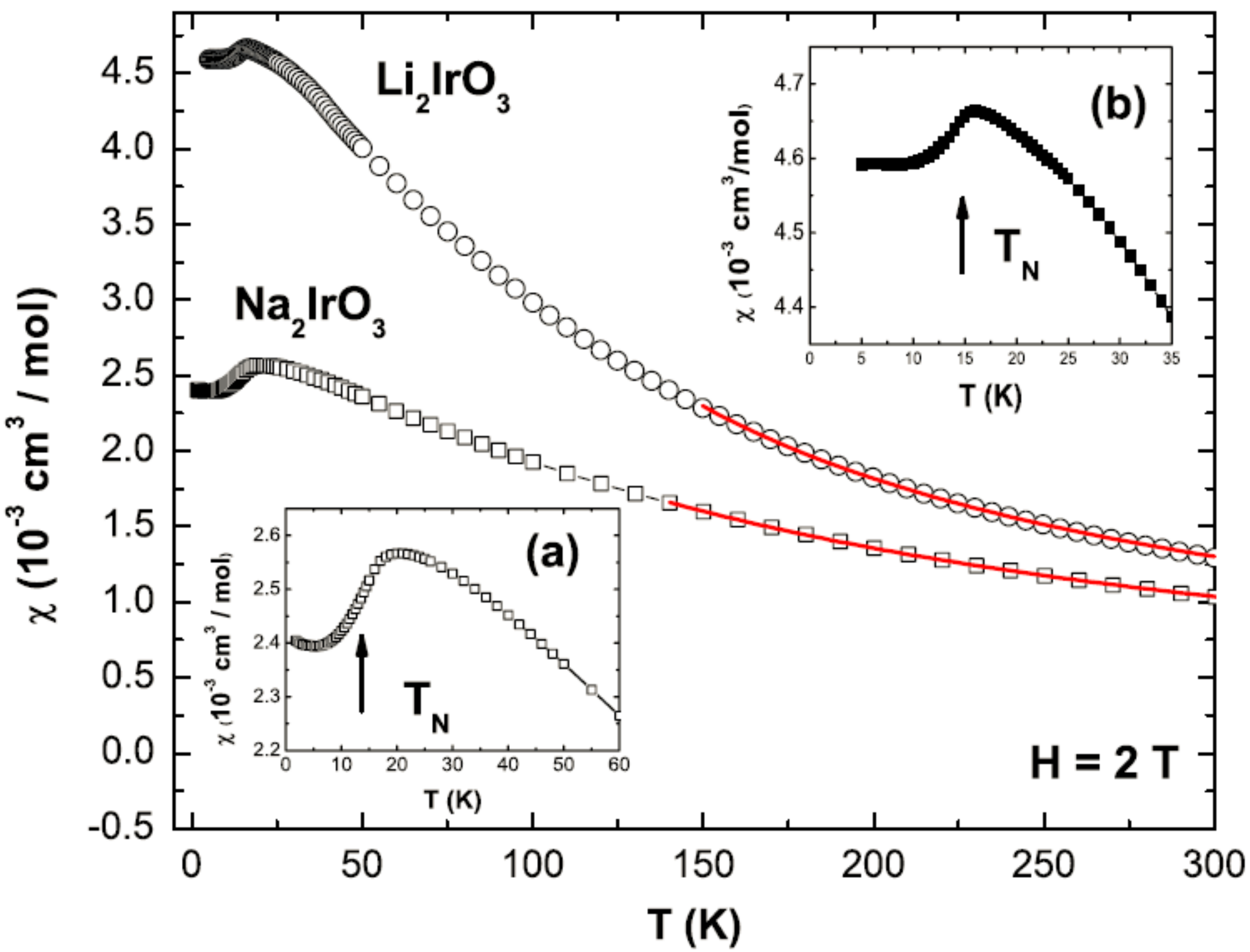


→ what are the most relevant perturbations in these materials ?

However: Kitaev spin liquid has not been observed so far

- Na213, Li213 & a-$RuCl_3$: all show magnetic LRO at low T

Singh & Gegenwart ('10), Singh et al ('12)



→ what are the most relevant perturbations in these materials ?

→ how close are we to the Kitaev spin liquid?

Current status on Na_2IrO_3

Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

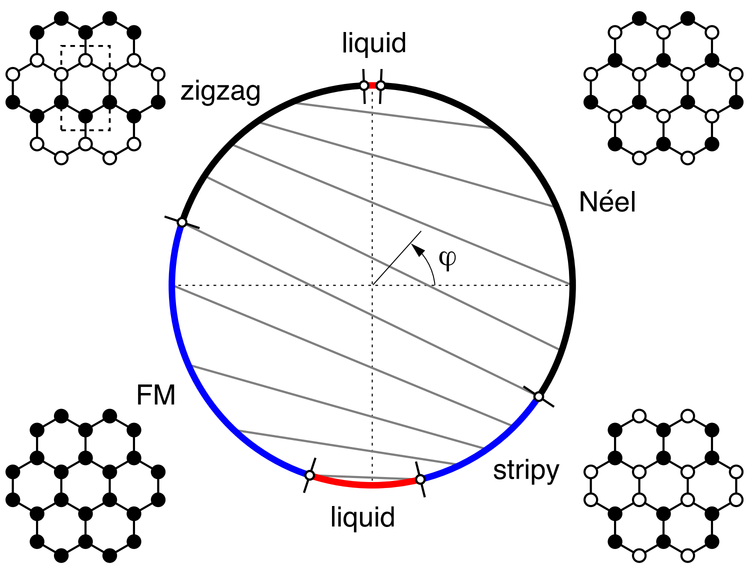
  X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

Chaloupka, Jackeli & Khaliullin (2013)



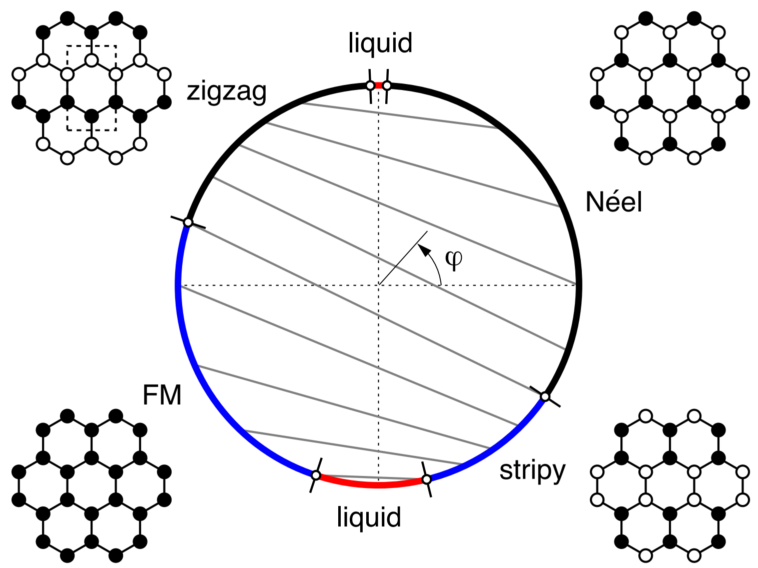
Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

  X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

→ J_1 is not the relevant perturbation !

Chaloupka, Jackeli & Khaliullin (2013)



Current status on Na_2IrO_3

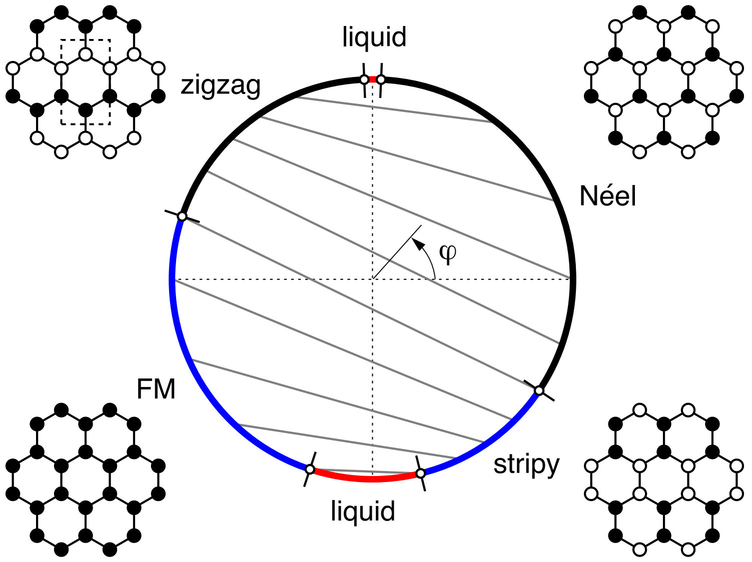
- Na213 orders in the **zigzag** phase (T_N~15 K)

X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

→ J_1 is not the relevant perturbation !

- Θ_{CW}=-125 K
(whereas dominant K_1 coupling is large and FM)

Chaloupka, Jackeli & Khaliullin (2013)



Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

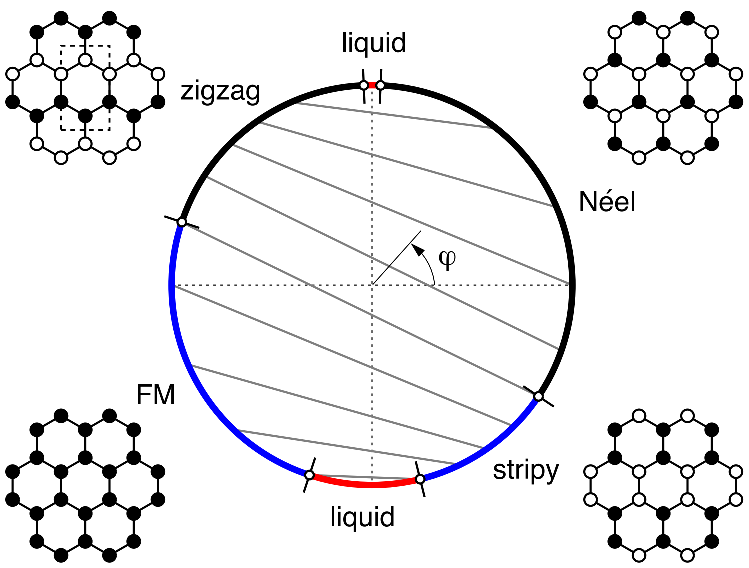
→ J_1 is not the relevant perturbation !

- θ_{CW}=-125 K
(whereas dominant K_1 coupling is large and FM)

→ further-neighbor exchange (J_2, J_3) ?

Kimchi & You (2011), …

Chaloupka, Jackeli & Khaliullin (2013)



Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

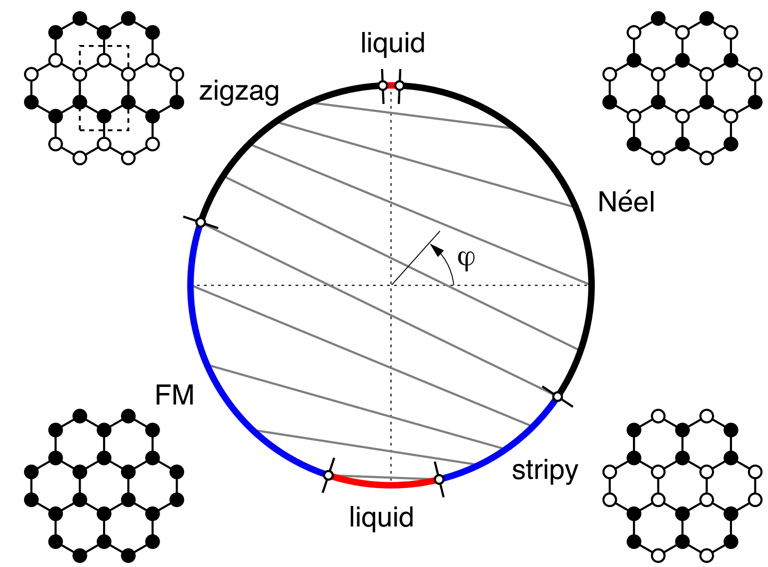
→ J_1 is not the relevant perturbation !

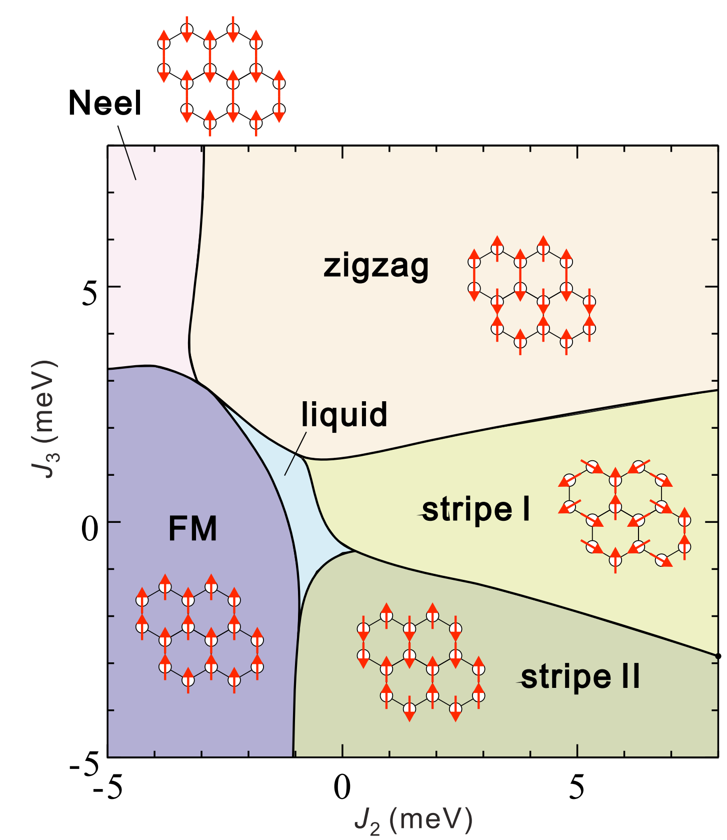
- θ_{CW}=-125 K
(whereas dominant K_1 coupling is large and FM)

→ further-neighbor exchange (J_2, J_3) ?

Kimchi & You (2011), …

Chaloupka, Jackeli & Khaliullin (2013)





Katukuri et al (2014)

Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

→ J_1 is not the relevant perturbation !

- Θ_{CW}=-125 K
(whereas dominant K_1 coupling is large and FM)

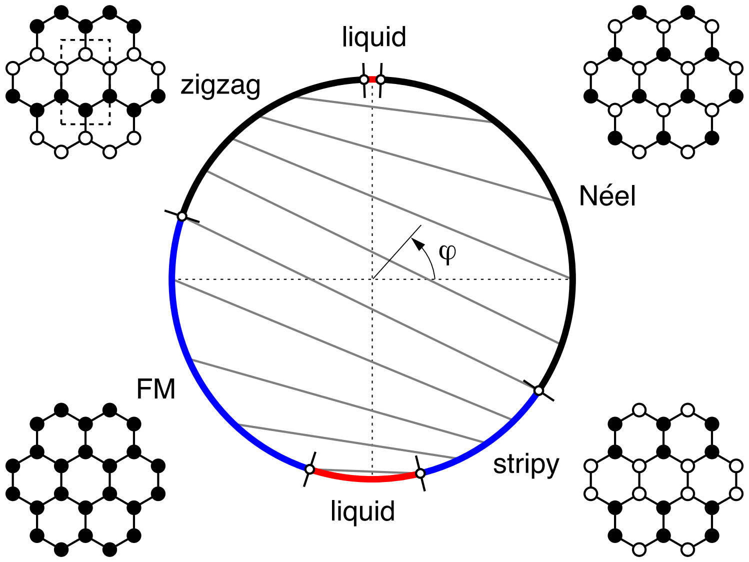
→ further-neighbor exchange (J_2, J_3) ?

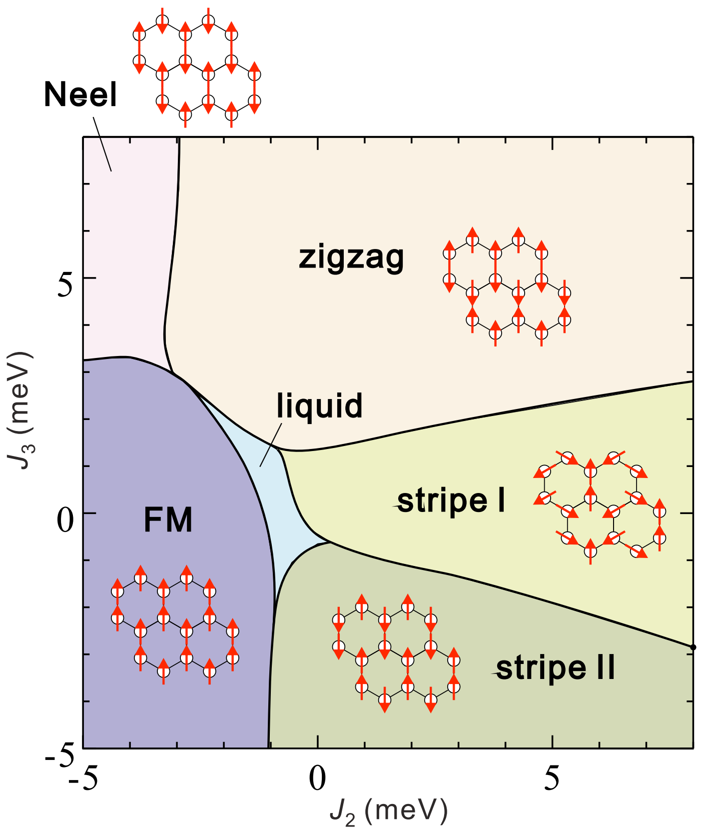
Kimchi & You (2011), …

→ other anisotropic interactions?

S. H. Chun et al, Nature Physics 2015

Chaloupka, Jackeli & Khaliullin (2013)





Katukuri et al (2014)

Current status on Na_2IrO_3

- Na213 orders in the **zigzag** phase (T_N~15 K)

  X. Liu et al ('11), Feng Ye et al ('12), S. K. Choi et al ('12)

  → J_1 is not the relevant perturbation !

- Θ_{CW}=-125 K
  (whereas dominant K_1 coupling is large and FM)

  → further-neighbor exchange (J_2, J_3) ?

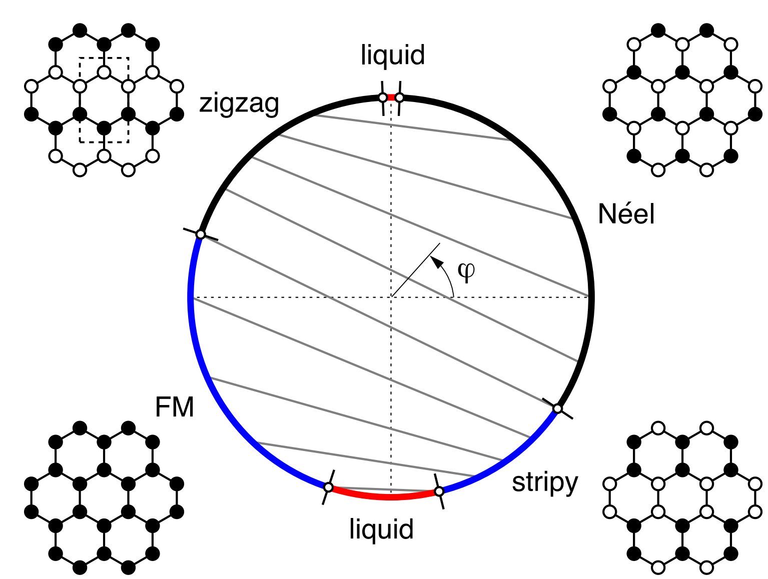
  Kimchi & You (2011), …

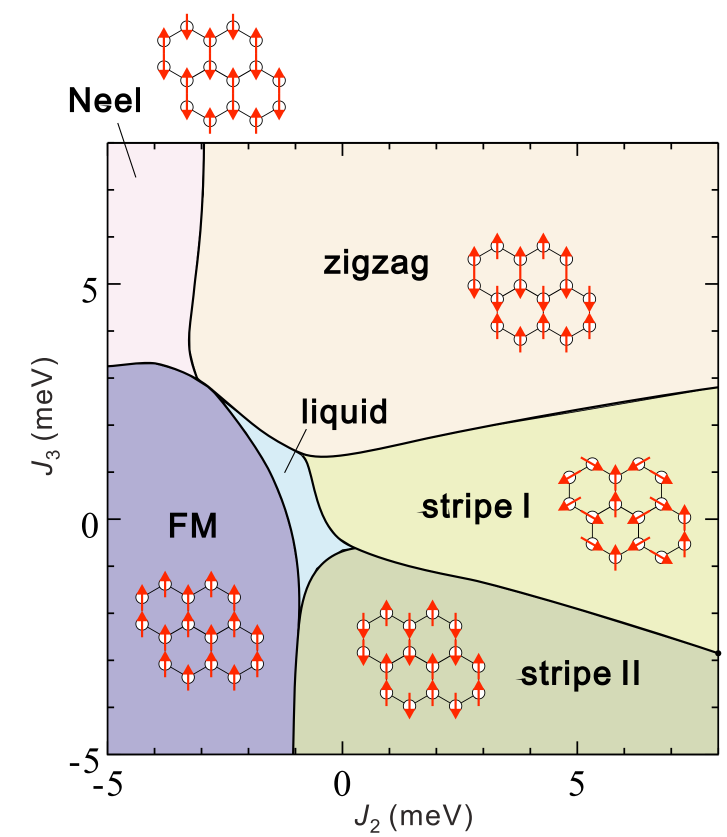
  → other anisotropic interactions?

  S. H. Chun et al, Nature Physics 2015

- superexchange expansion: Sizyuk, Perkins, et al (2014)
  - NN Γ is very small in Na213 (consistent with Q. Chem.)
  - K_2 is the largest coupling after K_1

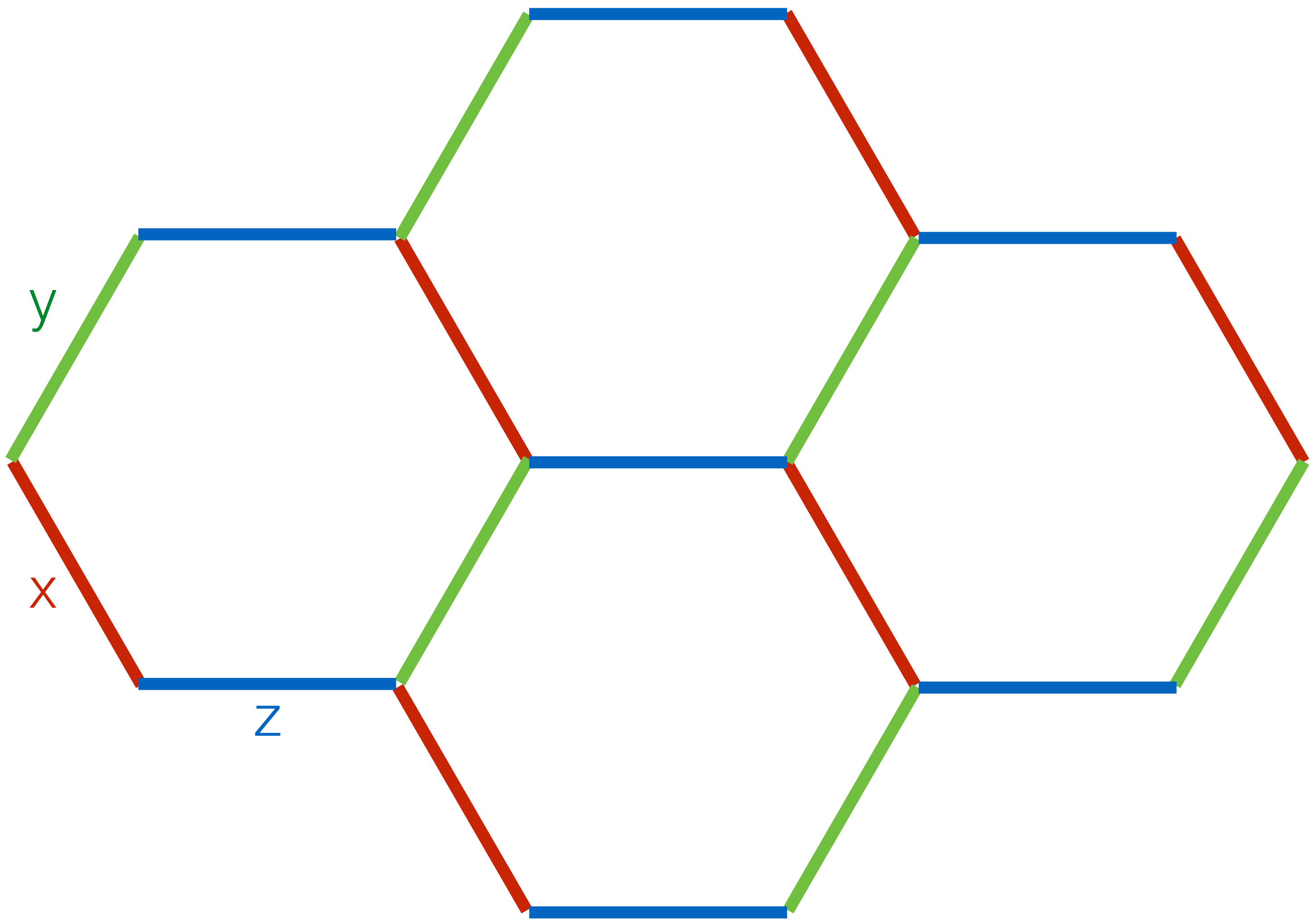
Chaloupka, Jackeli & Khaliullin (2013)



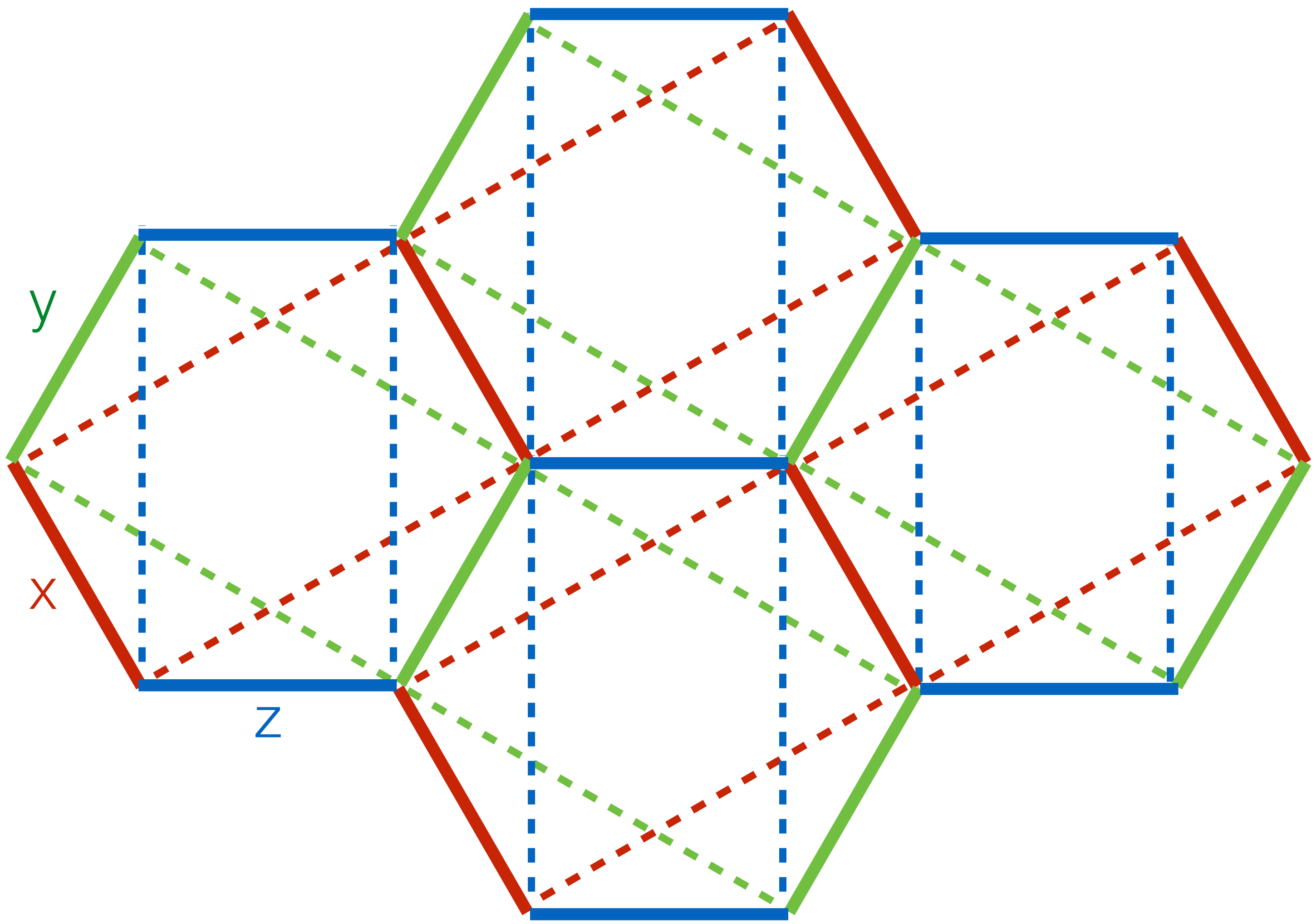


Katukuri et al (2014)

The quantum K_1-K_2 model: general

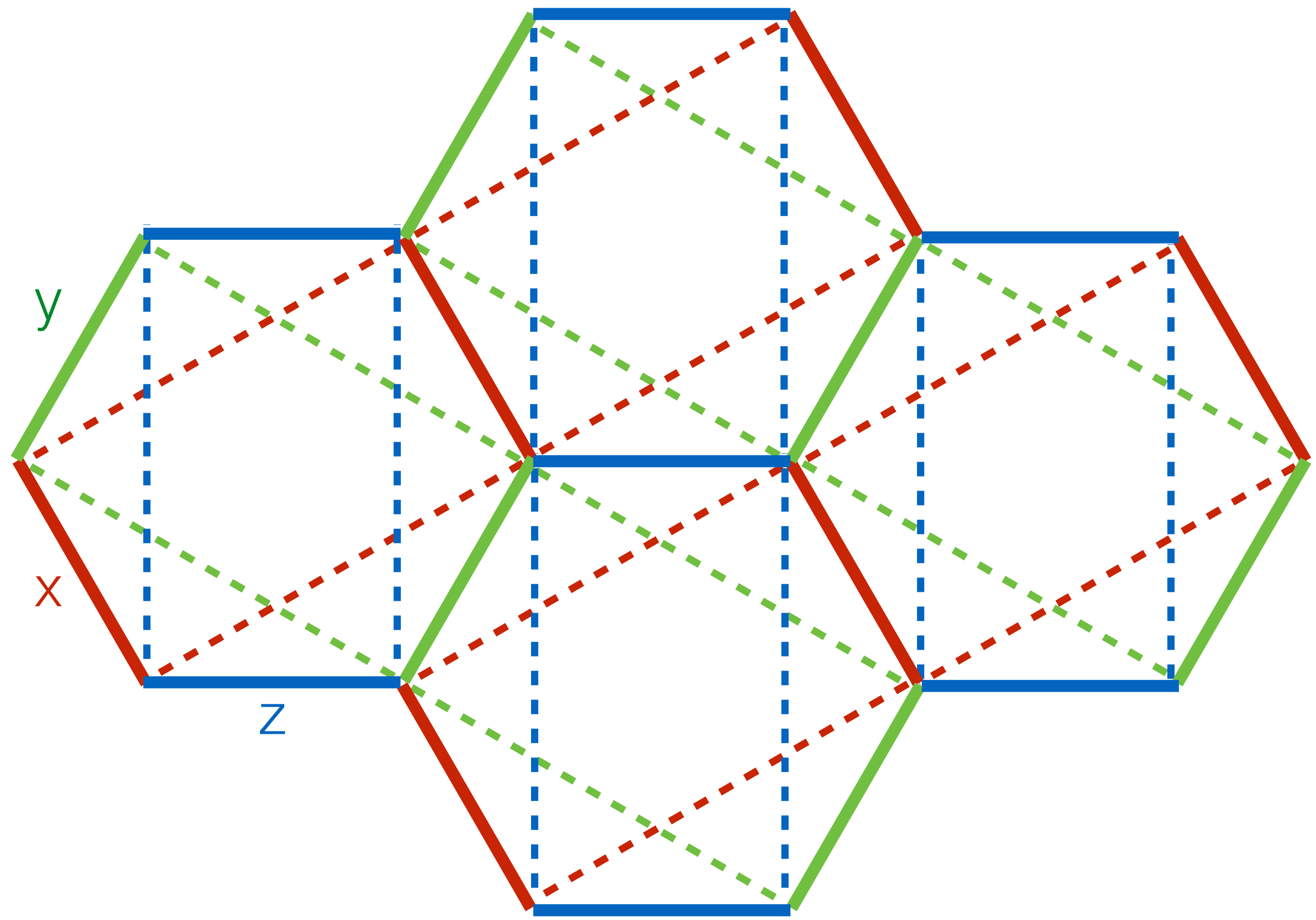


The quantum K1-K2 model: general



$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + K_2 \sum_{\langle\langle ij \rangle\rangle \in \alpha} S_i^\alpha S_j^\alpha$$

The quantum K_1-K_2 model: general

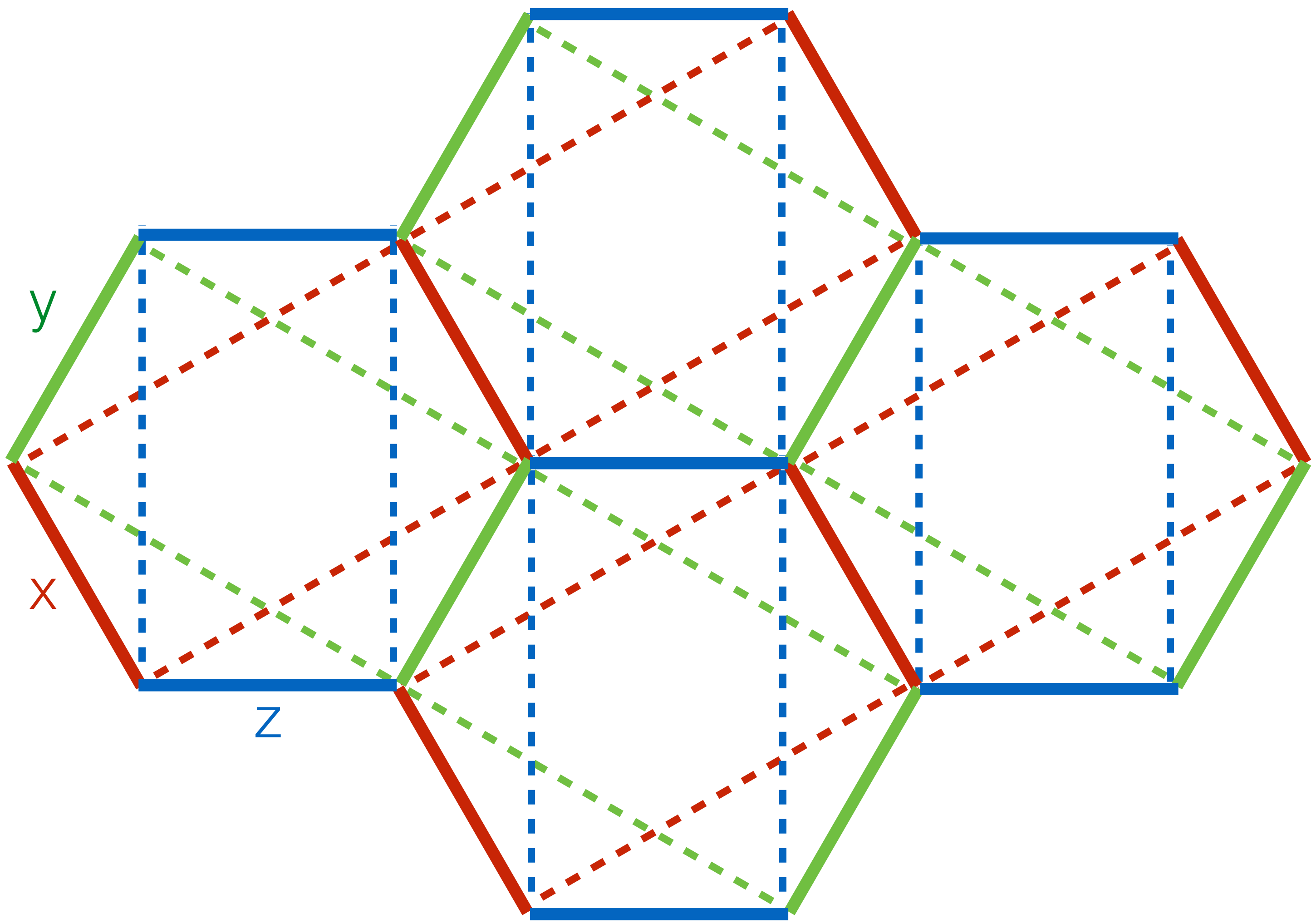


$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + K_2 \sum_{\langle\langle ij \rangle\rangle \in \alpha} S_i^\alpha S_j^\alpha$$

- interpolates between honeycomb Kitaev & triangular Kitaev

I.R. et al (2012), Becker et al (2015)
Jackeli & Avella(2015)

The quantum K_1-K_2 model: general



$$\mathcal{H} = K_1 \sum_{\langle ij \rangle \in \alpha} S_i^\alpha S_j^\alpha + K_2 \sum_{\ll ij \gg \in \alpha} S_i^\alpha S_j^\alpha$$

- interpolates between honeycomb Kitaev & triangular Kitaev

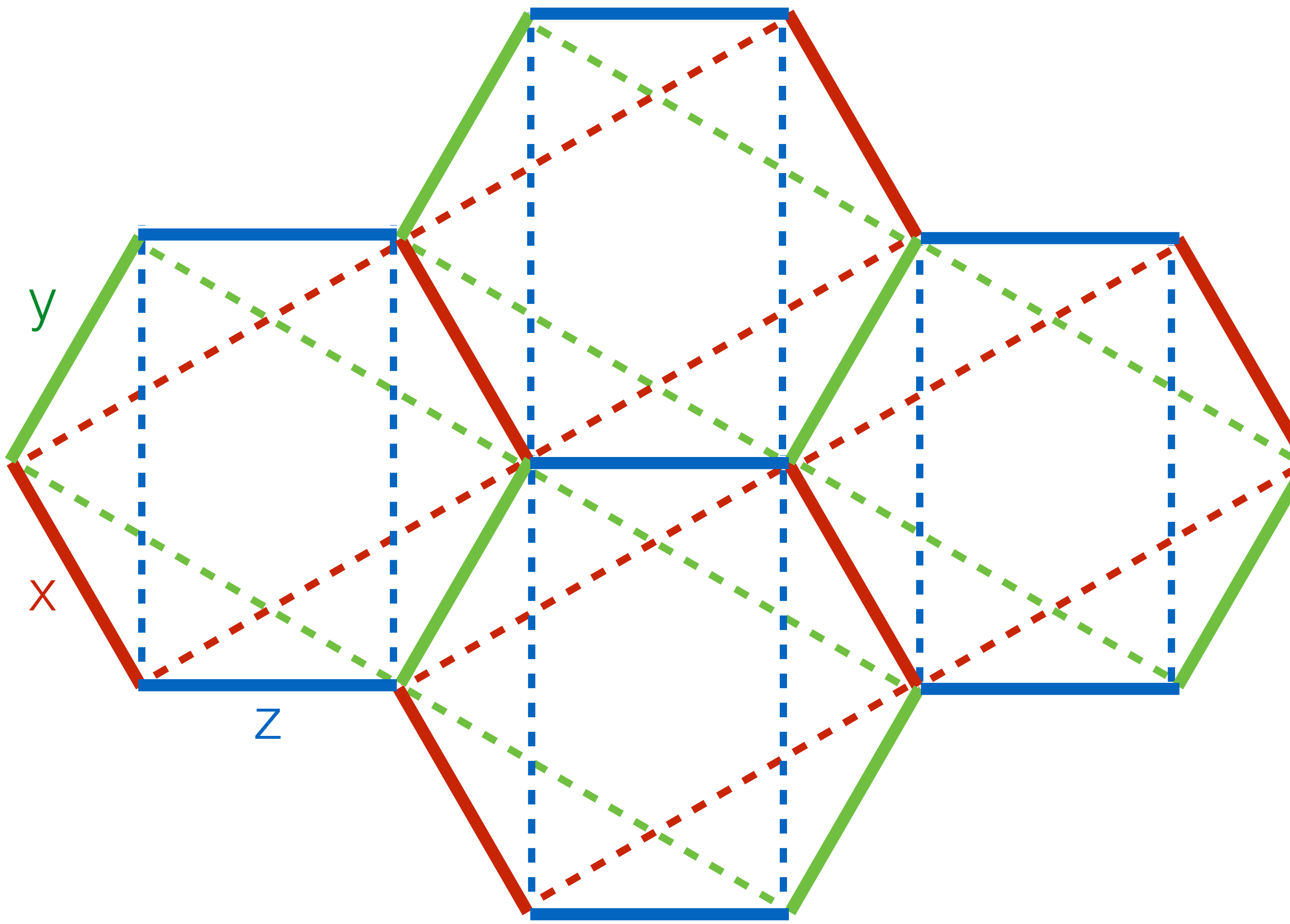
I.R. et al (2012), Becker et al (2015)
Jackeli & Avella(2015)

- K_2 kills the exact solvability: W_h are no longer conserved

The quantum K_1-K_2 model: hidden duality & symmetries

The quantum K_1-K_2 model: hidden duality & symmetries

- global symmetry in real+spin space: double cover of C_{3v}



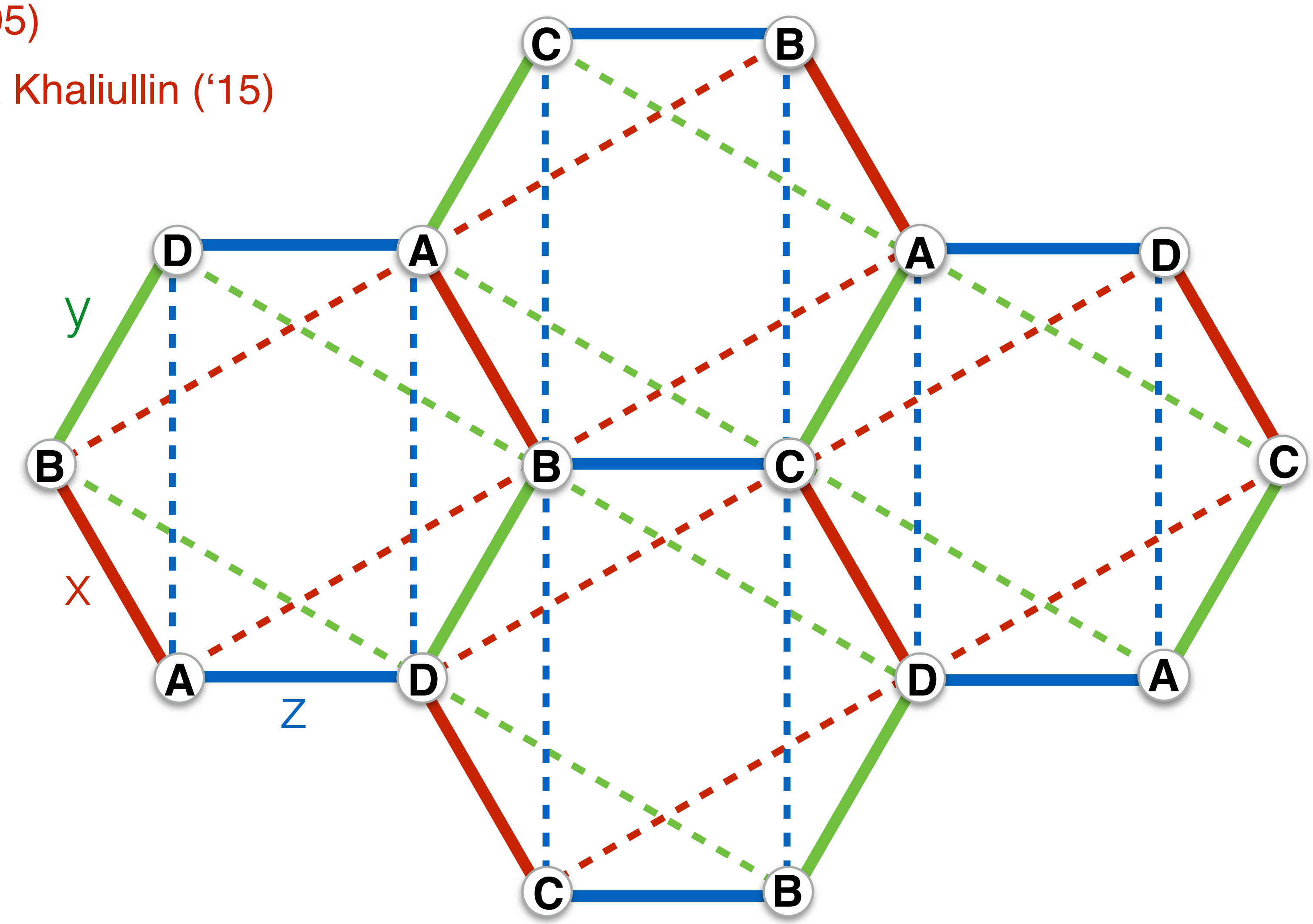
The quantum K_1-K_2 model: hidden duality & symmetries

- global symmetry in real+spin space: double cover of C_{3v}

- 4-sublattice decomposition (letters ABCD)

Khaliullin ('05)

Chaloupka & Khaliullin ('15)

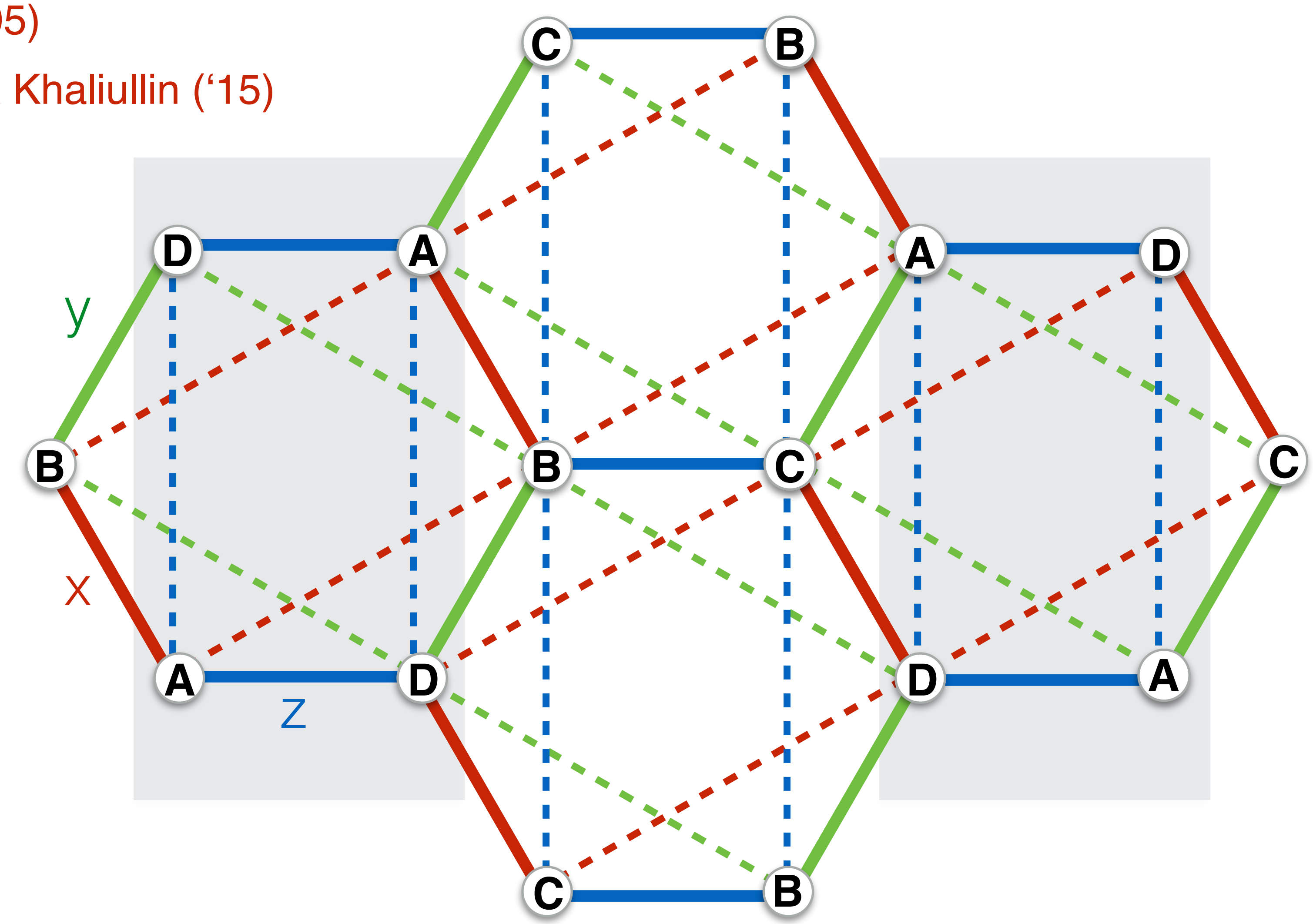


The quantum K_1-K_2 model: hidden duality & symmetries

- global symmetry in real+spin space: double cover of C_{3v}

- 4-sublattice decomposition (letters ABCD)

Khaliullin ('05)
Chaloupka & Khaliullin ('15)



The quantum K_1-K_2 model: hidden duality & symmetries

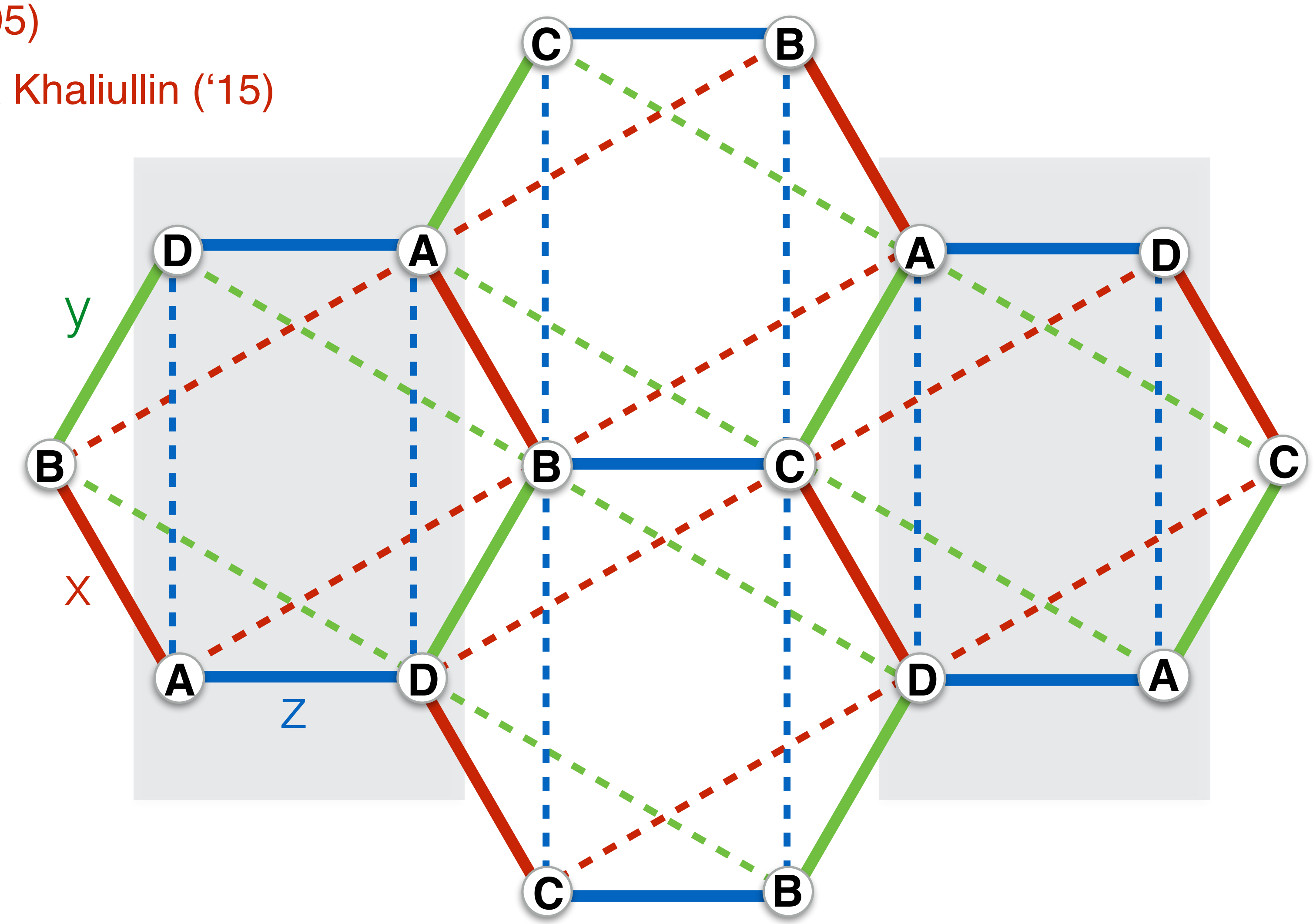
- global symmetry in real+spin space: double cover of C_{3v}

- 4-sublattice decomposition (letters ABCD)

Khaliullin ('05)
Chaloupka & Khaliullin ('15)

- duality that maps $(K_1,K_2) \rightarrow (-K_1,-K_2)$

$$D_{yzx} = \begin{pmatrix} A & B & C & D \\ \mathbf{1} & C_{2y} & C_{2z} & C_{2x} \end{pmatrix}$$

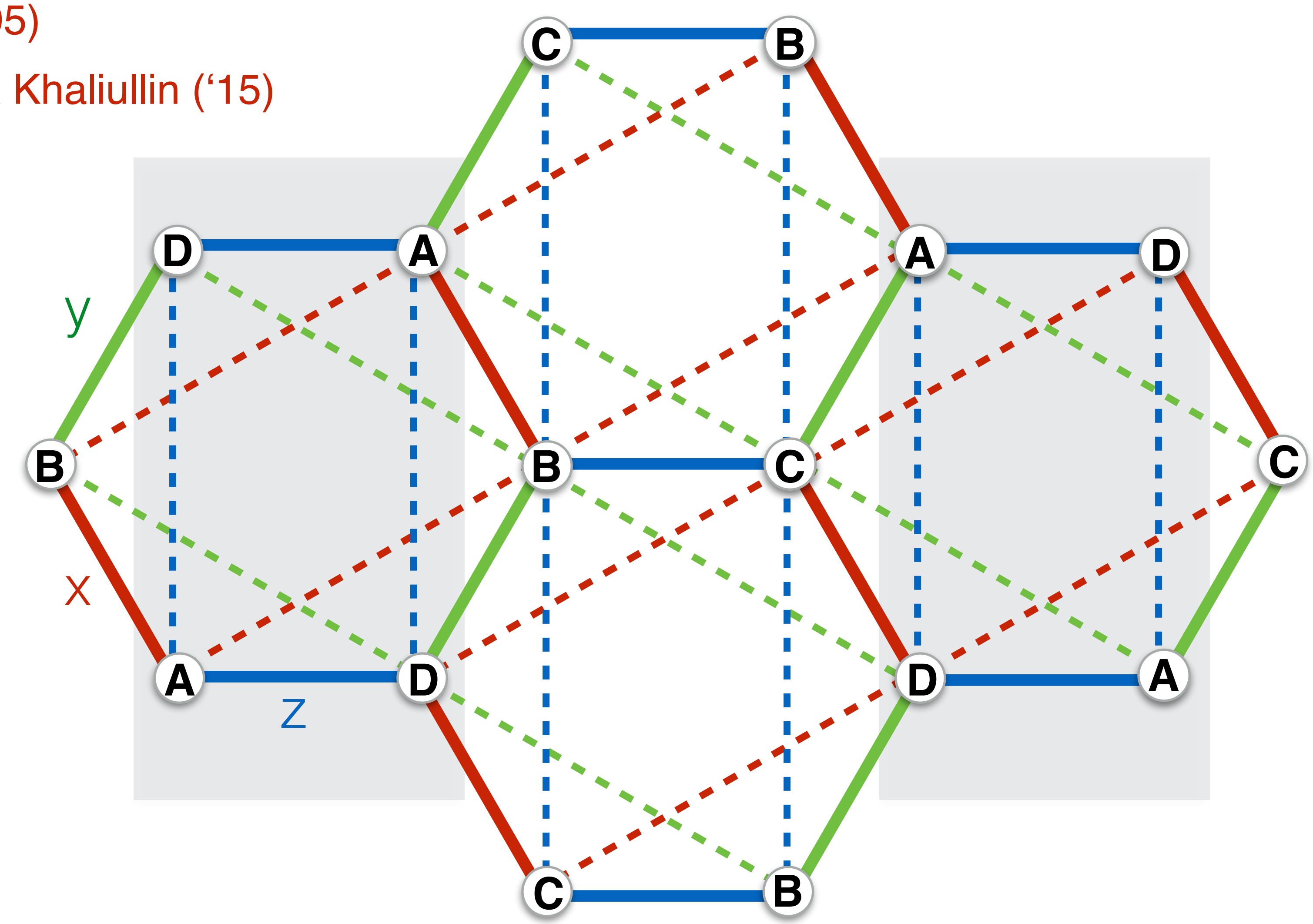


The quantum K_1-K_2 model: hidden duality & symmetries

- global symmetry in real+spin space: double cover of C_{3v}

- 4-sublattice decomposition (letters ABCD)

Khaliullin ('05)
Chaloupka & Khaliullin ('15)



- duality that maps $(K_1,K_2)\rightarrow(-K_1,-K_2)$

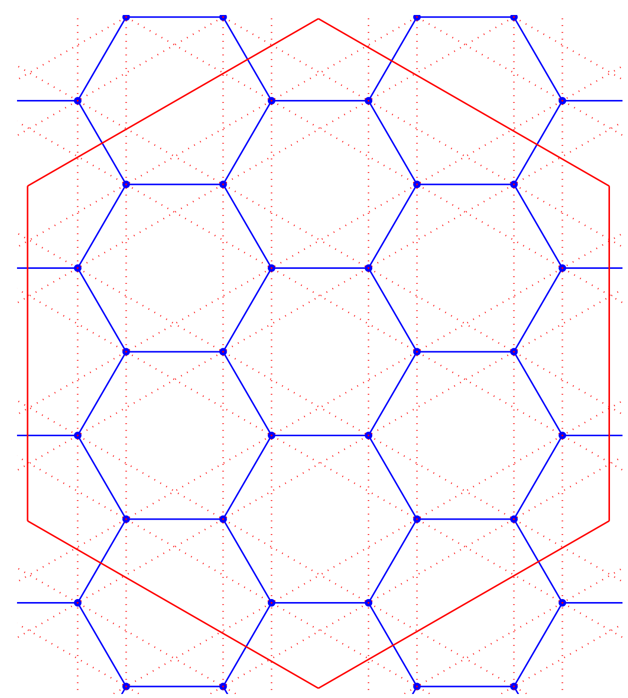
$$D_{yzx} = \begin{pmatrix} A & B & C & D \\ \mathbf{1} & C_{2y} & C_{2z} & C_{2x} \end{pmatrix}$$

- gauge-like symmetry

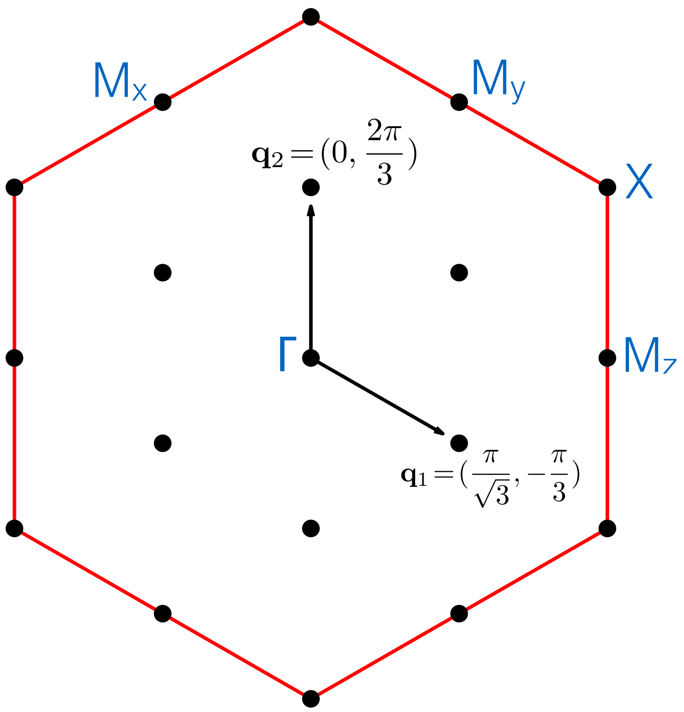
$$S_{xyz} = \begin{pmatrix} A & B & C & D \\ \mathbf{1} & C_{2x} & C_{2y} & C_{2z} \end{pmatrix}$$

Exact diagonalizations: finite-size clusters

24-sites

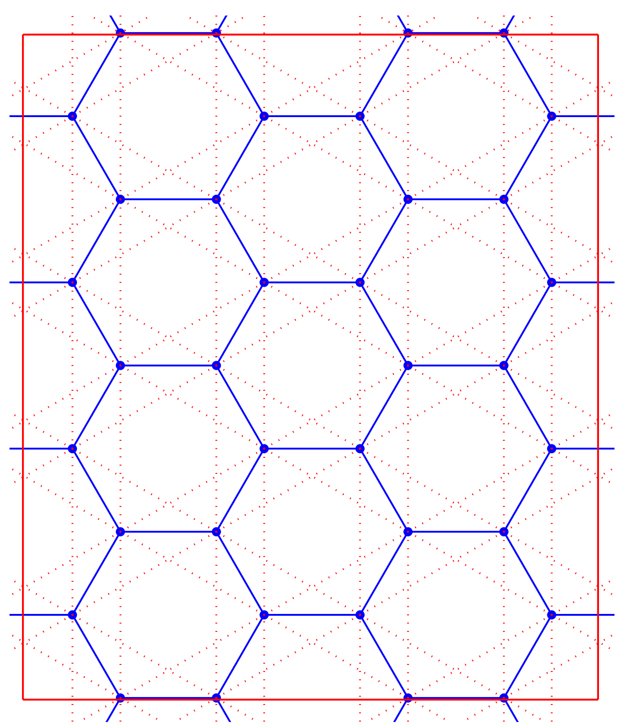


(a)

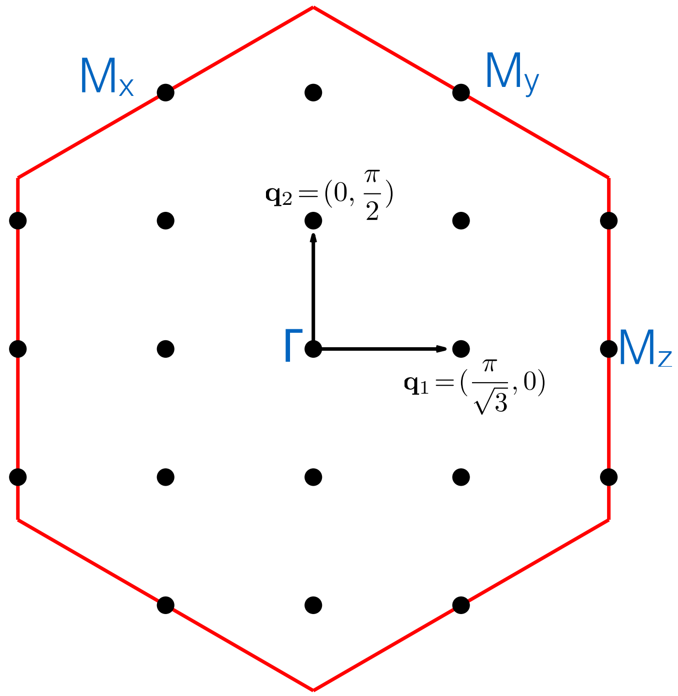


(b)

32-sites

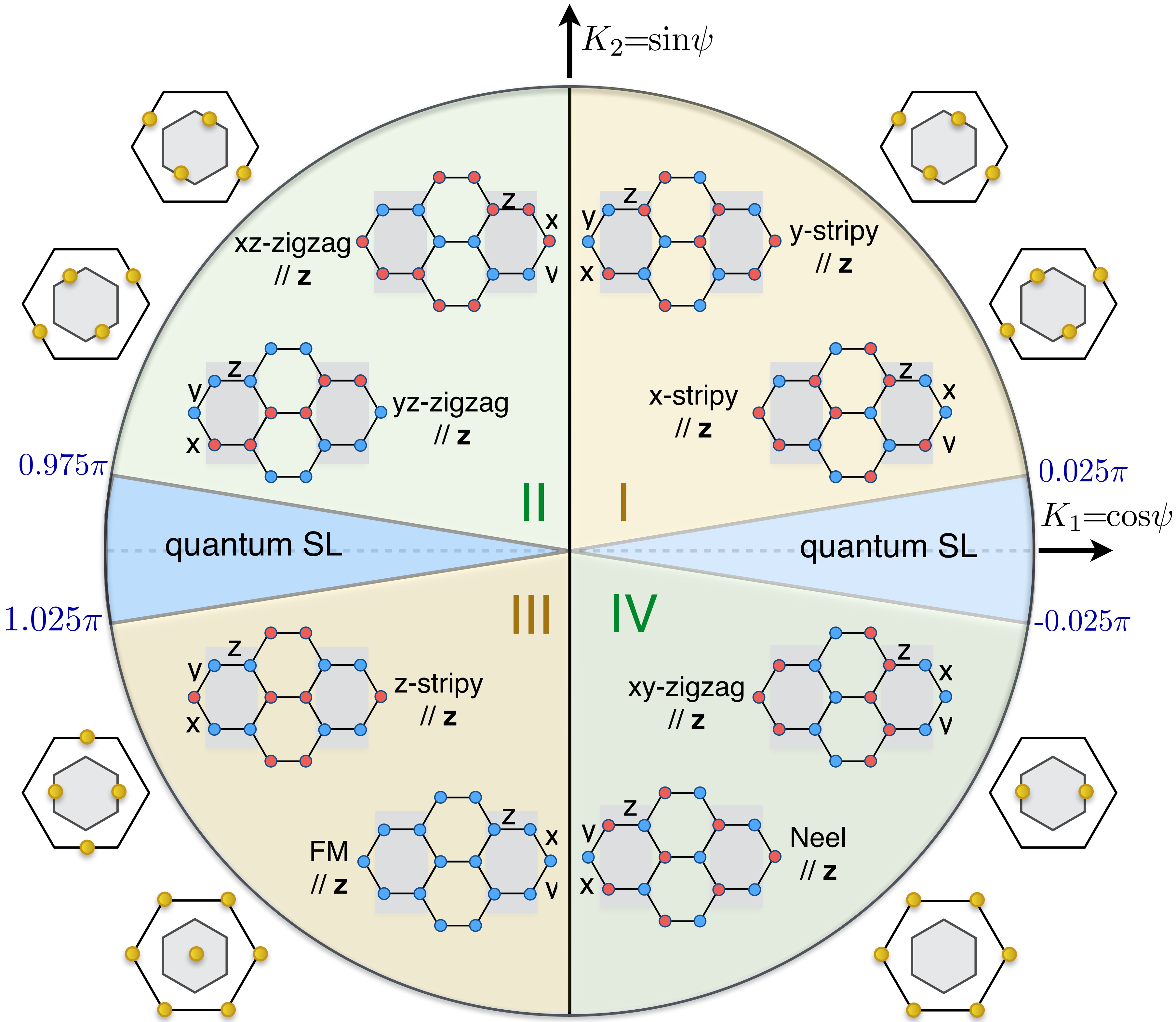


(c)



(d)

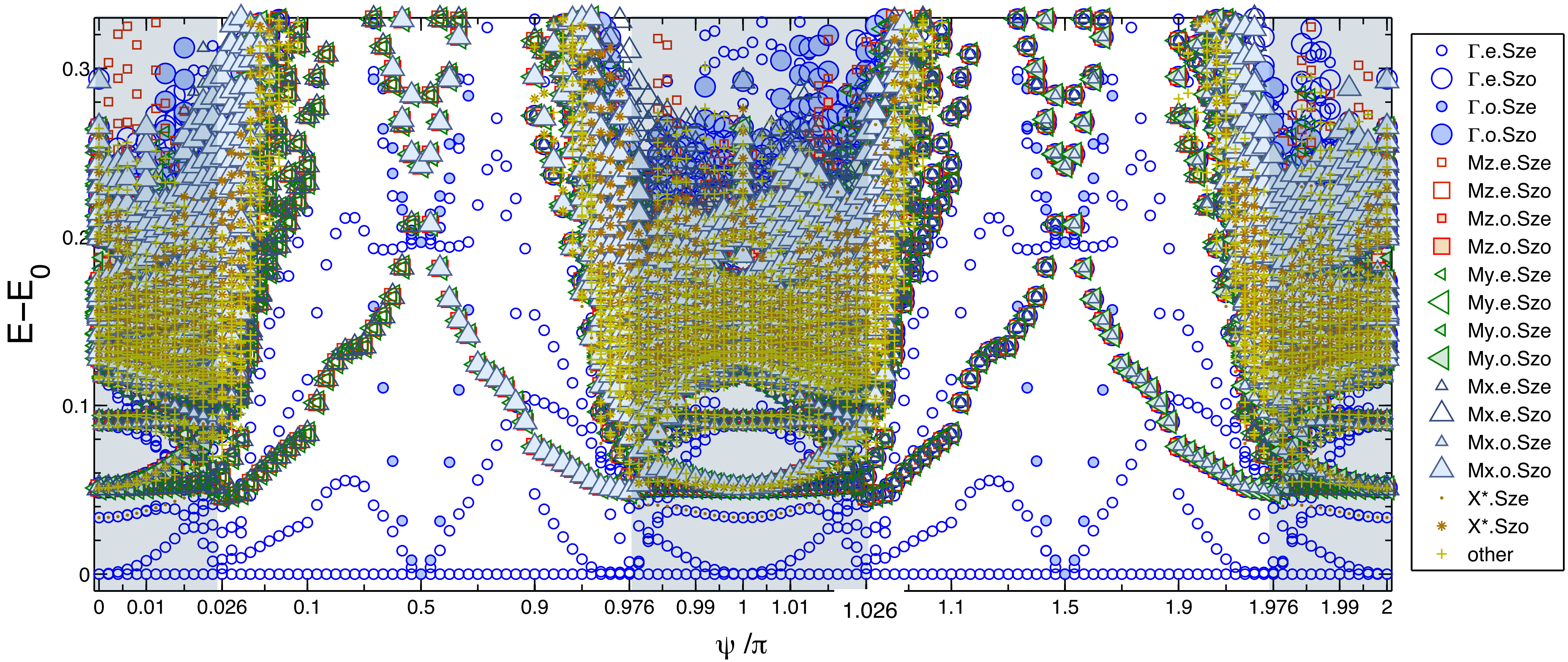
The quantum K_1-K_2 model: phase diagram obtained from ED



- QSL is very fragile !
- 4 magnetic LRO regions; related by duality
- zigzag in Na213
- 12 GS's in each region; some are very different !

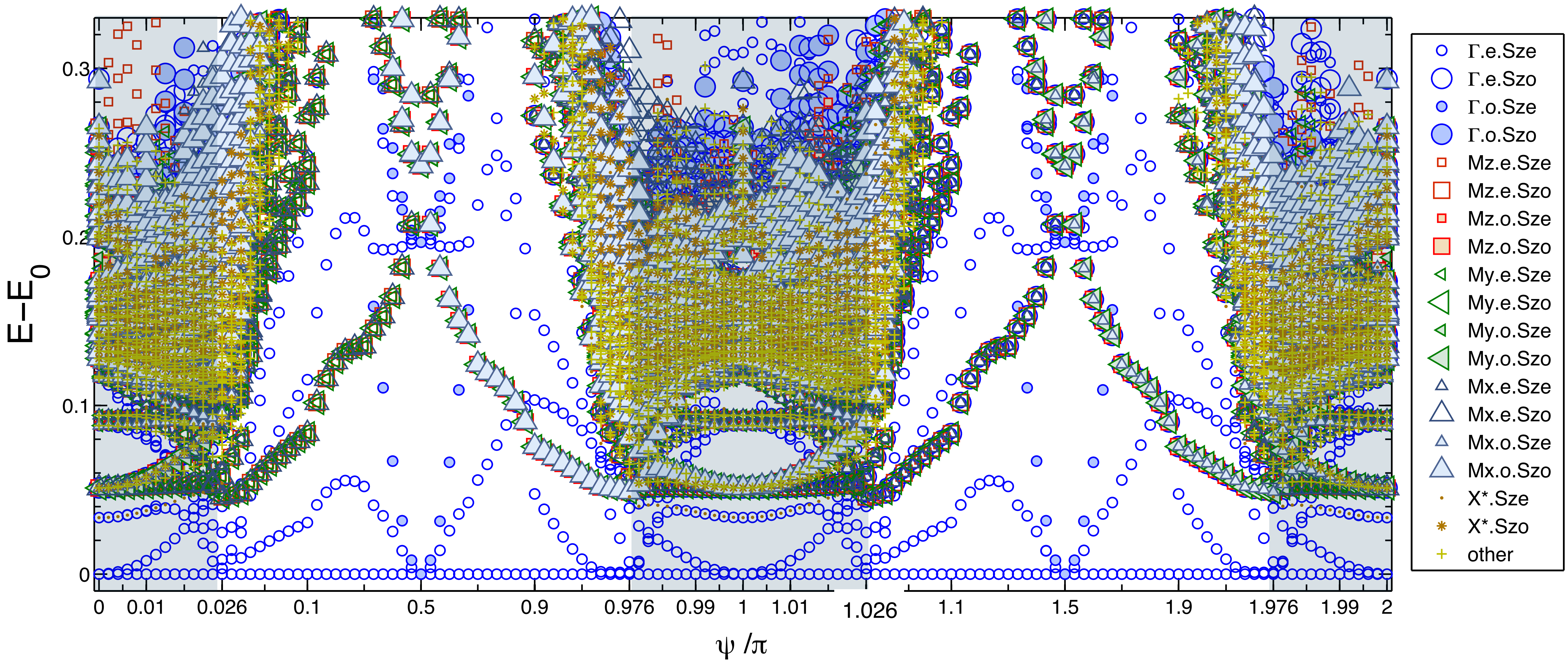
The quantum K_1-K_2 model: “low-energy spectroscopy”

24-site spectrum



The quantum K_1-K_2 model: “low-energy spectroscopy”

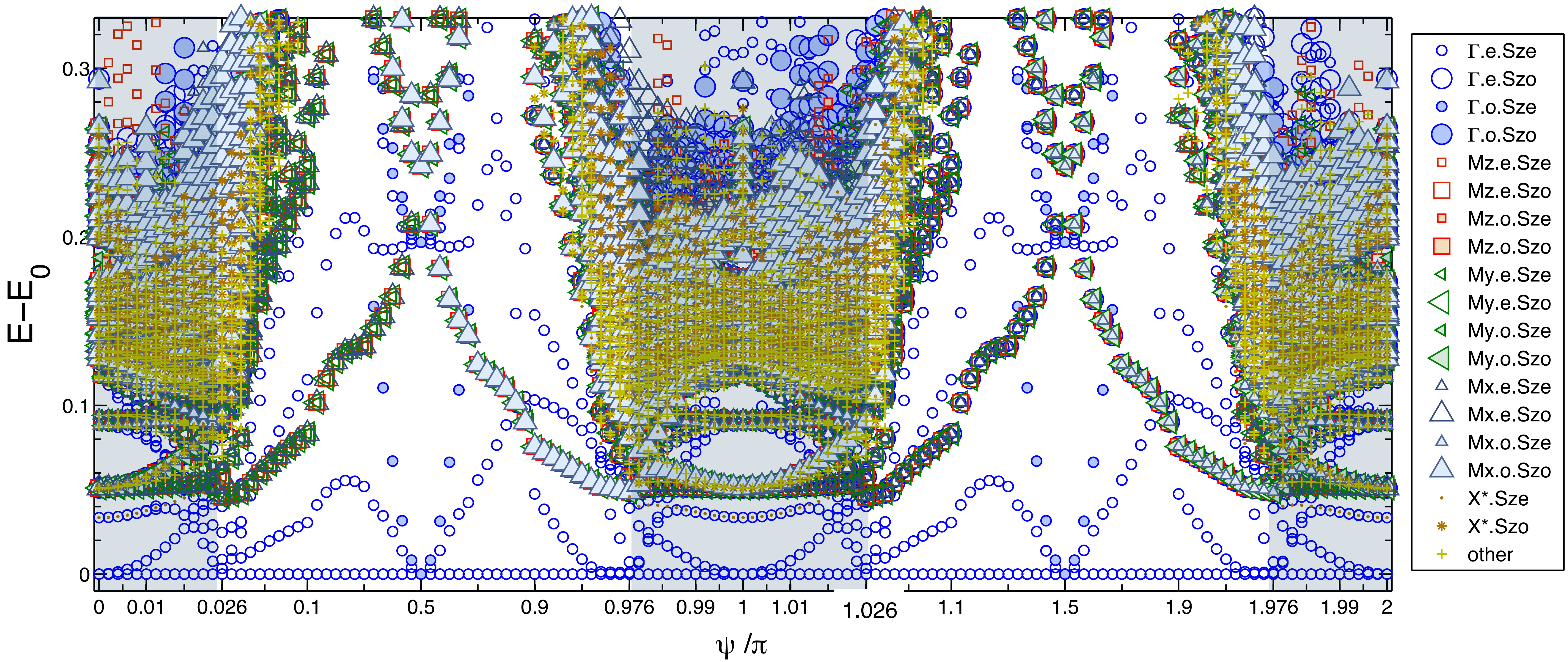
24-site spectrum



- Kitaev spin liquid regions: very dense spectra !

The quantum K_1-K_2 model: “low-energy spectroscopy”

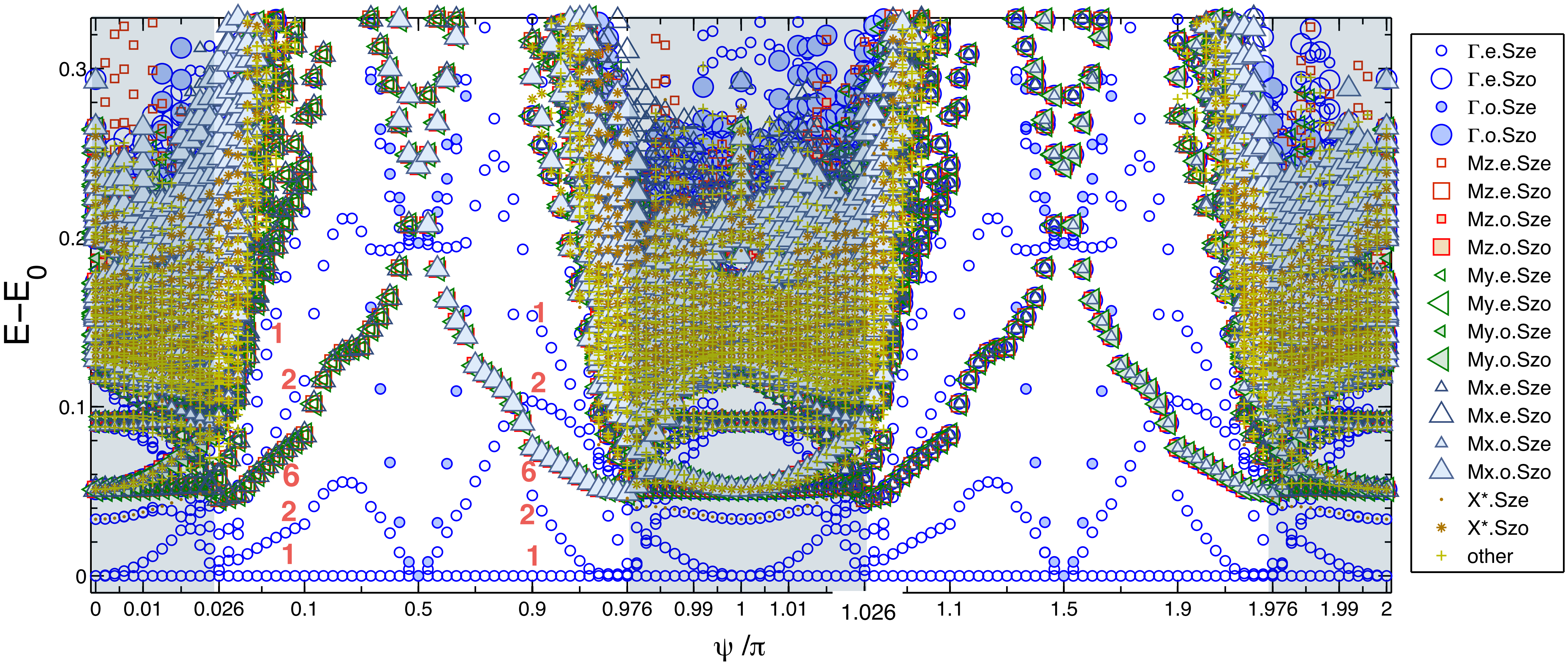
24-site spectrum



- Kitaev spin liquid regions: very dense spectra !
- Magnetic LRO regions: small number of low-lying states

The quantum K_1-K_2 model: "low-energy spectroscopy"

24-site spectrum



- Kitaev spin liquid regions: very dense spectra !
- Magnetic LRO regions: small number of low-lying states
- low-energy spectroscopy: multiplicity & symmetry quantum numbers; large spin gap

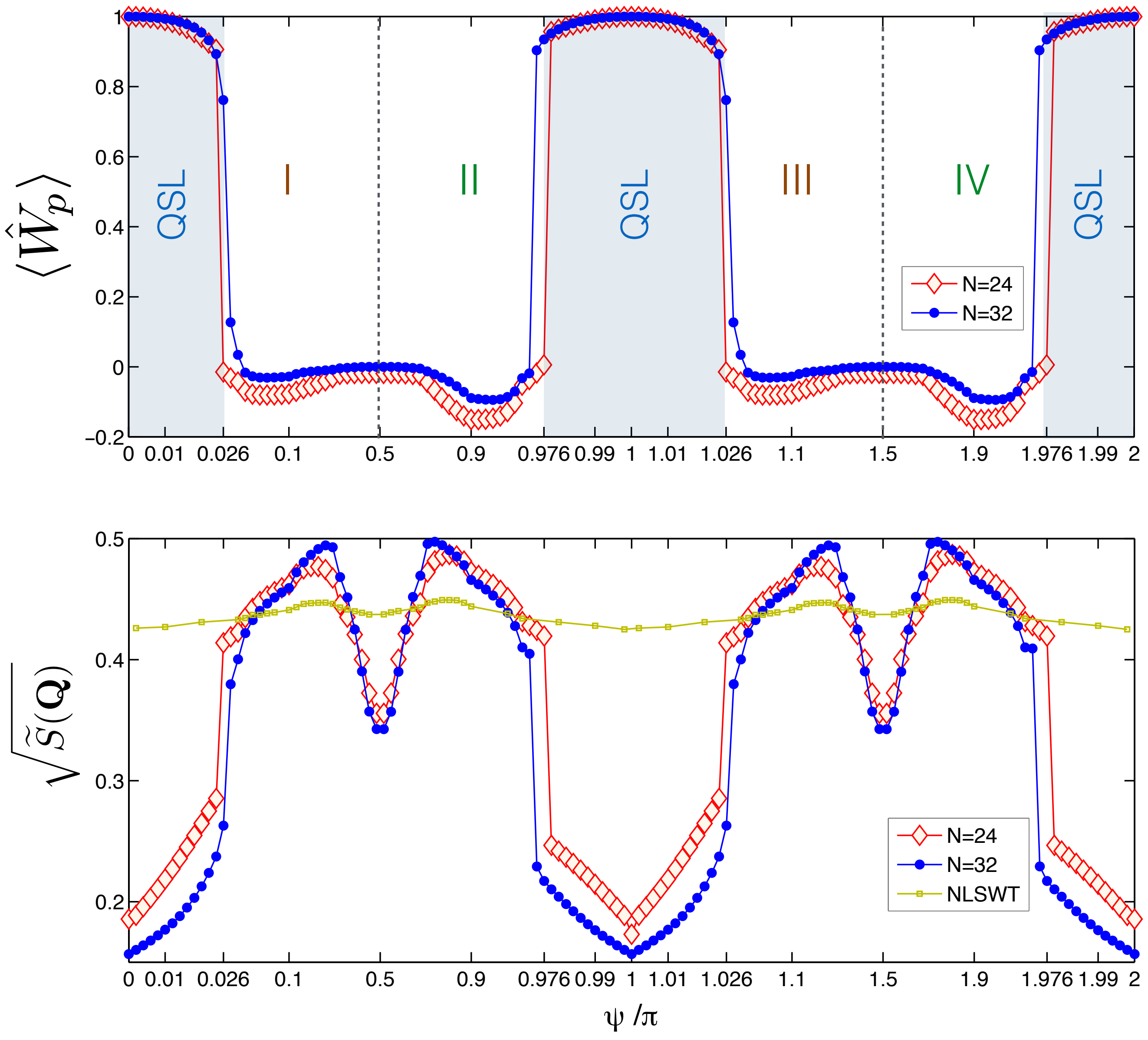
The quantum K_1-K_2 model: fluxes and spin structure factors

The quantum K_1-K_2 model: fluxes and spin structure factors

- transitions out of the Kitaev spin liquid are 1st-order

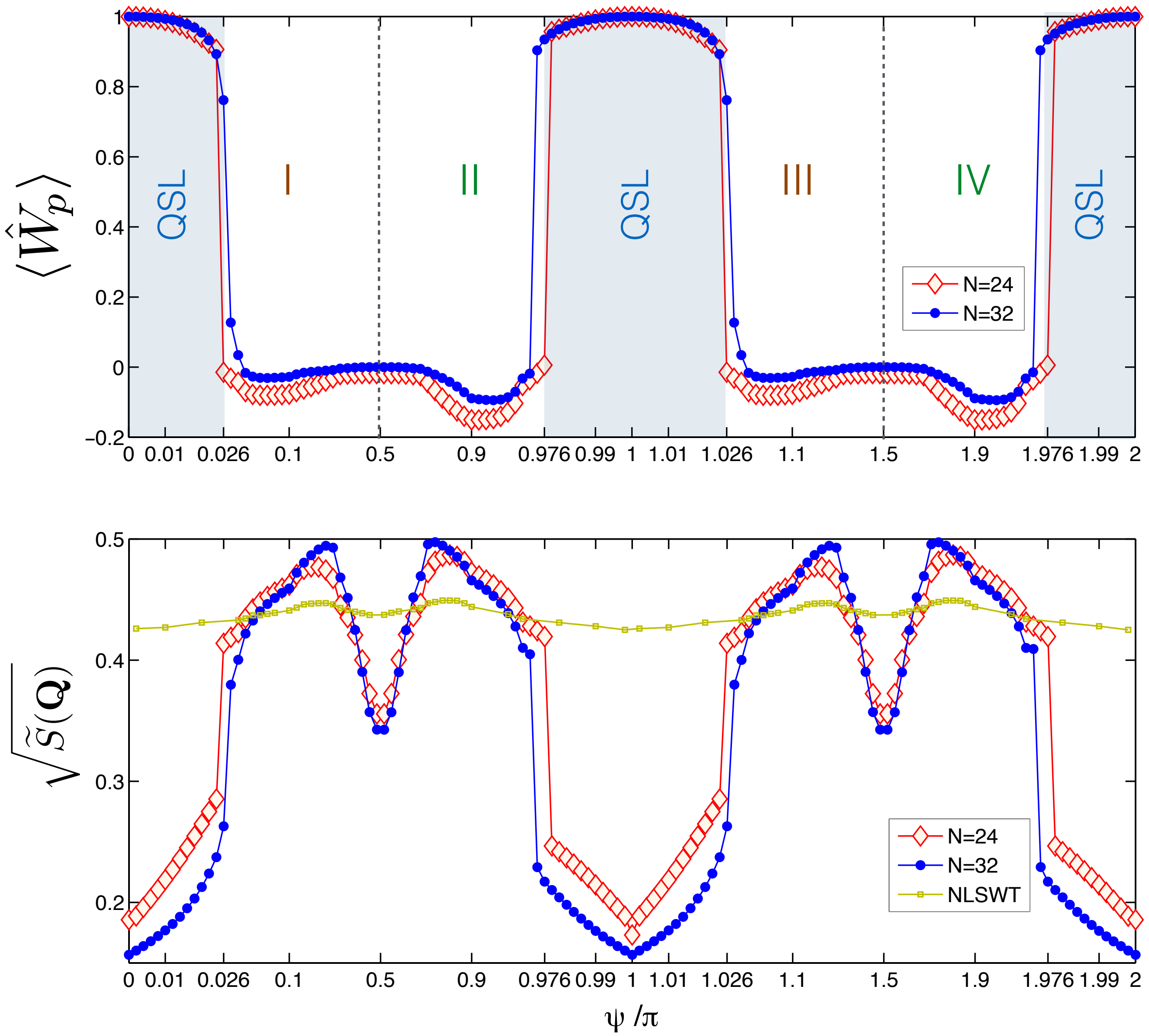
The quantum K_1-K_2 model: fluxes and spin structure factors

- transitions out of the Kitaev spin liquid are 1st-order



The quantum K_1-K_2 model: fluxes and spin structure factors

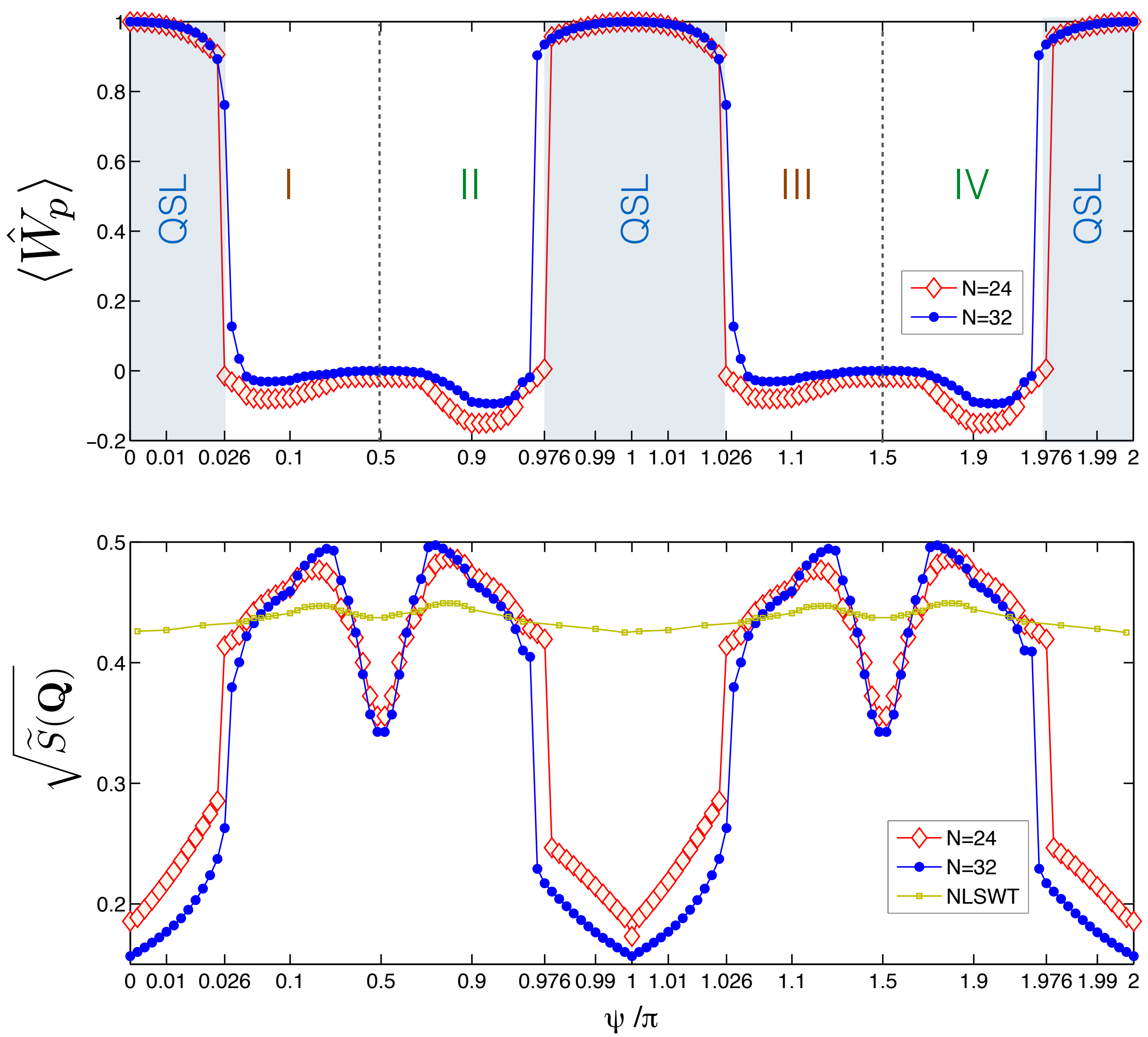
- transitions out of the Kitaev spin liquid are 1st-order

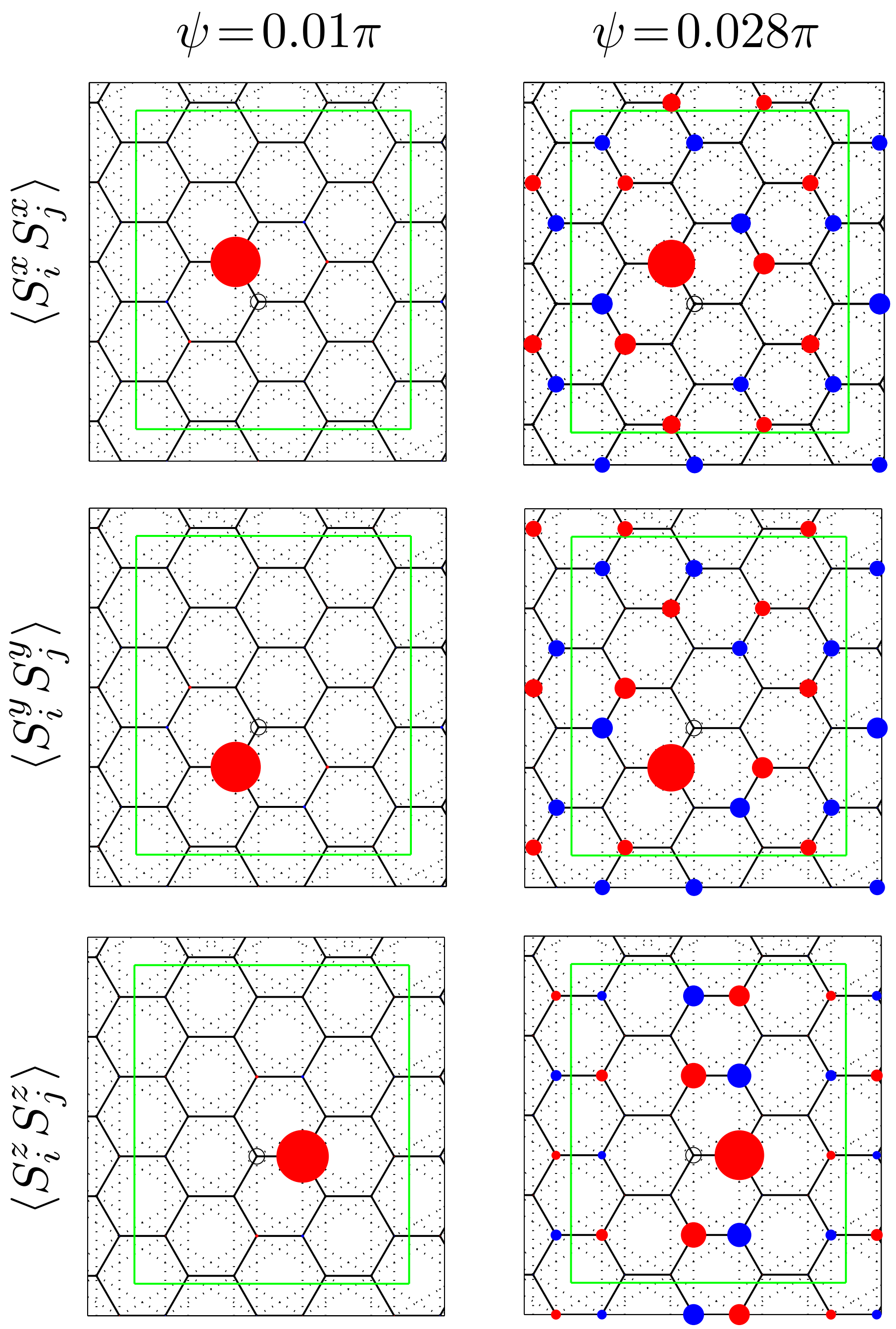


- short vs. long-range ordering

The quantum K_1-K_2 model: fluxes and spin structure factors

- transitions out of the Kitaev spin liquid are 1st-order

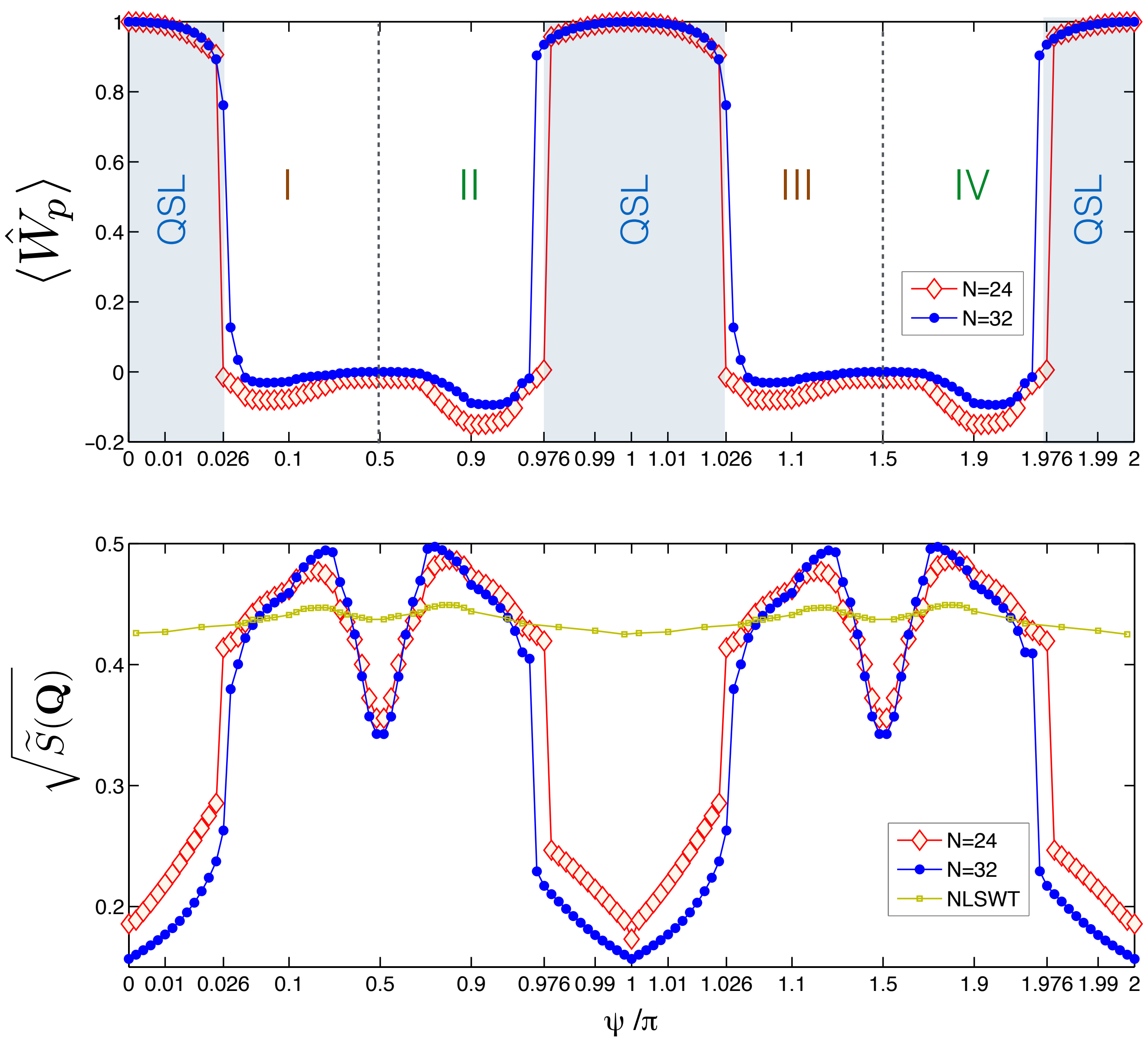


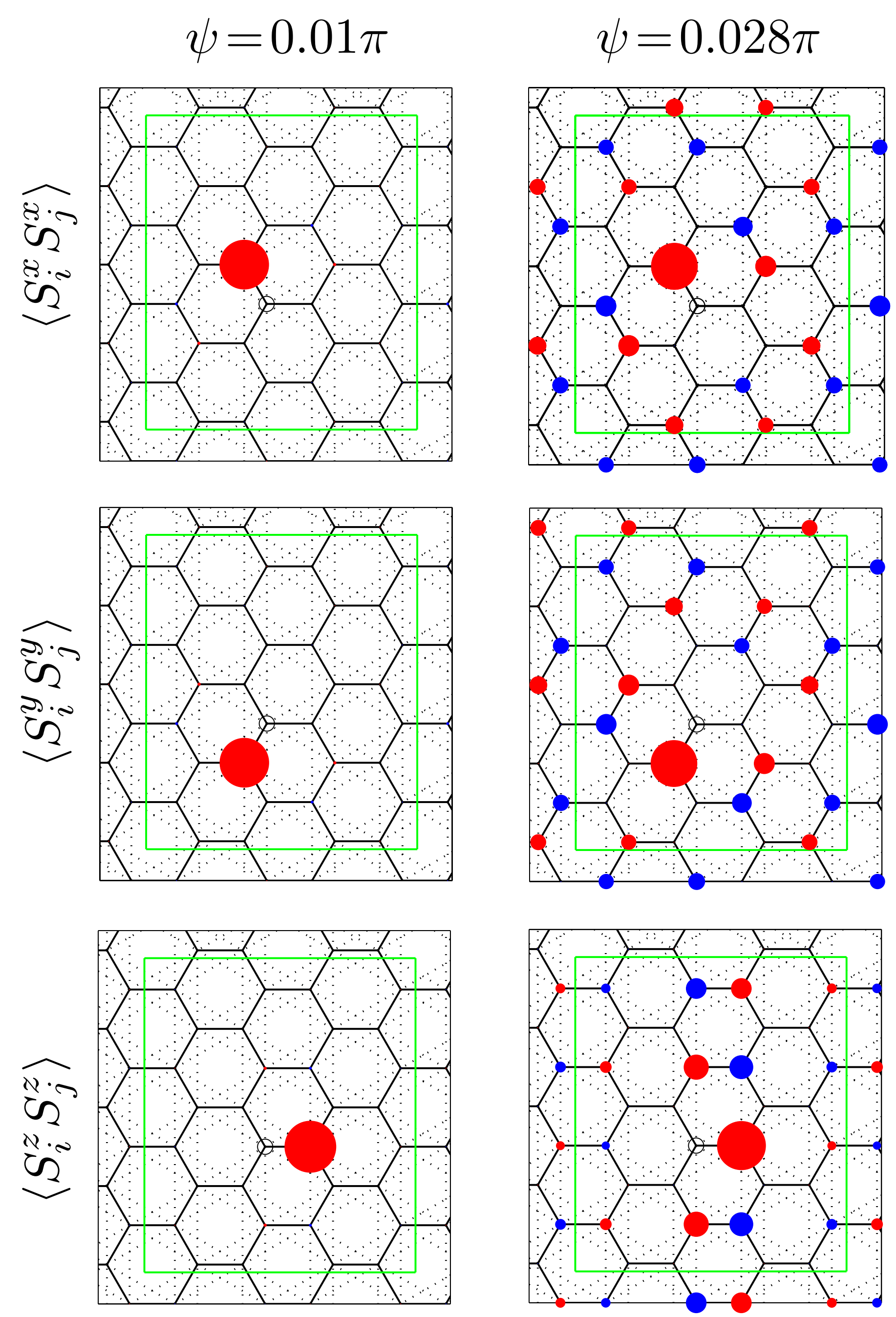


- short vs. long-range ordering

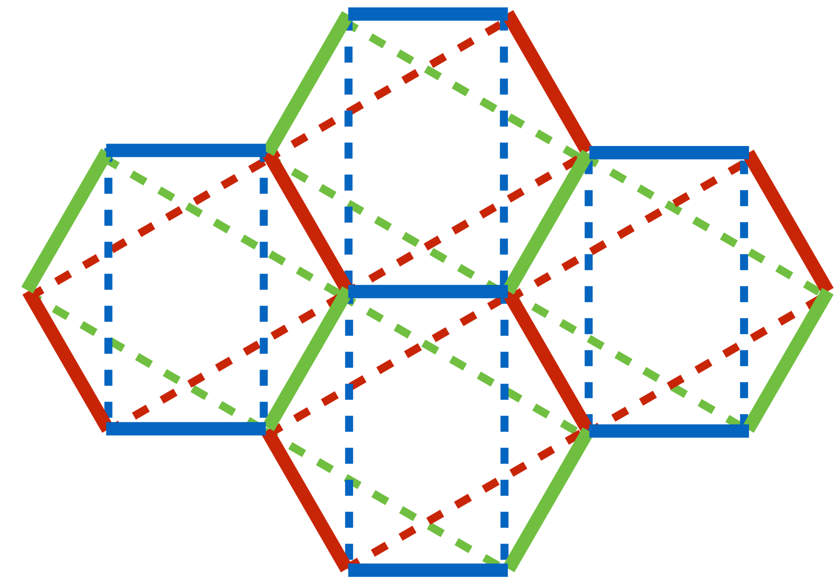
The quantum K_1-K_2 model: fluxes and spin structure factors

- transitions out of the Kitaev spin liquid are 1st-order





- short vs. long-range ordering
- local spin length is very close to 1/2; states seem very classical !



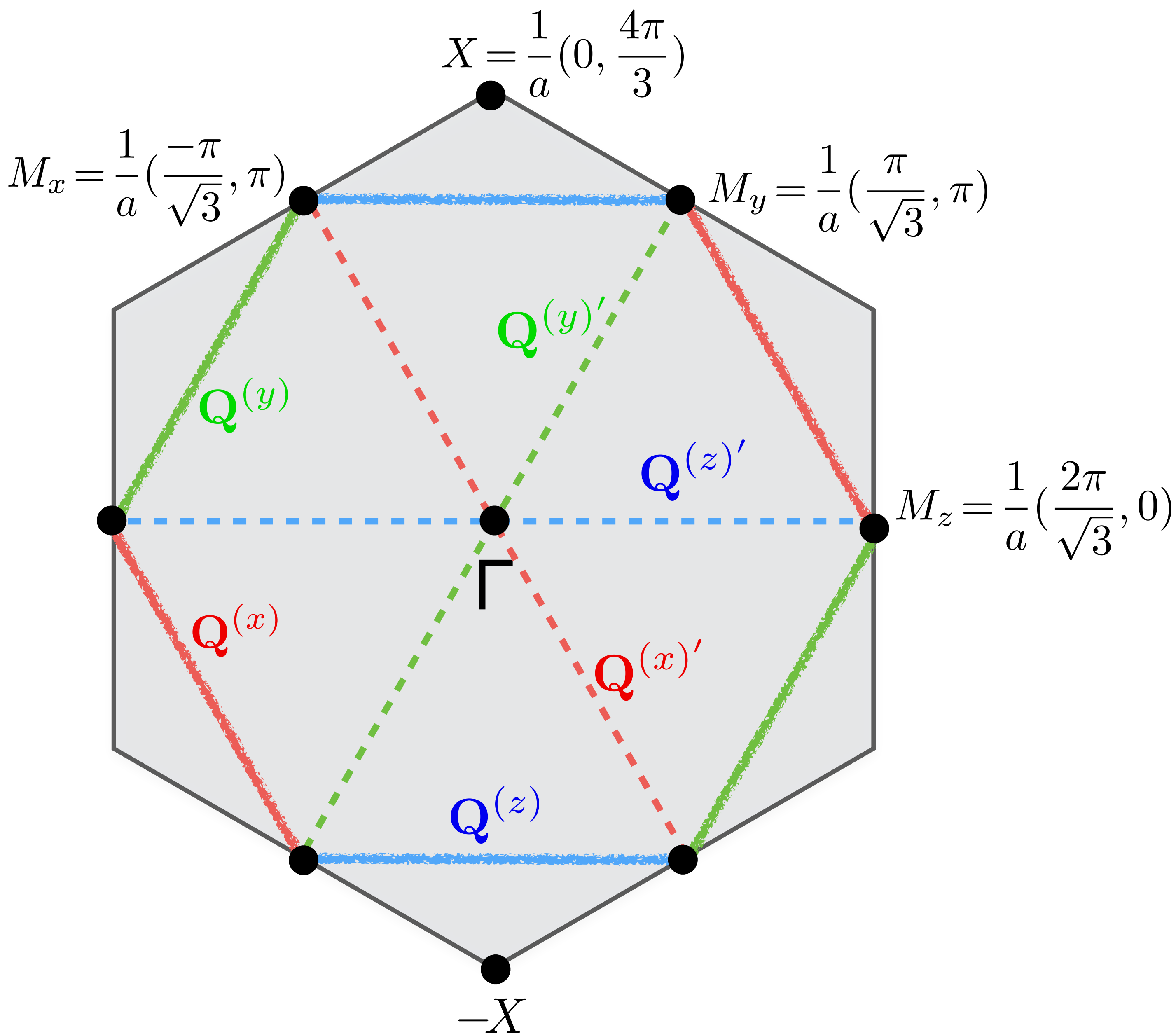
What is the physical mechanism of instability?

Striking aspect:

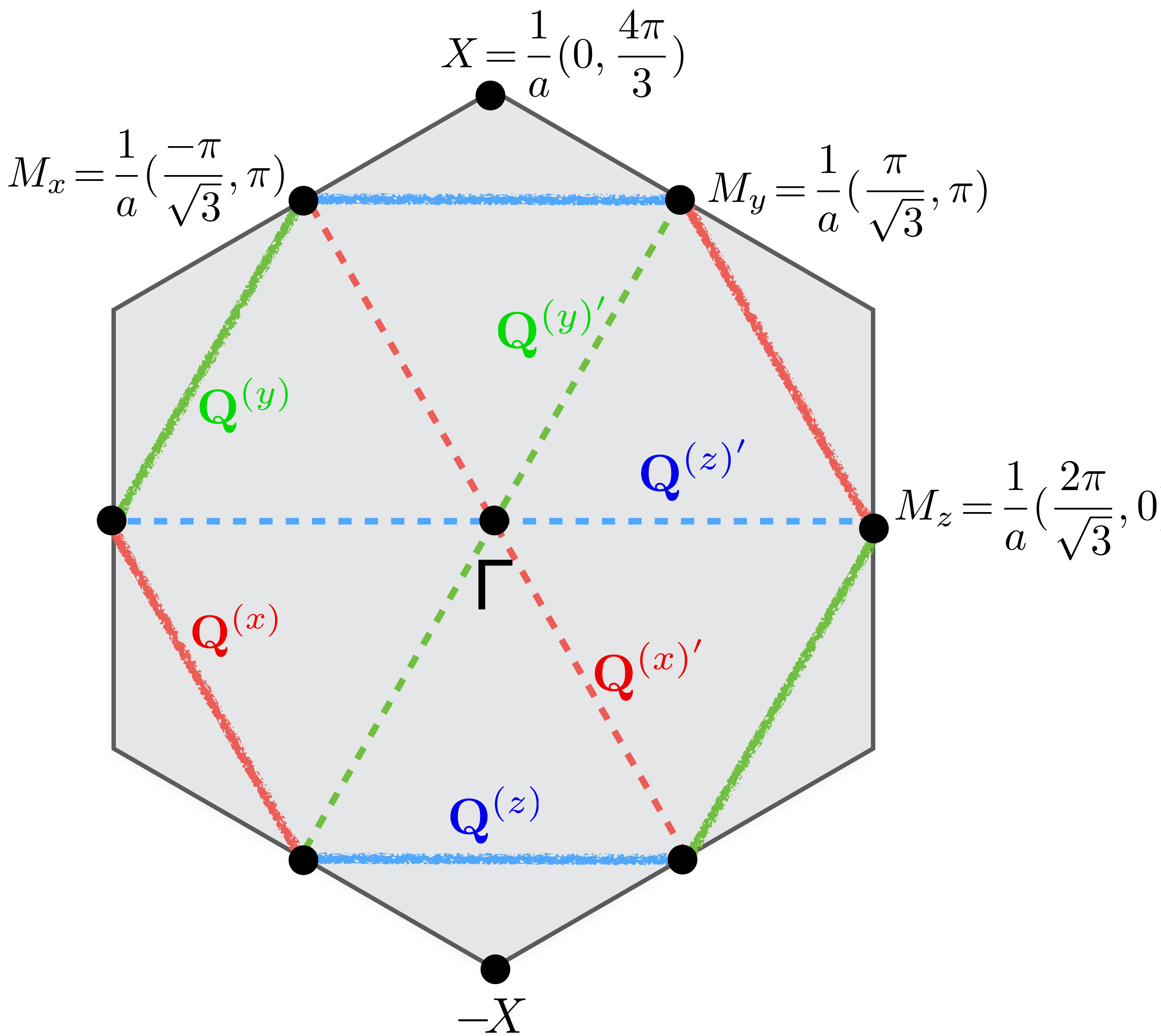
States **seem** very classical (local spin length almost 1/2, large spin gap, etc).

Yet, classical limit hosts **qualitatively** different physics !

Classical minima form lines in momentum space



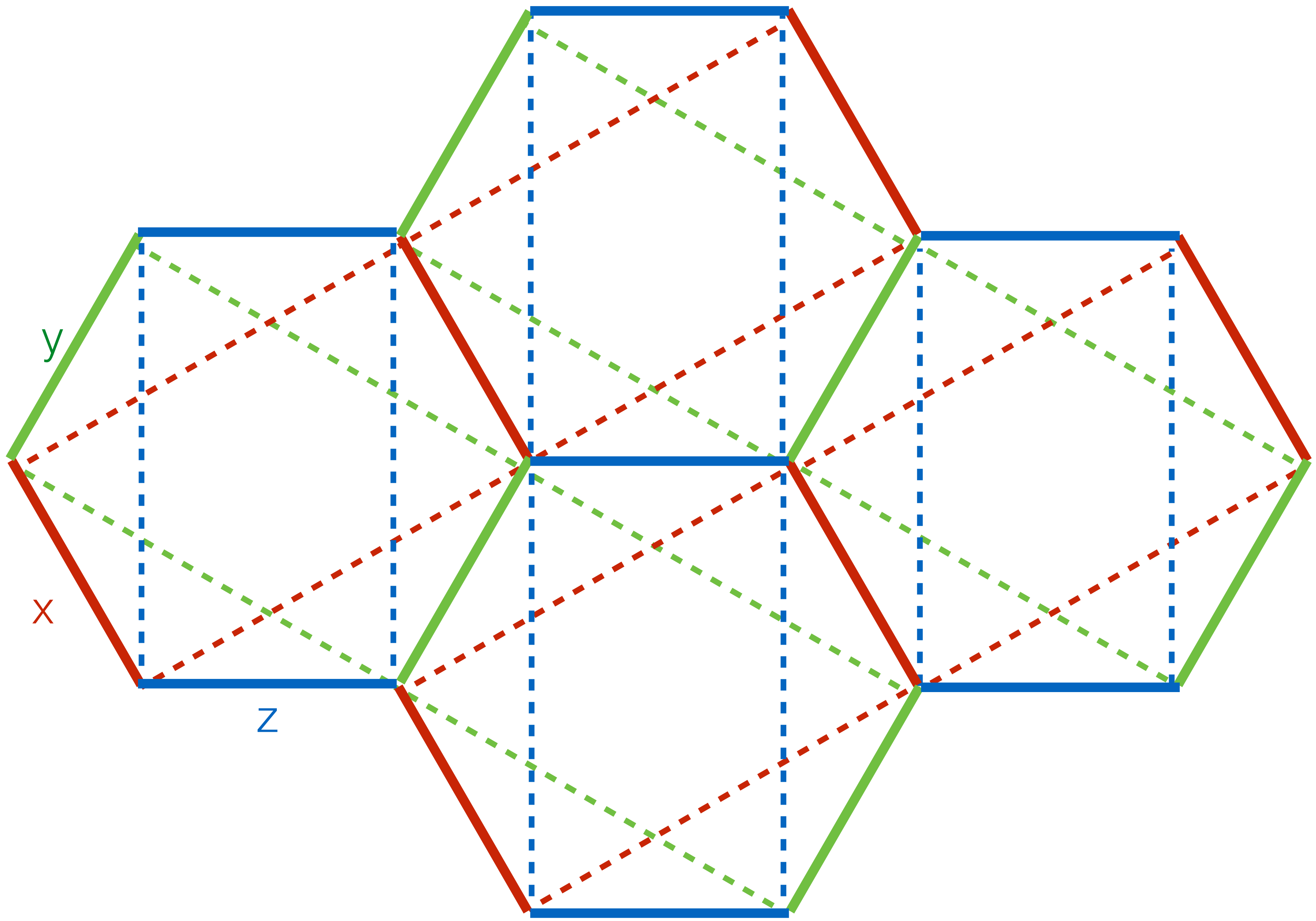
Classical minima form lines in momentum space



→ sub-extensive number of classical GS's

Origin of sub-extensive degeneracy: *sliding* symmetries

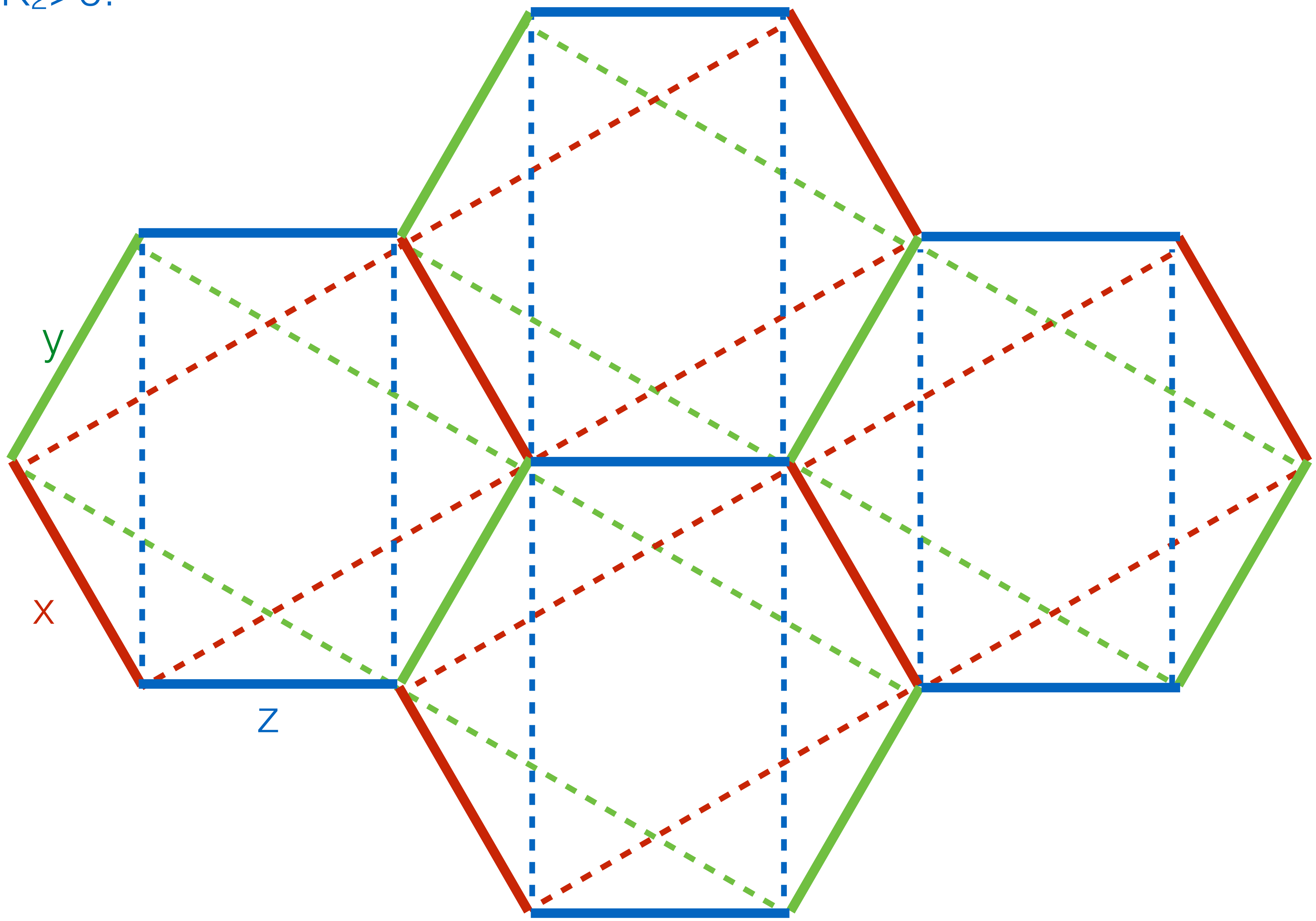
Batista & Nussinov (2005)



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

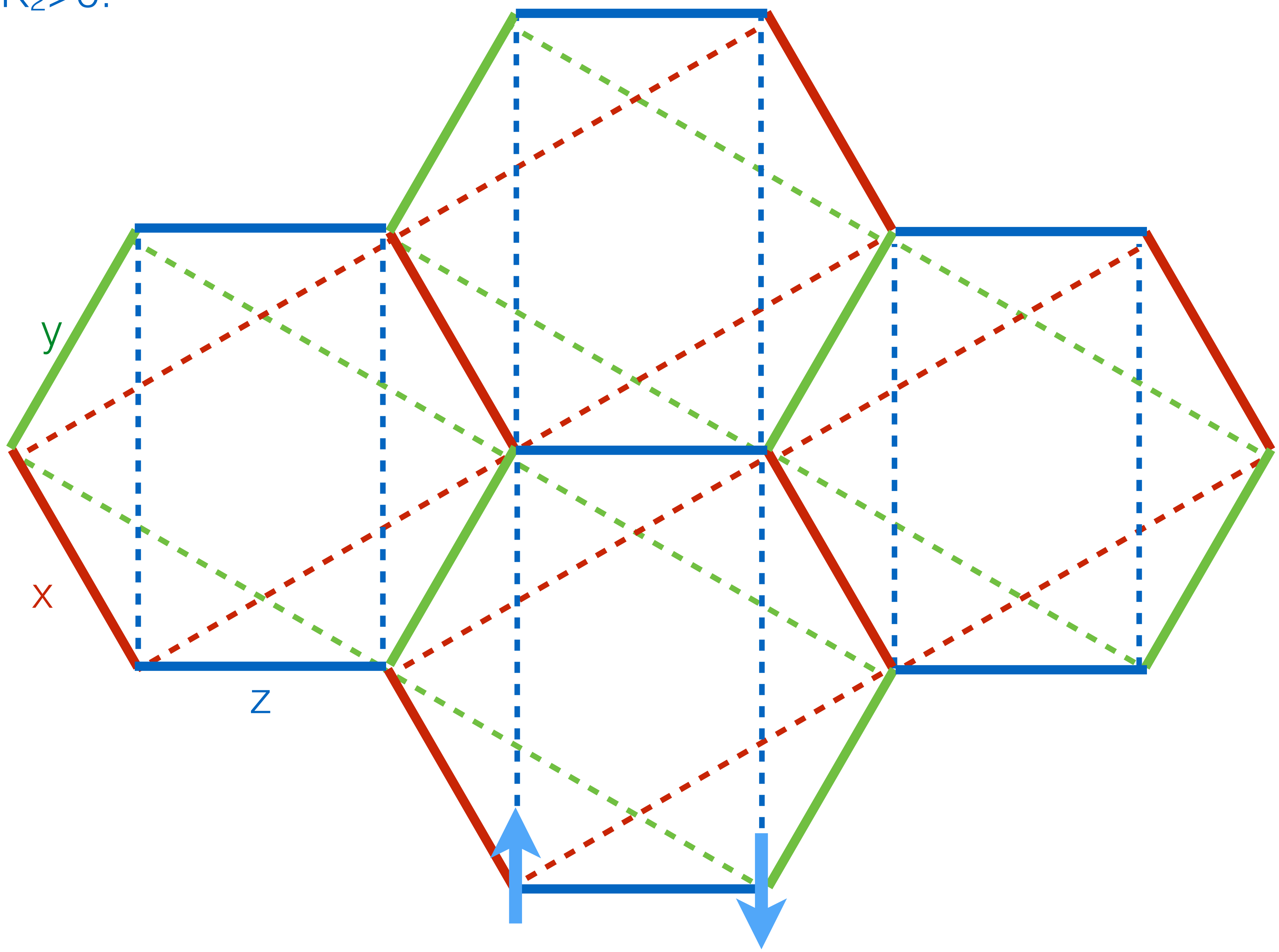
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

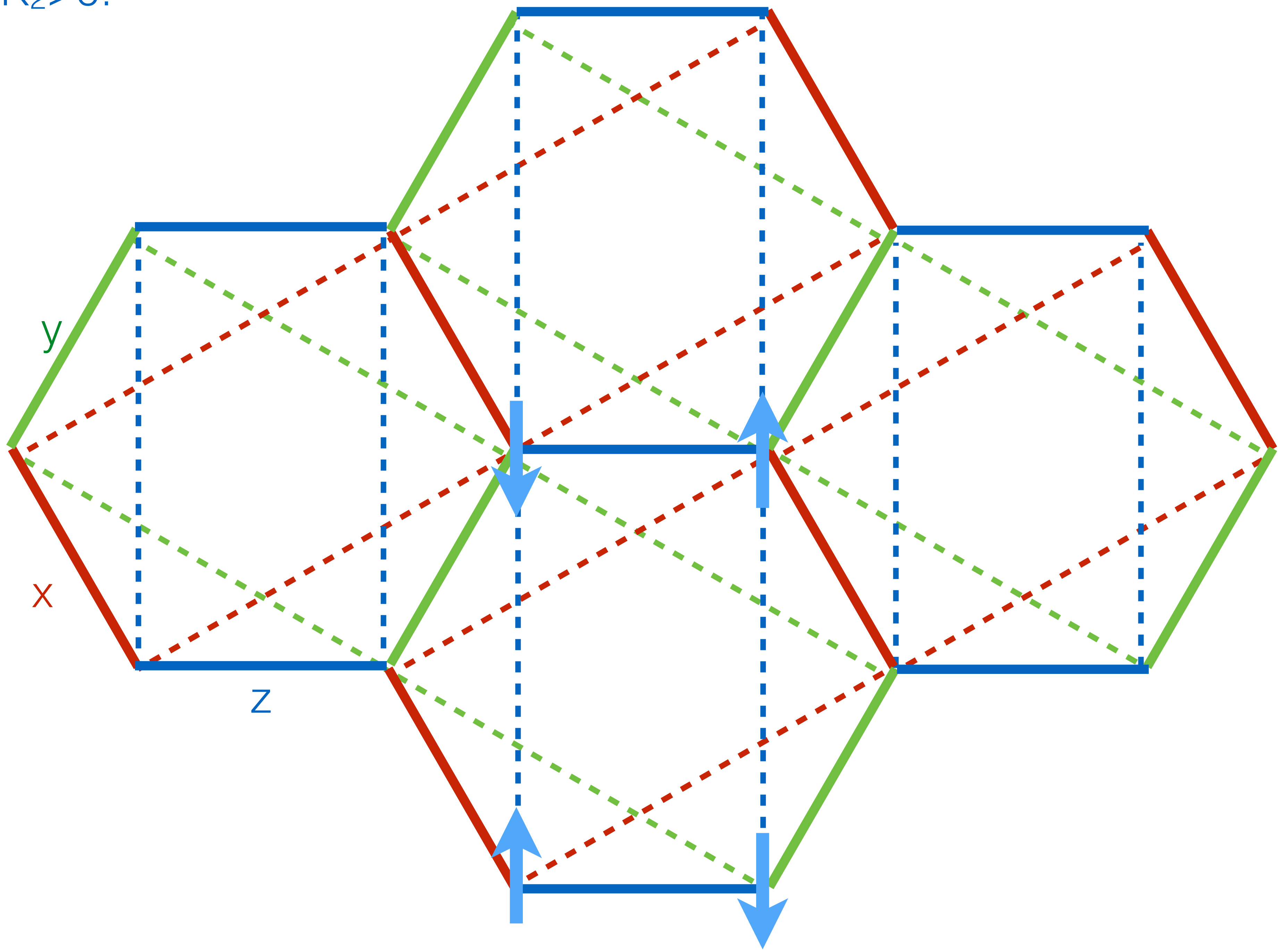
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

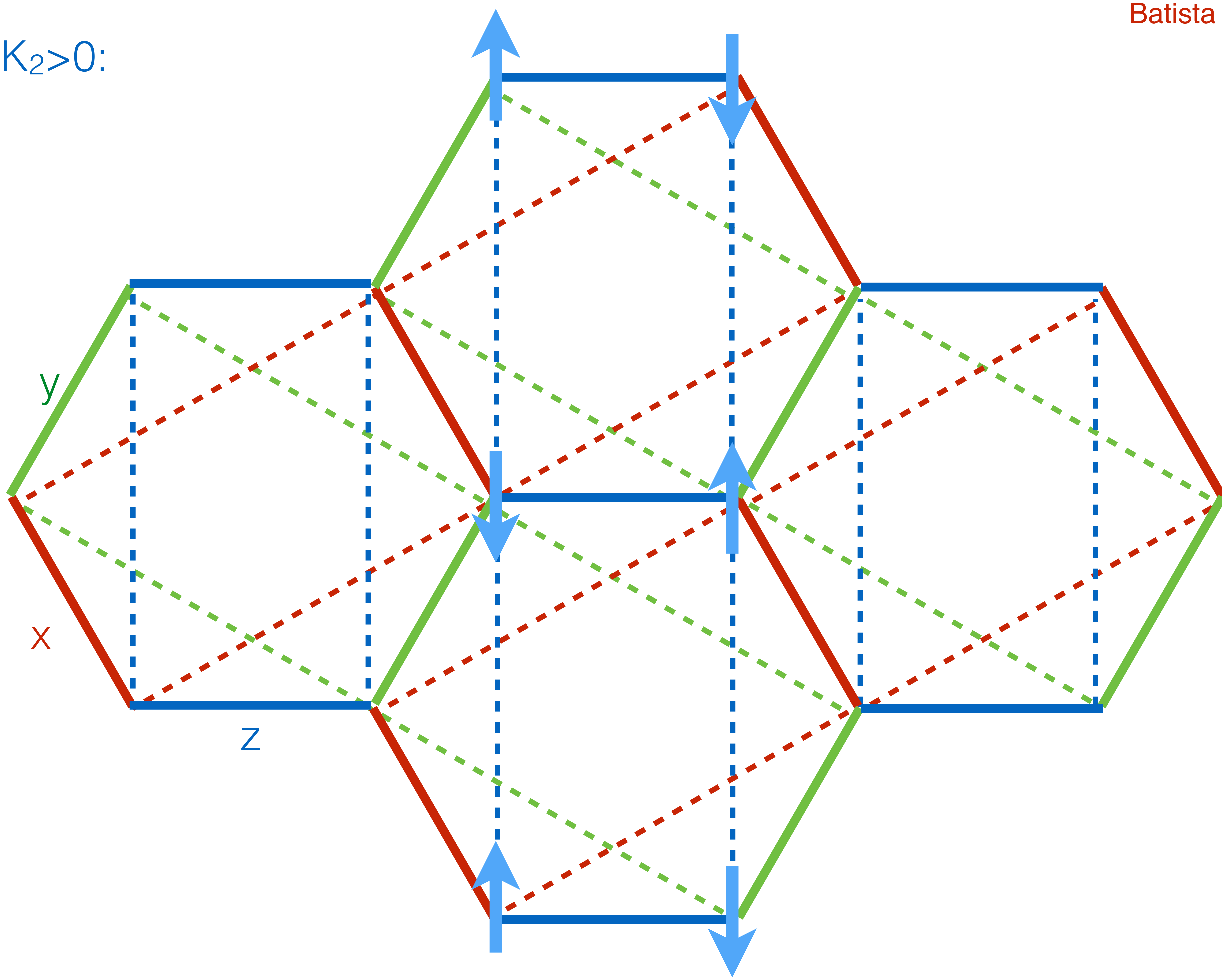
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

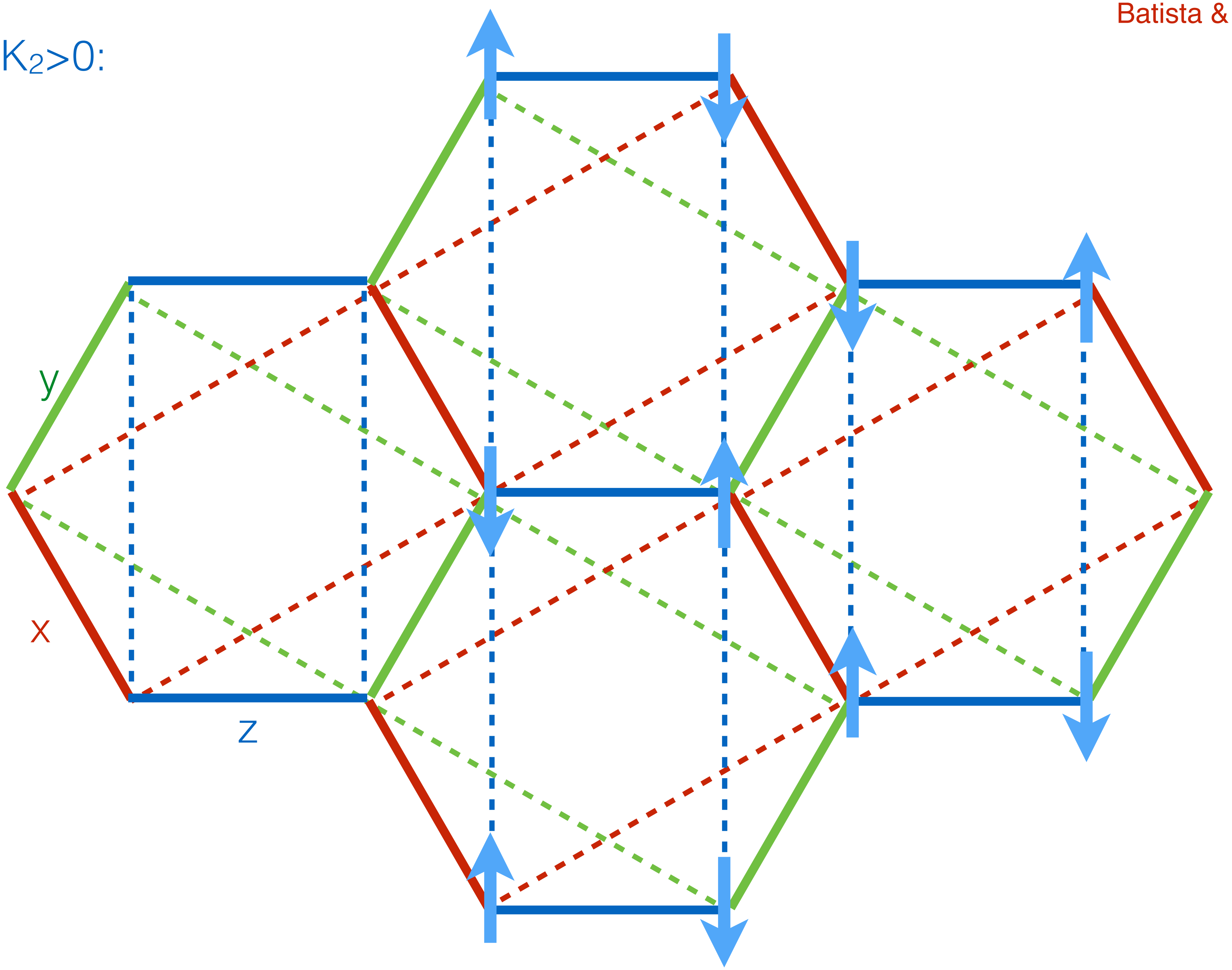
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

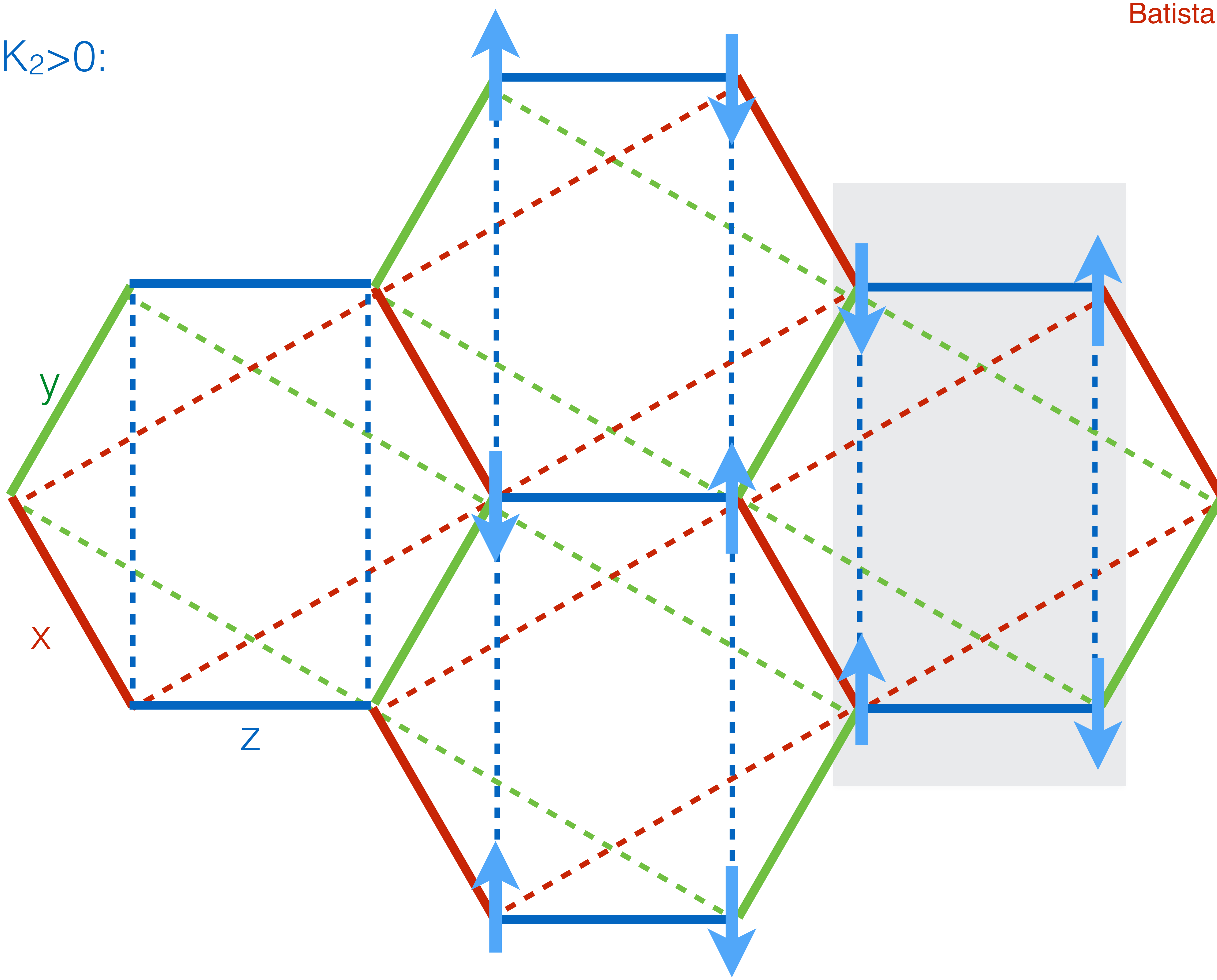
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

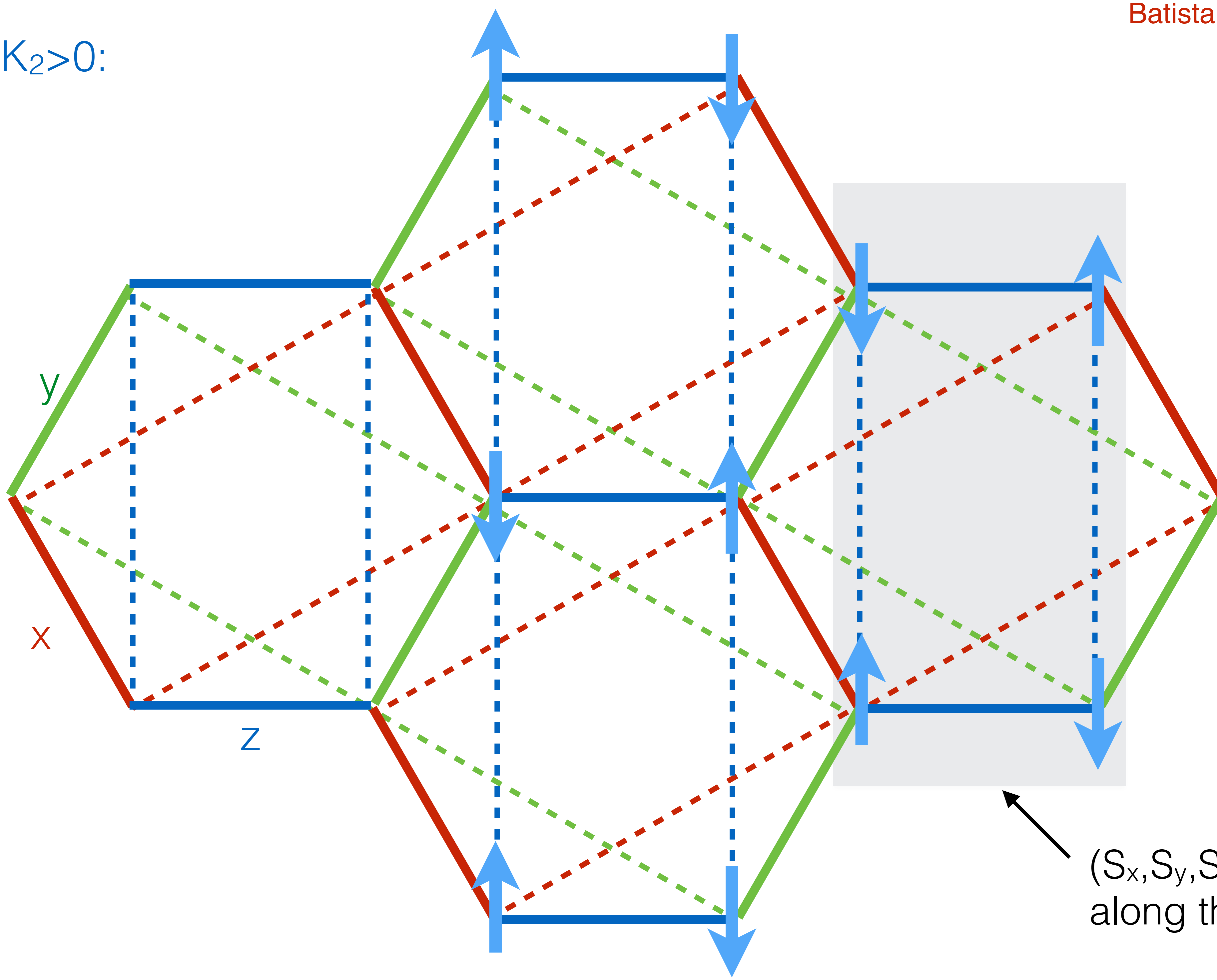
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

- Say $K_1>0$, $K_2>0$:

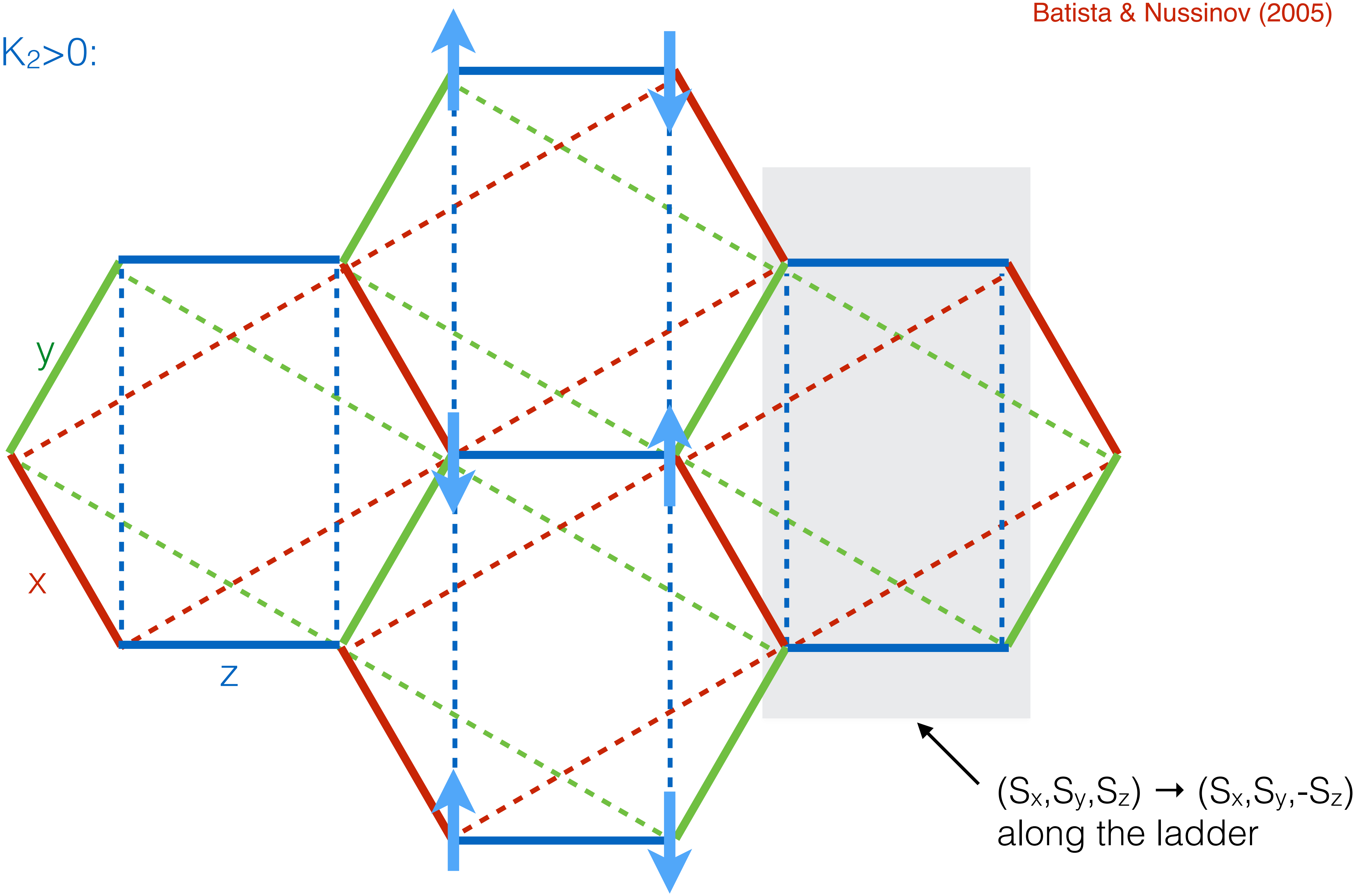


$(S_x,S_y,S_z) \rightarrow (S_x,S_y,-S_z)$
along the ladder

Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

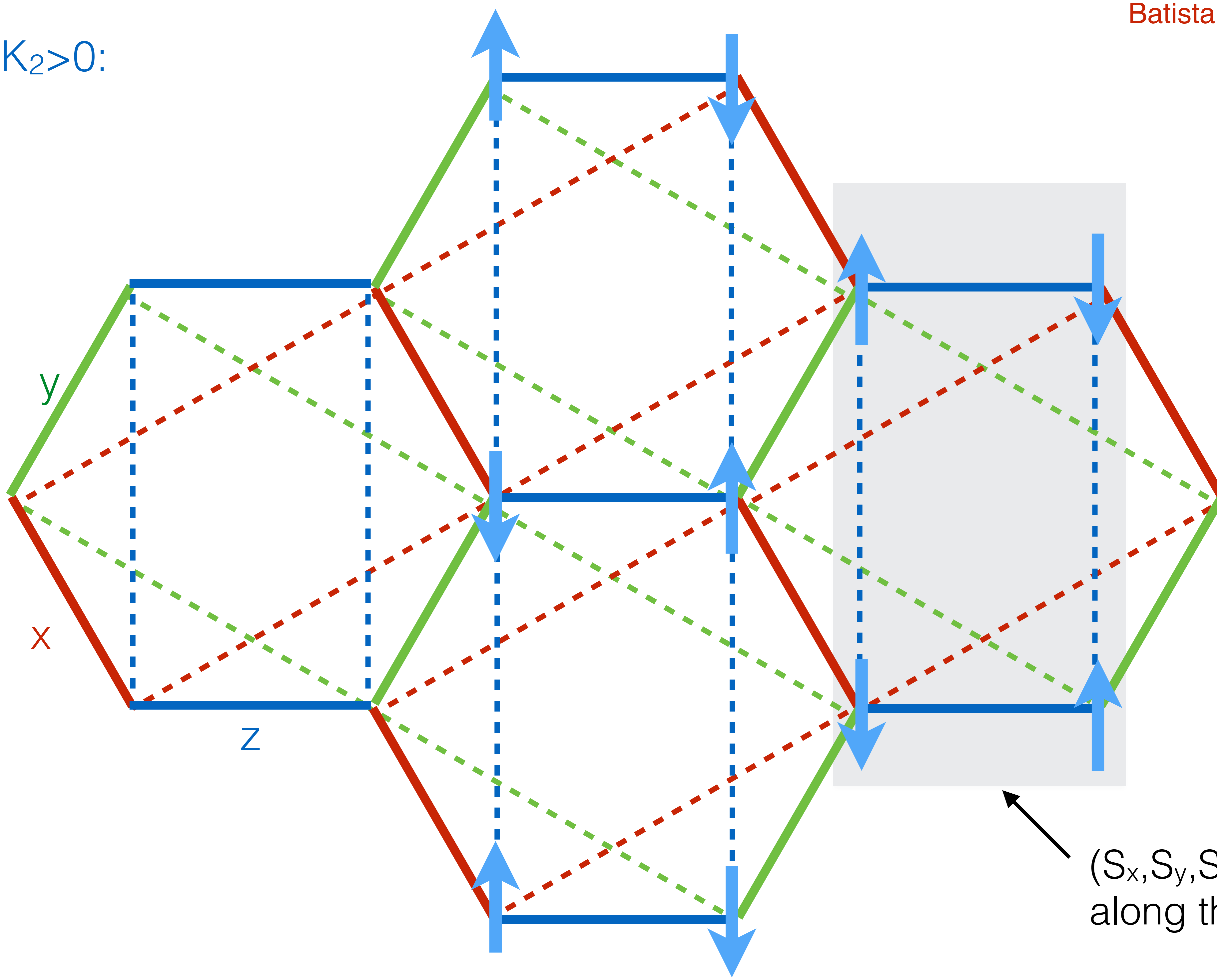
- Say $K_1>0$, $K_2>0$:



Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

- Say $K_1>0$, $K_2>0$:

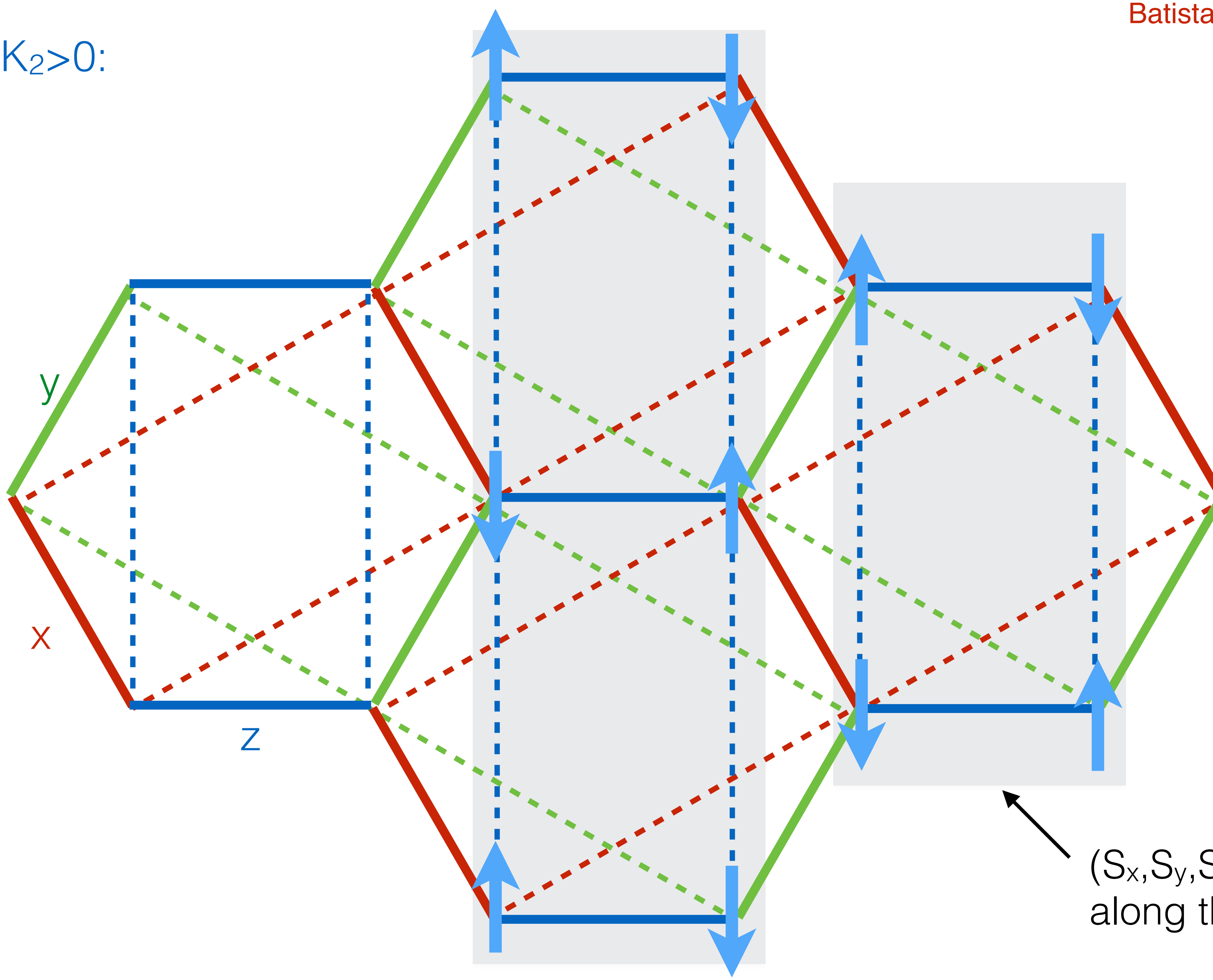


$(S_x,S_y,S_z) \to (S_x,S_y,-S_z)$
along the ladder

Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

- Say $K_1>0$, $K_2>0$:

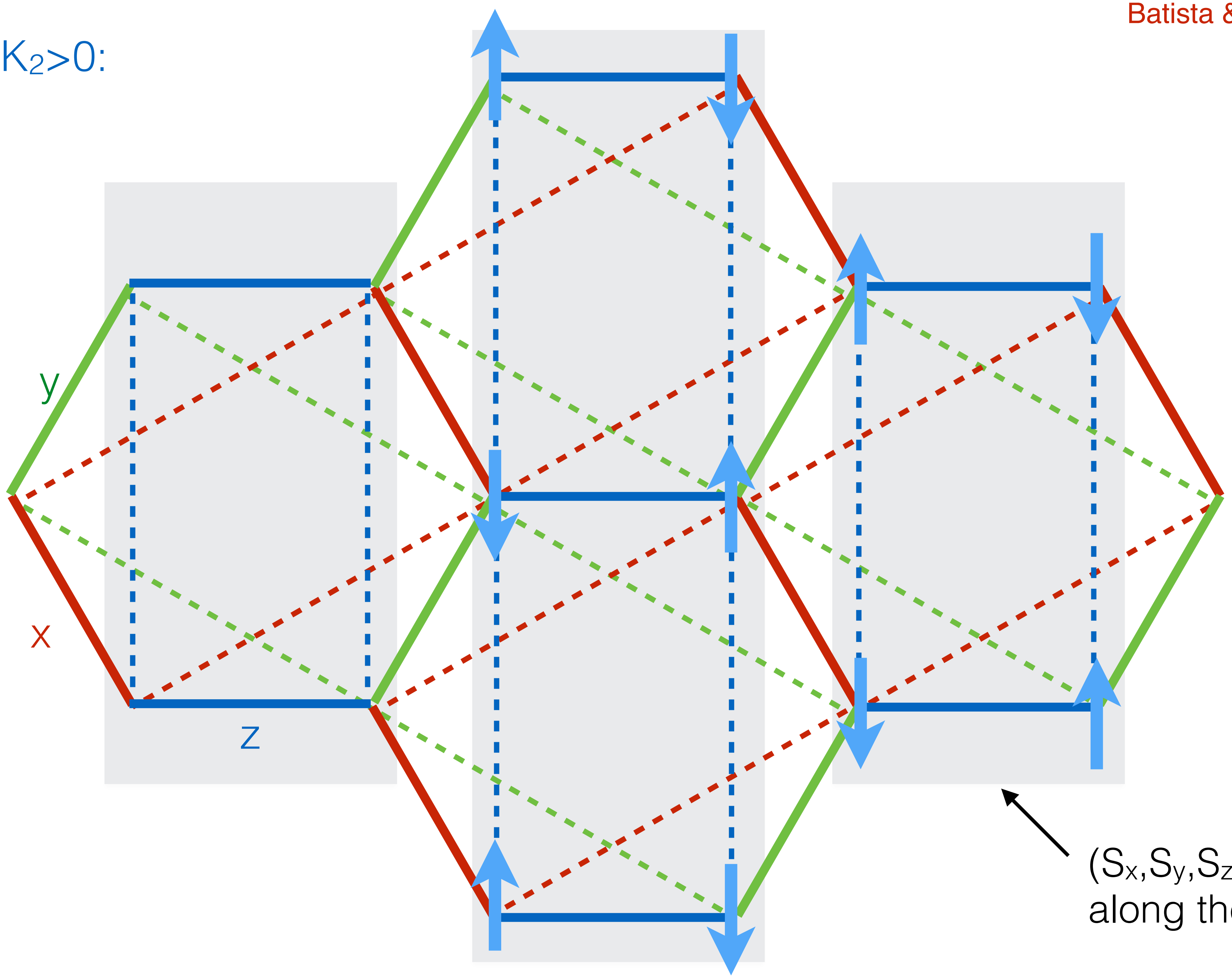


$(S_x,S_y,S_z) \rightarrow (S_x,S_y,-S_z)$ along the ladder

Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

- Say $K_1>0$, $K_2>0$:

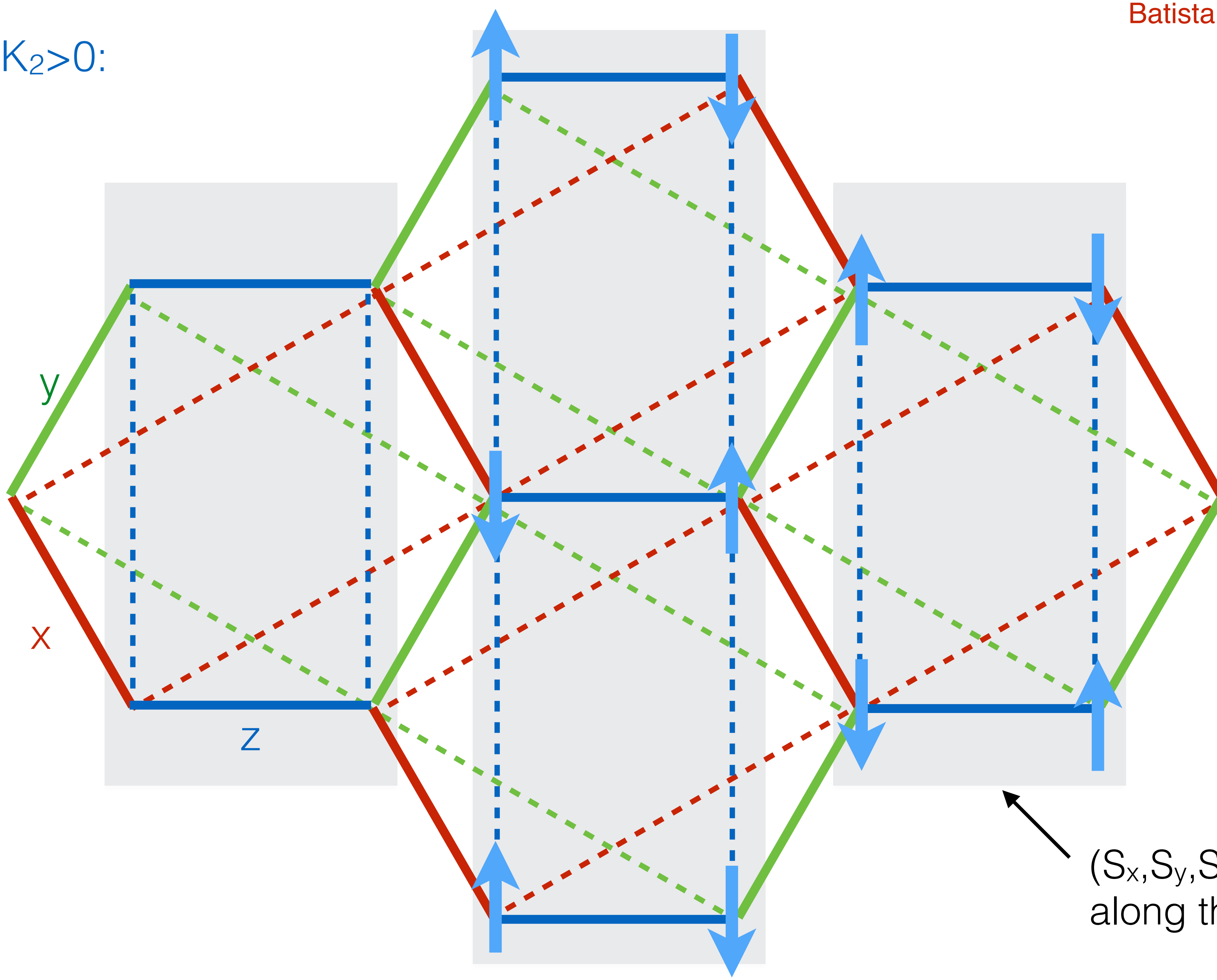


$(S_x,S_y,S_z) \rightarrow (S_x,S_y,-S_z)$
along the ladder

Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

- Say $K_1>0$, $K_2>0$:



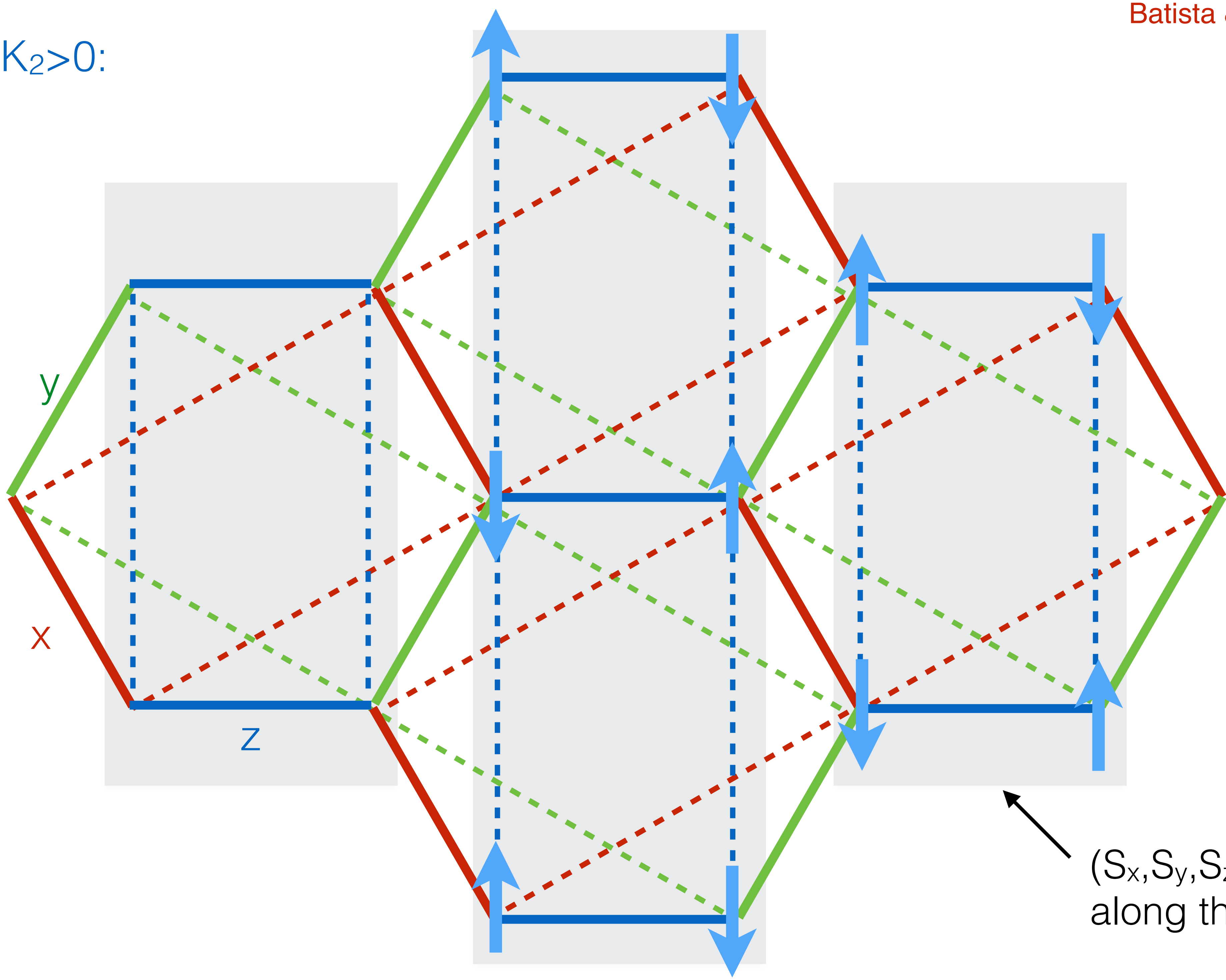
$(S_x,S_y,S_z) \rightarrow (S_x,S_y,-S_z)$
along the ladder

- sub-extensive number of **sliding** symmetries → $3\text{x}2^L$ classical minima

Origin of sub-extensive degeneracy: *sliding* symmetries

Batista & Nussinov (2005)

- Say $K_1>0$, $K_2>0$:



$(S_x,S_y,S_z) \to (S_x,S_y,-S_z)$
along the ladder

- sub-extensive number of **sliding** symmetries → $3x2^L$ classical minima

- Actually, there many more ground states: **accidental** continuous degeneracy

All these features are very common in compass-like models

All these features are very common in compass-like models

- *accidental degeneracy: lifted by thermal fluctuations, as usual*

All these features are very common in compass-like models

- *accidental degeneracy: lifted by thermal fluctuations, as usual*
- *sliding* symmetries: **cannot** break spontaneously at any finite T !

Generalized Elitzur's theorem, Batista & Nussinov (2005)

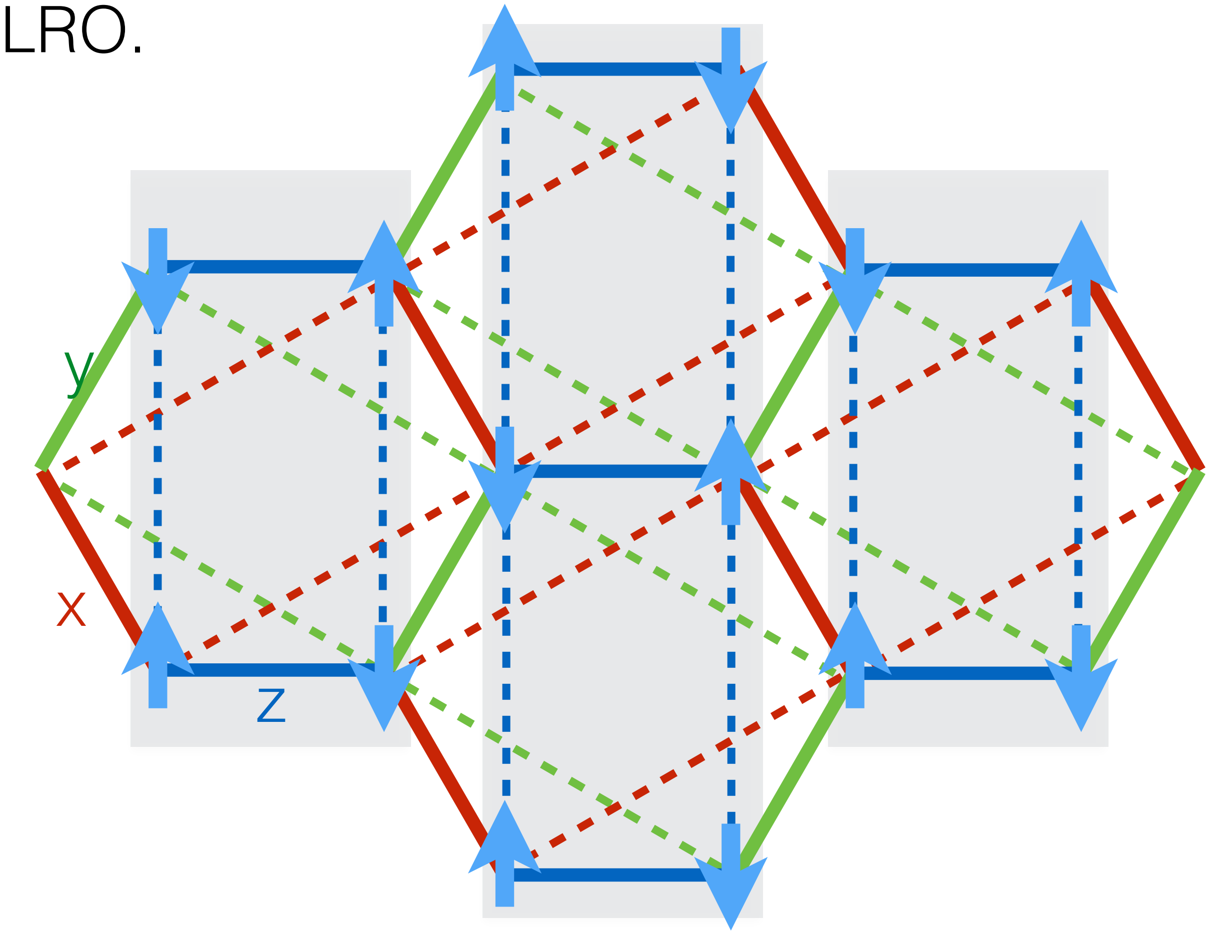
All these features are very common in compass-like models

- *accidental degeneracy: lifted by thermal fluctuations, as usual*
- *sliding* symmetries: **cannot** break spontaneously at any finite T !

Generalized Elitzur's theorem, Batista & Nussinov (2005)

→ this leaves one possibility (finite T): **nematic** ordering

C_3 symmetry breaks, system samples all 2^L states of one of the three types of bonds. But no magnetic LRO.



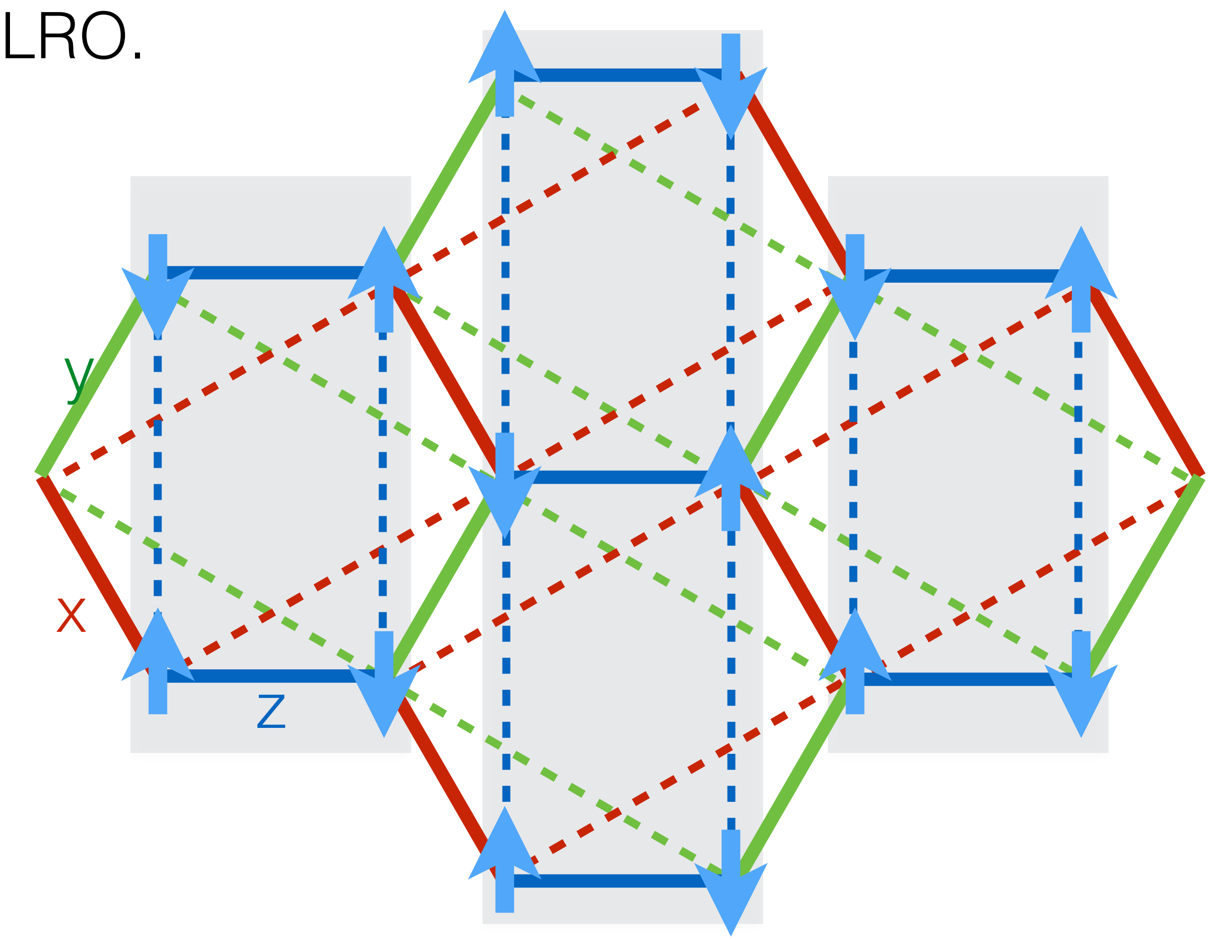
All these features are very common in compass-like models

- *accidental degeneracy: lifted by thermal fluctuations, as usual*
- *sliding* symmetries: **cannot** break spontaneously at any finite T !

Generalized Elitzur's theorem, Batista & Nussinov (2005)

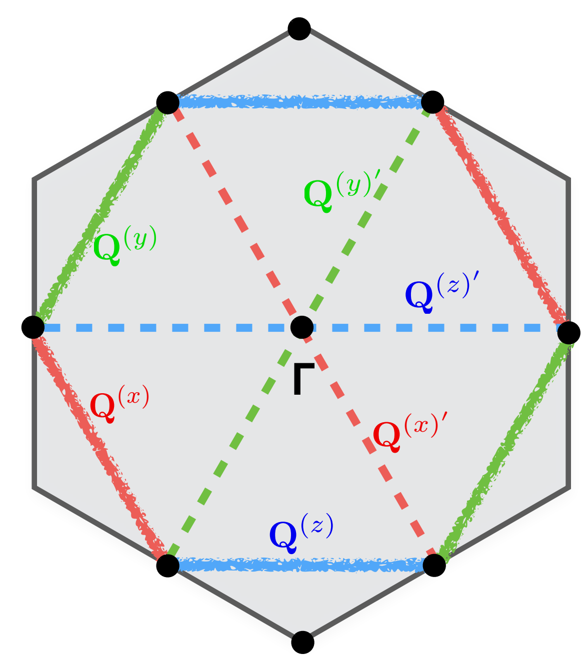
→ this leaves one possibility (finite T): **nematic** ordering

C_3 symmetry breaks, system samples all 2^L states of one of the three types of bonds. But no magnetic LRO.



- T=0: sliding symmetries can break, but in all possible ways!

→ no isolated Bragg peaks as we found with ED



what's happening?

what's happening?

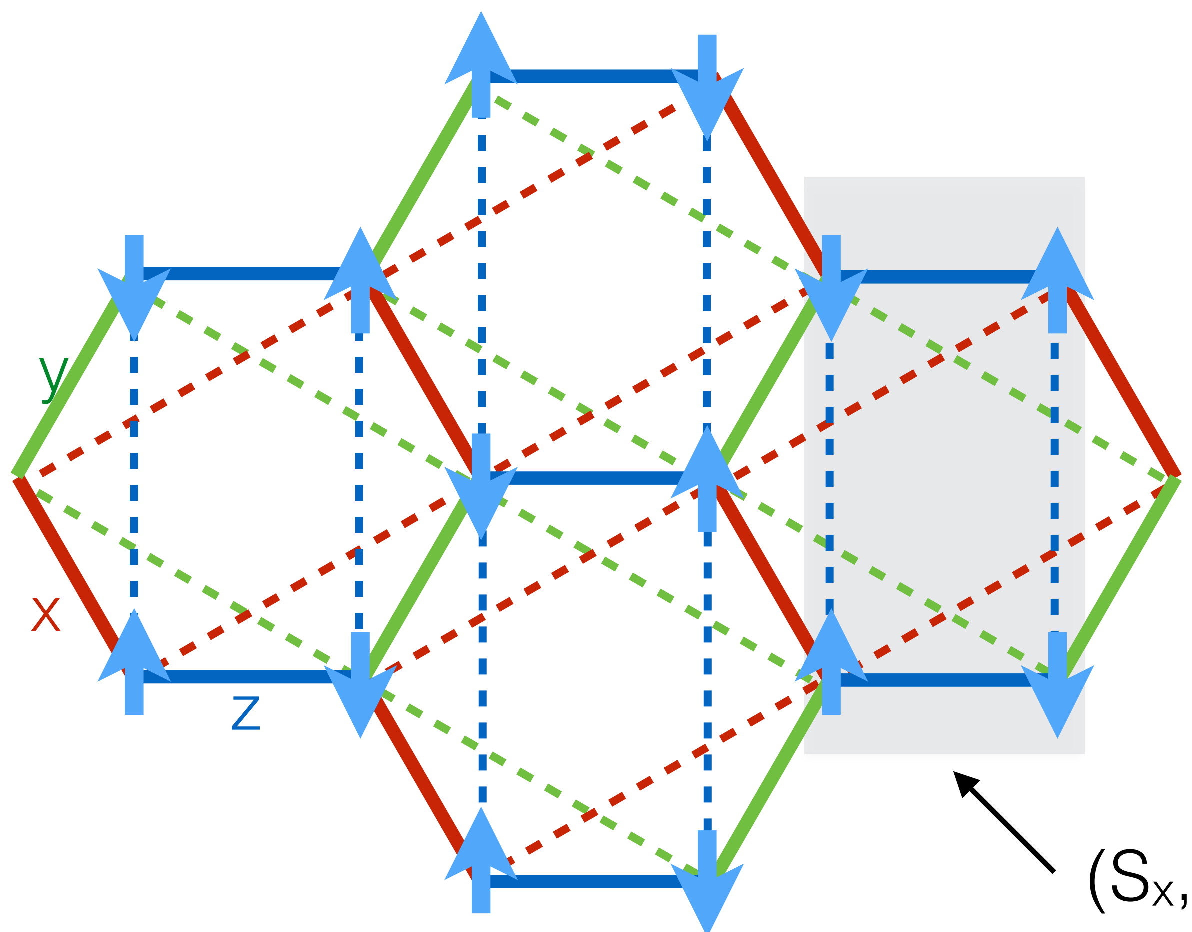
- *sliding* symmetries: **not** present for quantum spins !

  because **local** time-reversal **not** possible in quantum mechanics !

what's happening?

- *sliding* symmetries: **not** present for quantum spins !

because **local** time-reversal **not** possible in quantum mechanics !

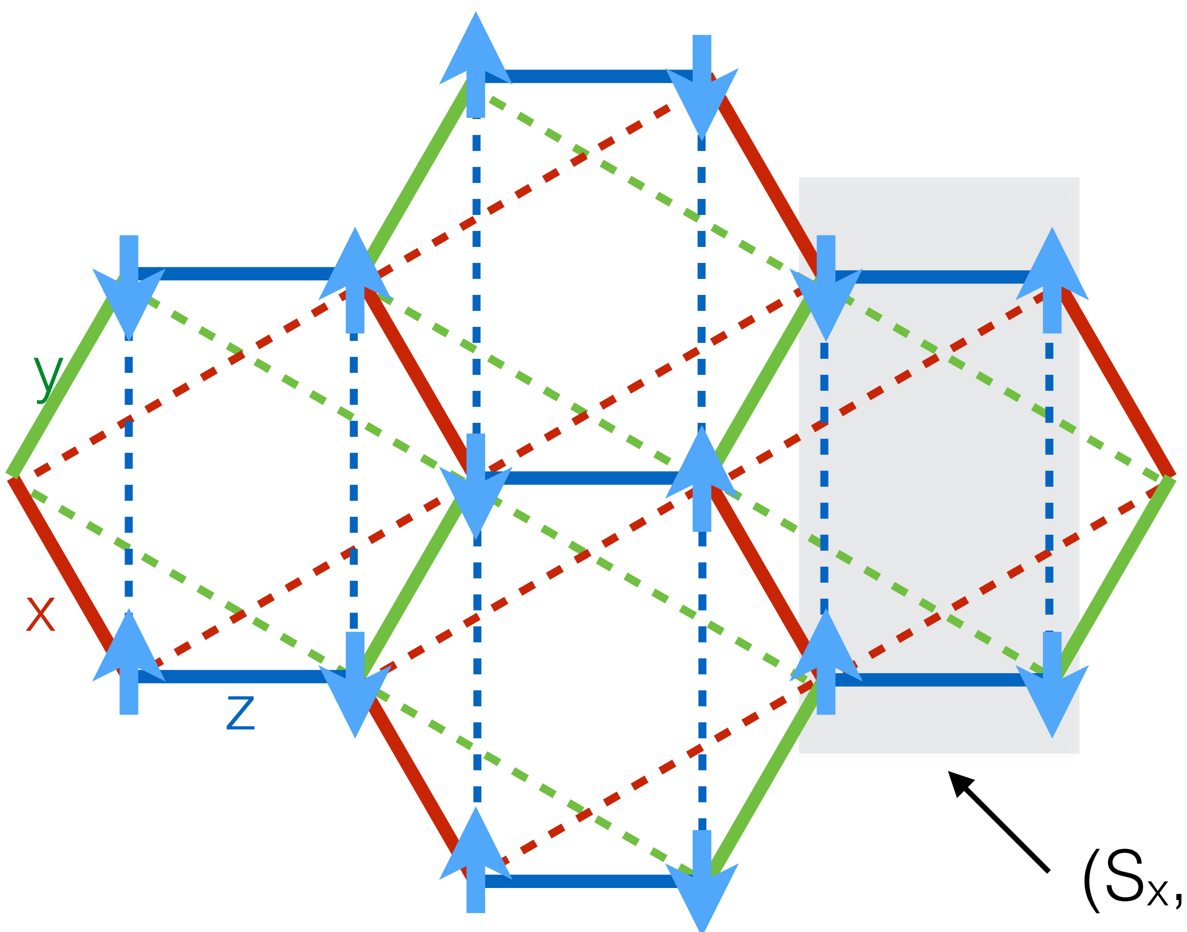


$(S_x,S_y,S_z) \rightarrow (S_x,S_y,-S_z)$
along the ladder

what's happening?

- *sliding* symmetries: **not** present for quantum spins !

because **local** time-reversal **not** possible in quantum mechanics !



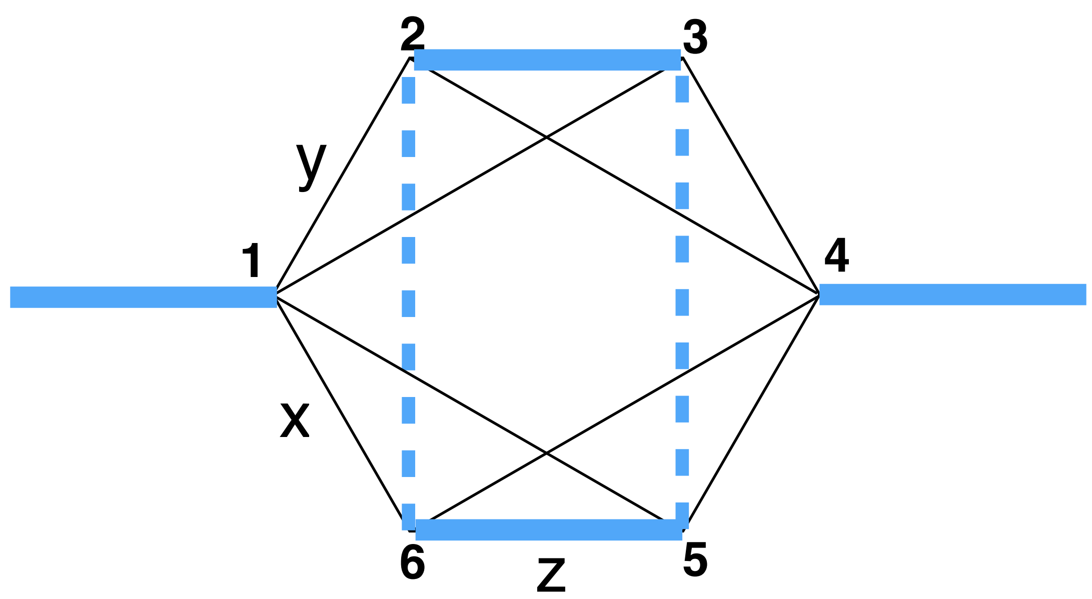
$(S_x, S_y, S_z) \rightarrow (S_x, S_y, -S_z)$
along the ladder

→ different ladders must talk to each other via quantum fluctuations !

Quantum order-by-disorder: strong coupling expansion

Quantum order-by-disorder: strong coupling expansion

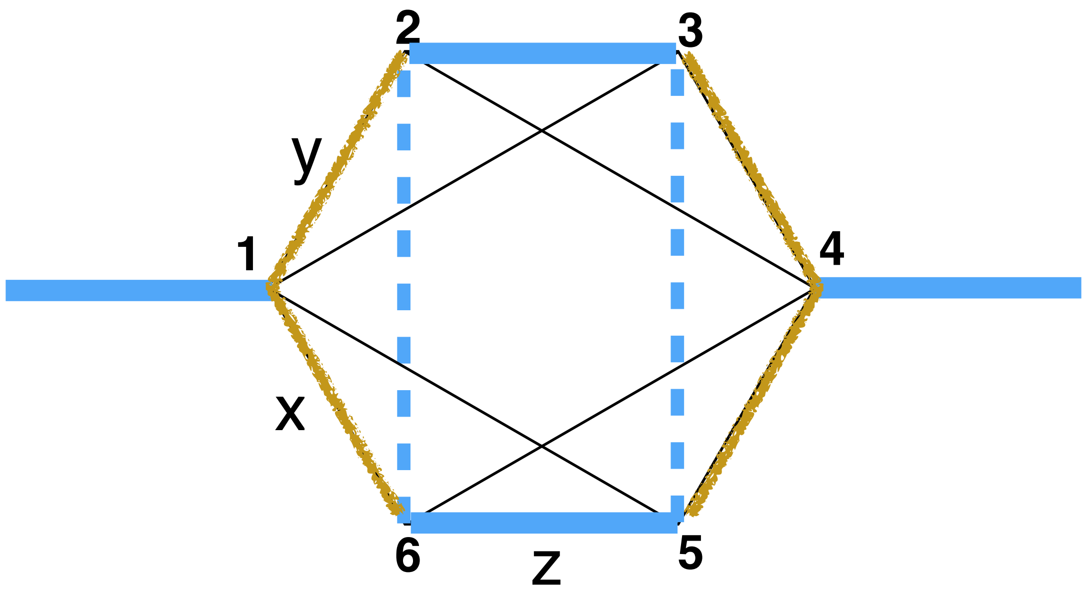
- Strong coupling limit: $K^z_{1,2} \gg K^{x,y}_{1,2}$



Quantum order-by-disorder: strong coupling expansion

- Strong coupling limit: $K^{z}_{1,2} \gg K^{x,y}_{1,2}$

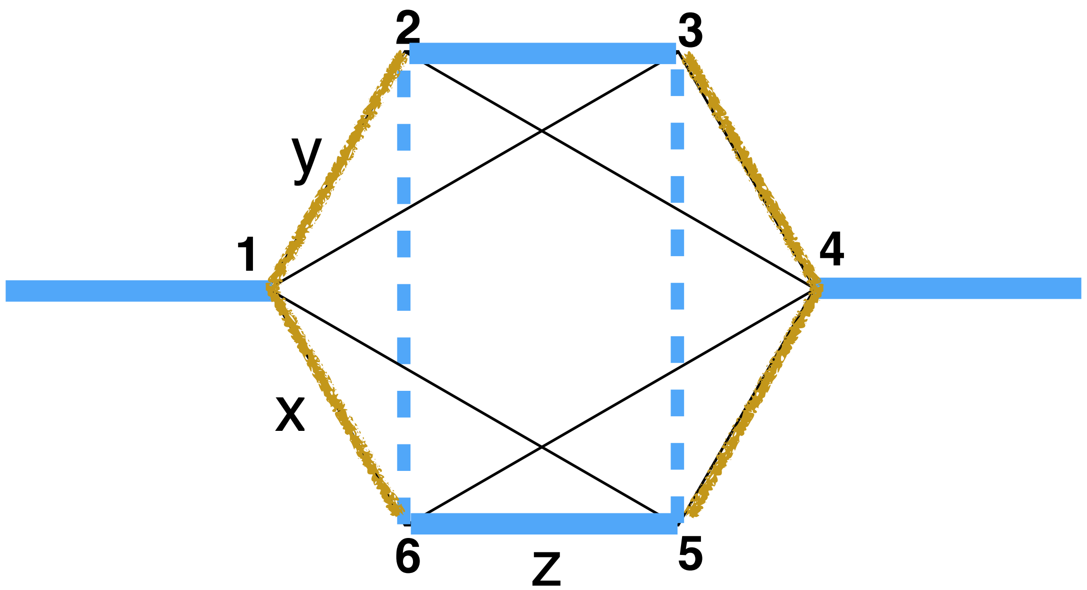
- **first** type of effective term: driven by K_1 perturbations only



Quantum order-by-disorder: strong coupling expansion

- Strong coupling limit: $K^{z}_{1,2} \gg K^{x,y}_{1,2}$

- **first** type of effective term: driven by K_1 perturbations only

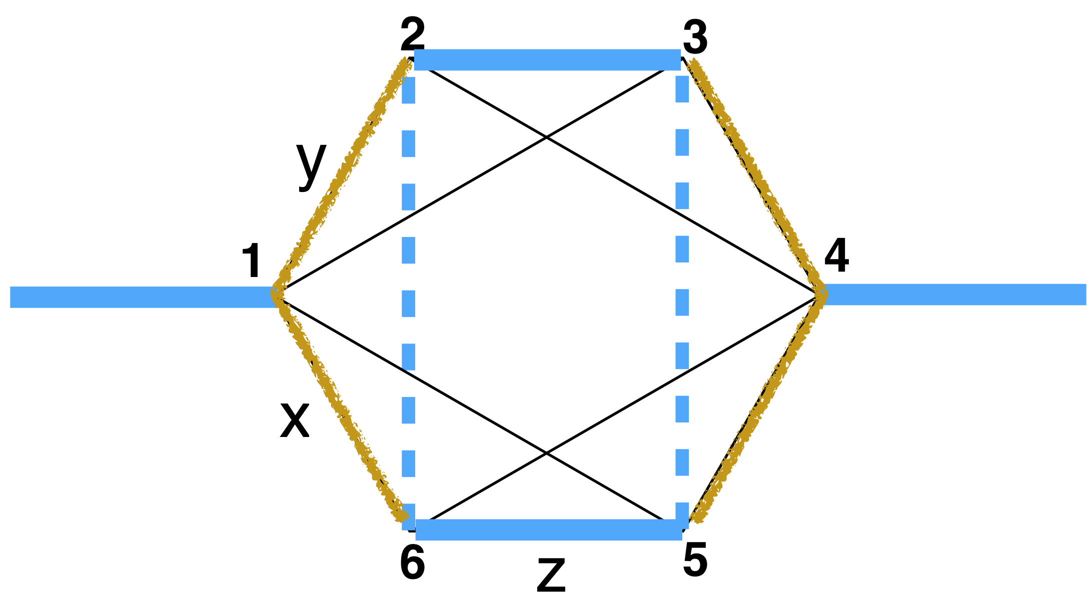


- *effective* term is a flux operator: $J_W \hat{W}_h = 2^6 J_W S^z_1 \; S^x_2 \; S^y_3 \; S^z_4 \; S^x_5 \; S^y_6$

$$J_W = \frac{-(K^x_1 K^y_1)^2 |K^z_1|}{64(|K^z_1| + 2|K^z_2|)^2 (|K^z_1| + 3|K^z_2|)(|K^z_1| + 4|K^z_2|)}$$

Quantum order-by-disorder: strong coupling expansion

- Strong coupling limit: $K_{1,2}^{z} \gg K_{1,2}^{x,y}$
- **first** type of effective term: driven by K_1 perturbations only



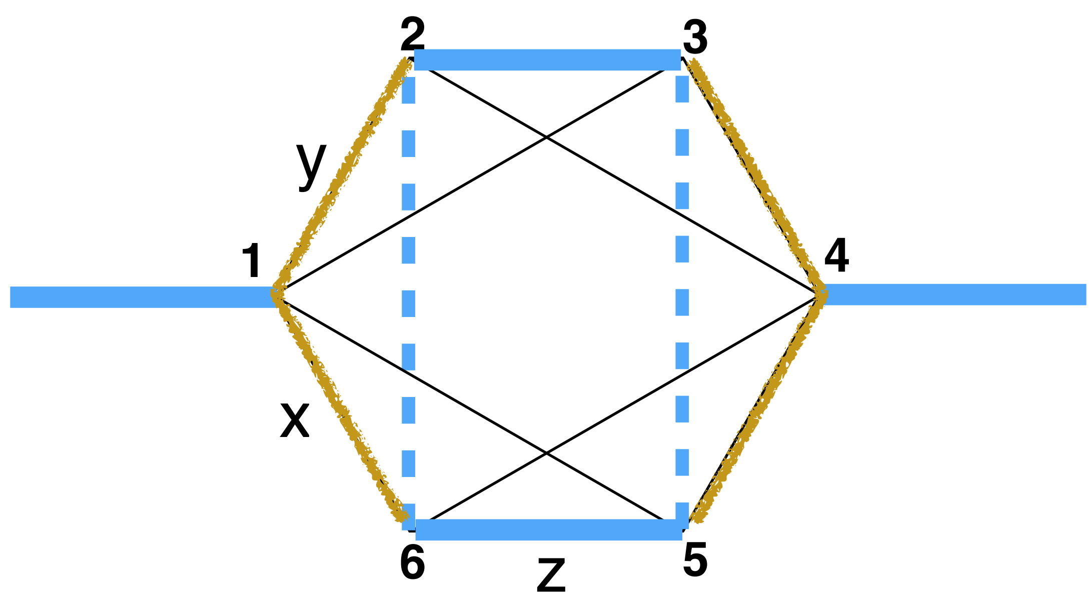
- *effective* term is a flux operator: $J_W \hat{W}_h = 2^6 J_W S_1^z \; S_2^x \; S_3^y \; S_4^z \; S_5^x \; S_6^y$

$$J_W = \frac{-(K_1^x K_1^y)^2 |K_1^z|}{64(|K_1^z| + 2|K_2^z|)^2(|K_1^z| + 3|K_2^z|)(|K_1^z| + 4|K_2^z|)}$$

Quantum order-by-disorder: strong coupling expansion

- Strong coupling limit: $K^z_{1,2} \gg K^{x,y}_{1,2}$

- **first** type of effective term: driven by K_1 perturbations only



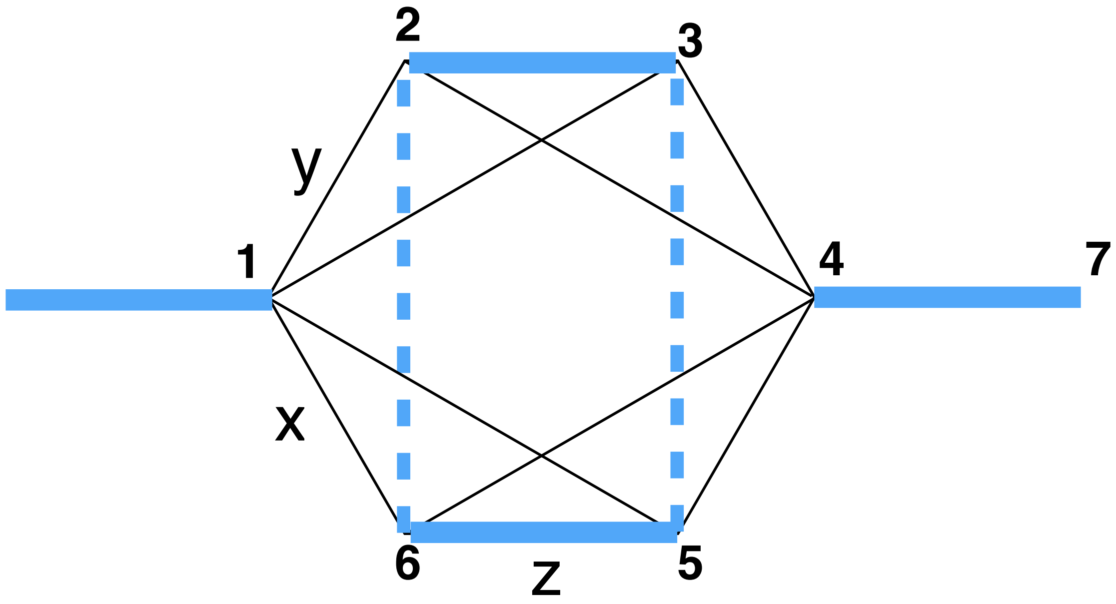
- *effective* term is a flux operator: $J_W \hat{W}_h = 2^6 J_W S^z_1 \; S^x_2 \; S^y_3 \; S^z_4 \; S^x_5 \; S^y_6$

$$J_W = \frac{-(K^x_1 K^y_1)^2 |K^z_1|}{64(|K^z_1| + 2|K^z_2|)^2(|K^z_1| + 3|K^z_2|)(|K^z_1| + 4|K^z_2|)}$$

- this term maps to the so-called **Toric code model** in the square lattice

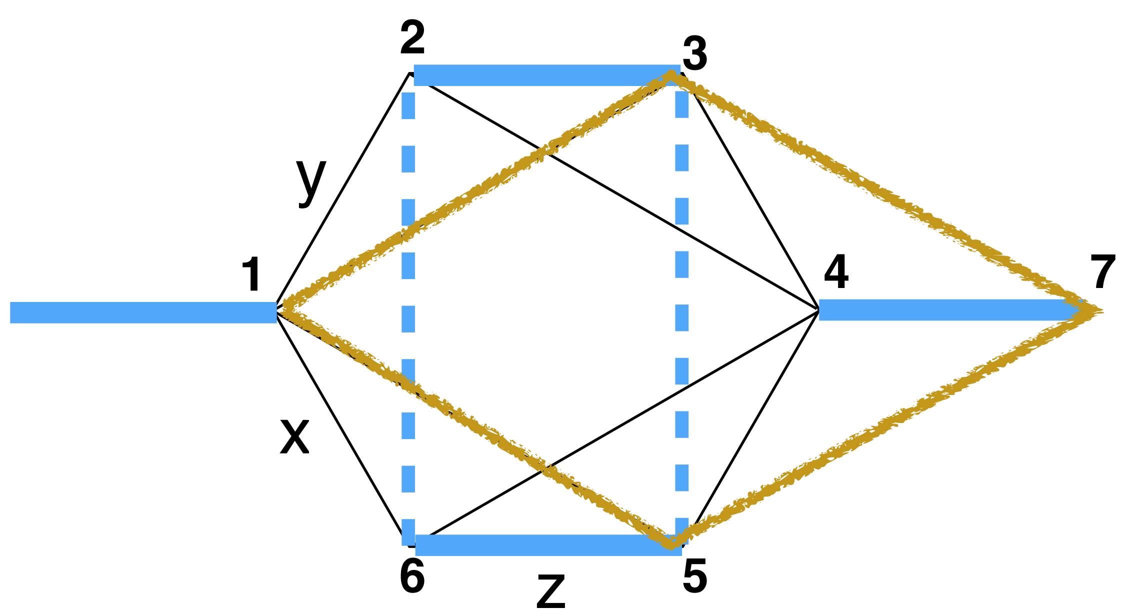
A. Kitaev ('03, '06)

Quantum order-by-disorder: strong coupling expansion



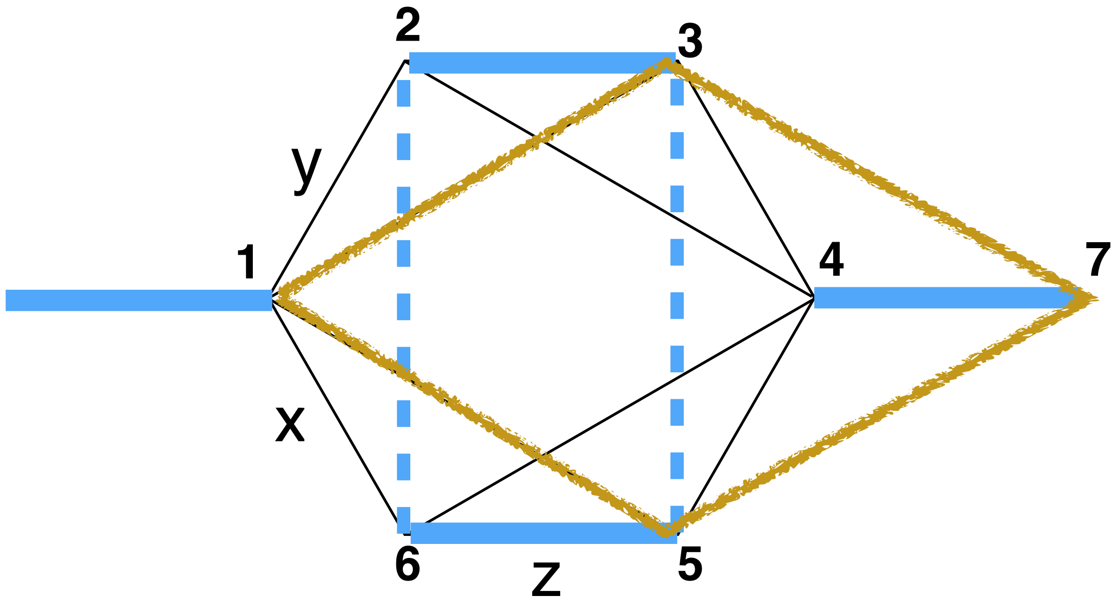
Quantum order-by-disorder: strong coupling expansion

- second type: driven by K_2 perturbations only



Quantum order-by-disorder: strong coupling expansion

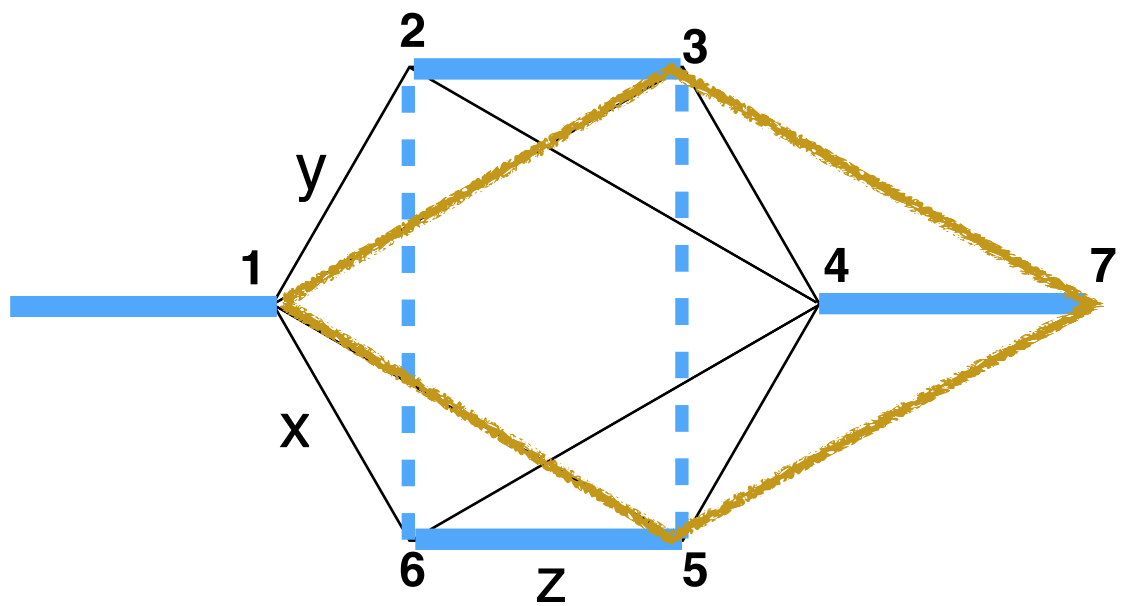
- **second** type: driven by K_2 perturbations only



- *effective* Ising coupling **J_1** between **NNN** ladders: $J_1 S_1^z S_7^z$

Quantum order-by-disorder: strong coupling expansion

- second type: driven by K_2 perturbations only

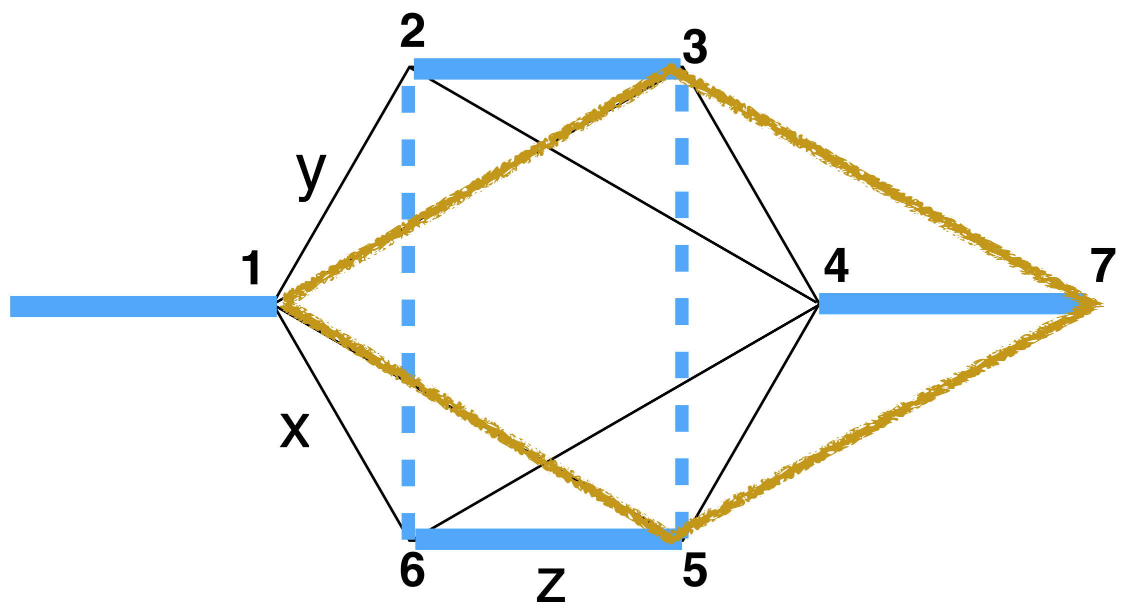


- *effective* Ising coupling J_1 between NNN ladders: $J_1 S_1^z S_7^z$

$$J_1 = \frac{(K_2^x K_2^y)^2}{8(|K_1^z| + 2|K_2^z|)^2(2|K_1^z| + 3|K_2^z|)} \mathrm{sgn}(K_2^z)$$

Quantum order-by-disorder: strong coupling expansion

- second type: driven by K_2 perturbations only

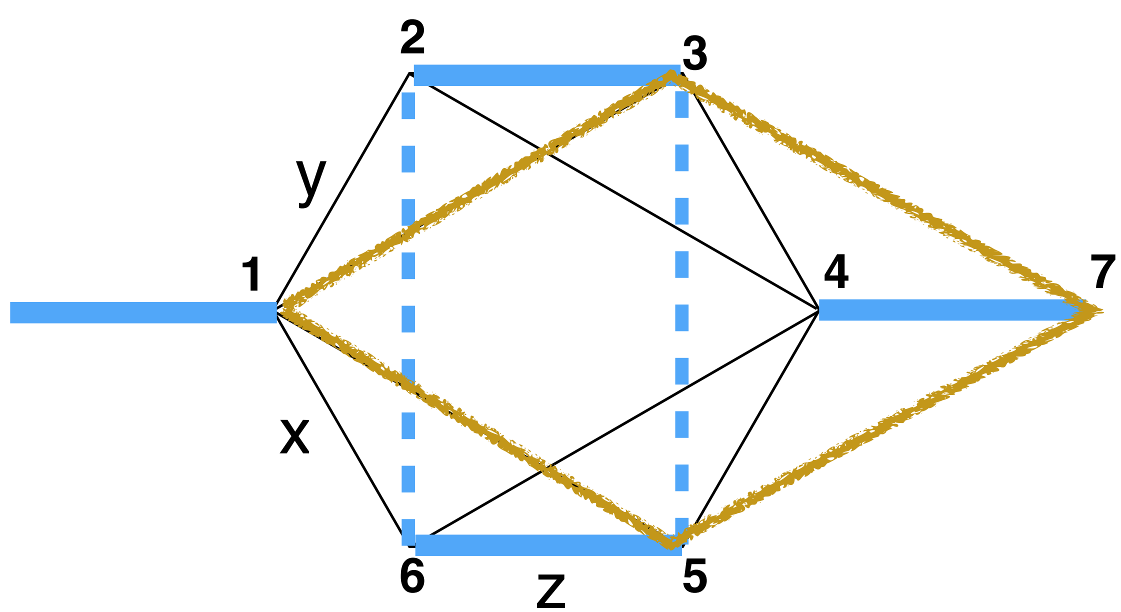


- *effective* Ising coupling J_1 between NNN ladders: $J_1 S_1^z S_7^z$

$$J_1 = \frac{(K_2^x K_2^y)^2}{8(|K_1^z| + 2|K_2^z|)^2(2|K_1^z| + 3|K_2^z|)} \mathrm{sgn}(K_2^z)$$

Quantum order-by-disorder: strong coupling expansion

- second type: driven by K_2 perturbations only



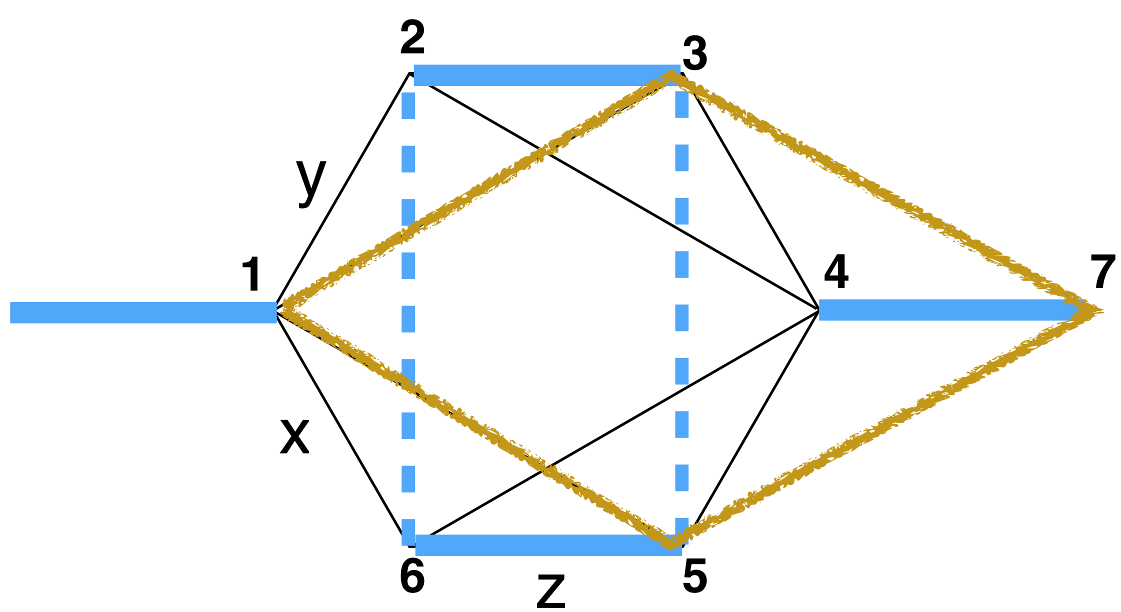
- *effective* Ising coupling J_1 between NNN ladders: $J_1 S_1^z S_7^z$

$$J_1 = \frac{(K_2^x K_2^y)^2}{8(|K_1^z| + 2|K_2^z|)^2(2|K_1^z| + 3|K_2^z|)} \mathrm{sgn}(K_2^z)$$

large for ψ=±π/2

Quantum order-by-disorder: strong coupling expansion

- second type: driven by K_2 perturbations only



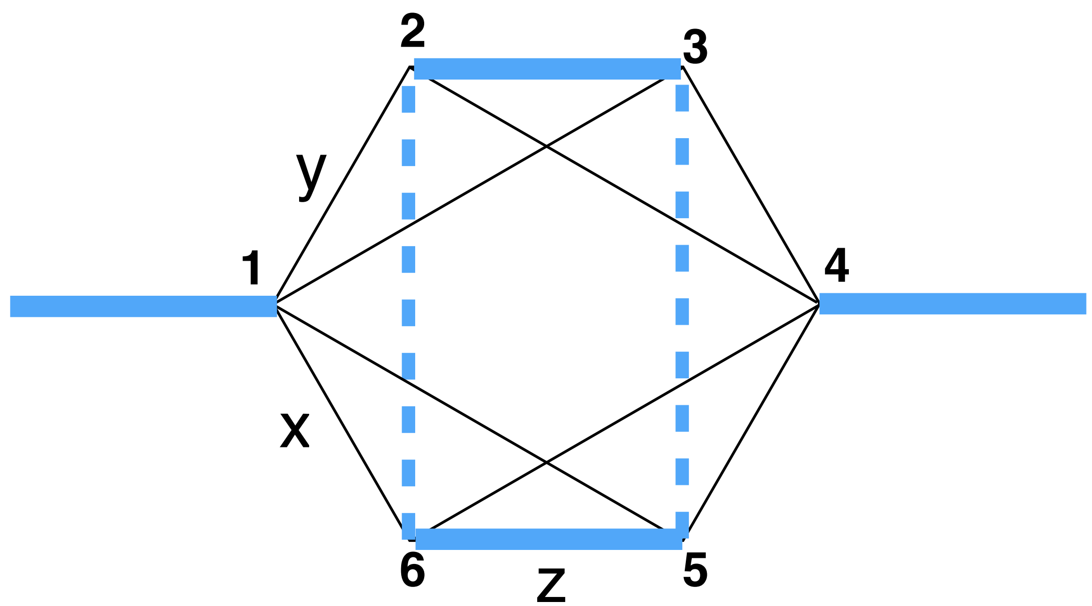
- *effective* Ising coupling J_1 between NNN ladders: $J_1 S_1^z S_7^z$

$$J_1 = \frac{(K_2^x K_2^y)^2}{8(|K_1^z| + 2|K_2^z|)^2(2|K_1^z| + 3|K_2^z|)} \mathrm{sgn}(K_2^z)$$

large for ψ=±π/2

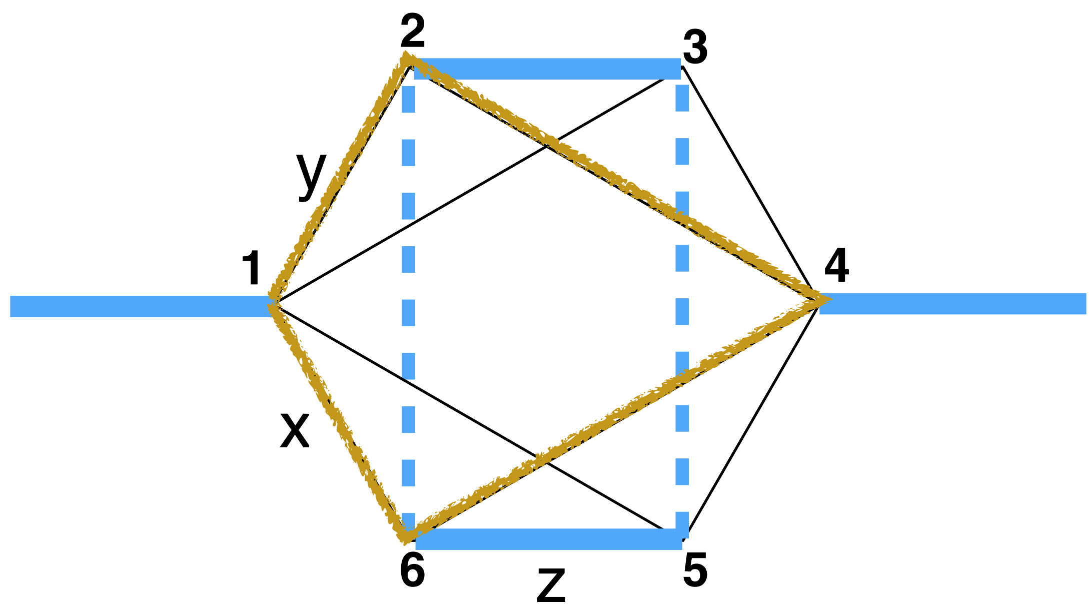
- same process in the triangular Kitaev model Jackeli & Avella(2015)

Quantum order-by-disorder: strong coupling expansion



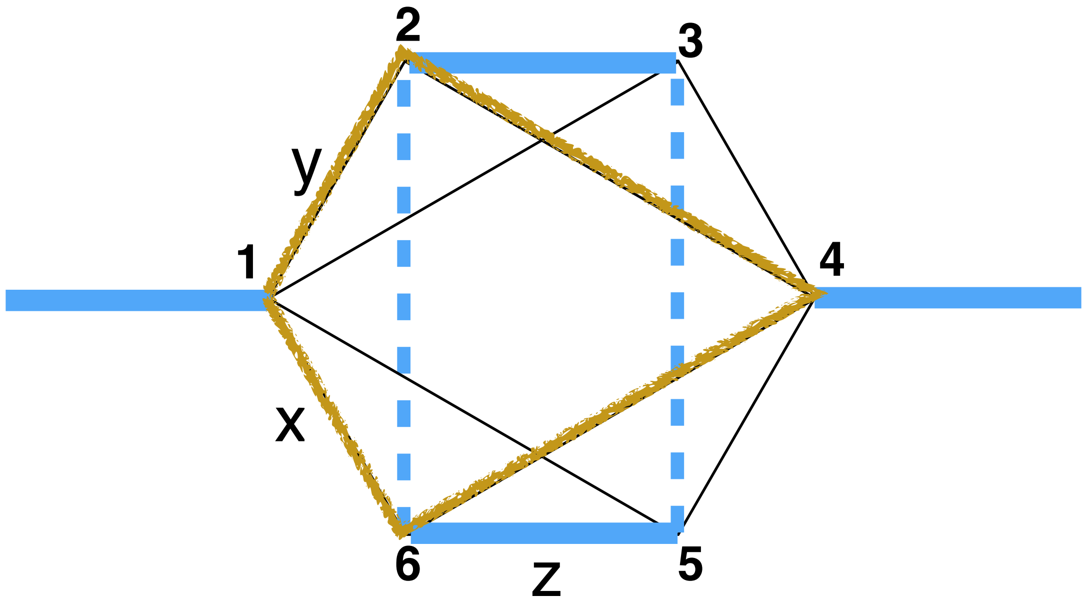
Quantum order-by-disorder: strong coupling expansion

- third type: driven by K_1 & K_2 perturbations



Quantum order-by-disorder: strong coupling expansion

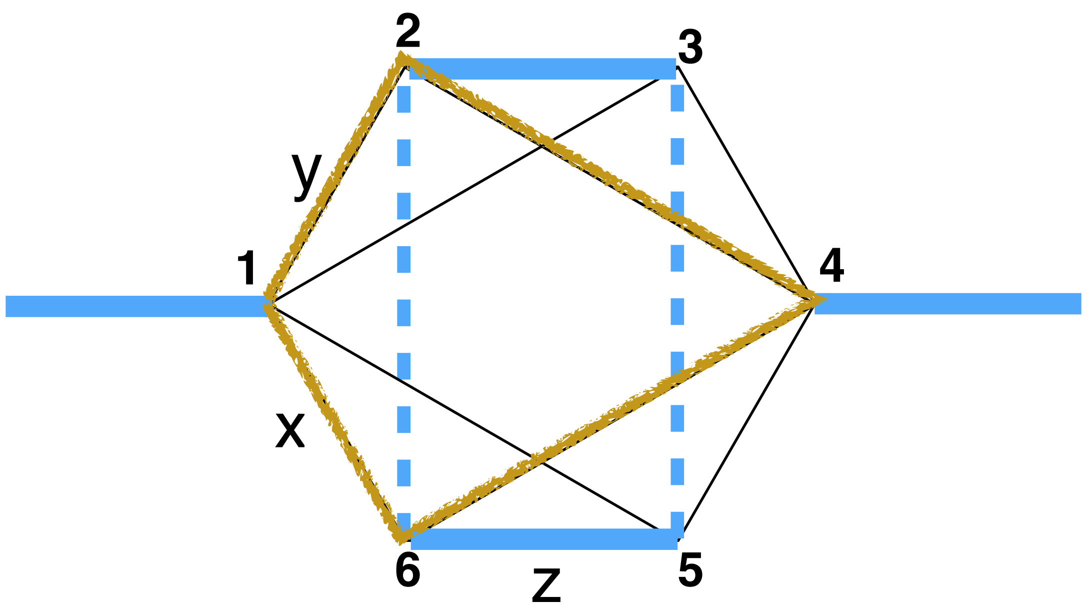
- **third** type: driven by K_1 & K_2 perturbations



- *effective* Ising coupling J_2 between **NNN** ladders: $J_2 S_1^z S_4^z$

Quantum order-by-disorder: strong coupling expansion

- third type: driven by K_1 & K_2 perturbations

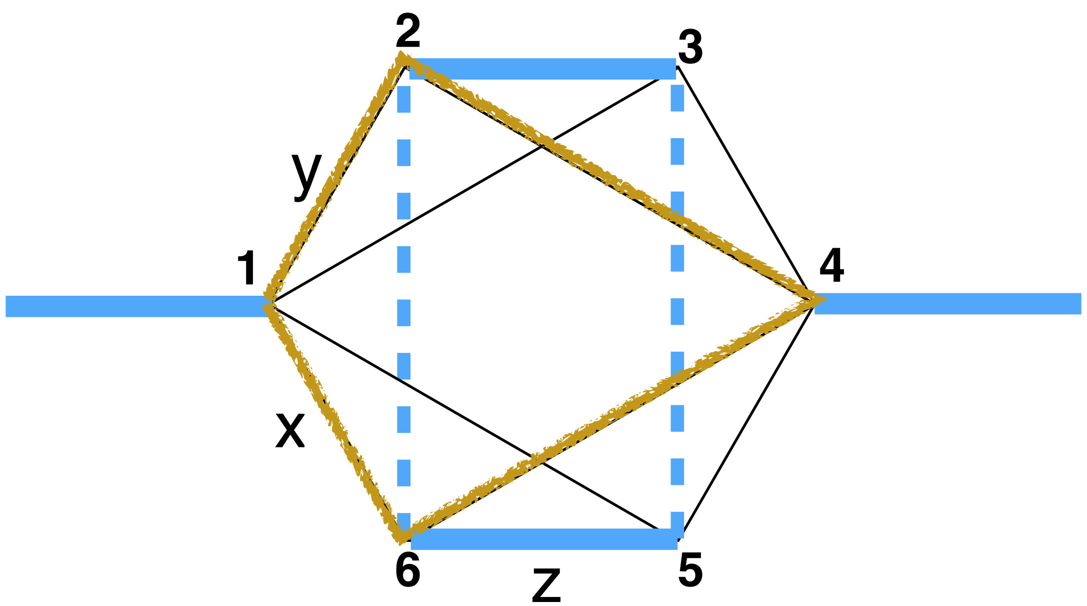


- *effective* Ising coupling J_2 between NNN ladders: $J_2 S_1^z S_4^z$

$$J_2 = \frac{K_1^x K_1^y K_2^x K_2^y}{4(|K_1^z| + 2|K_2^z|)^3} \left[\frac{|K_1^z| + |K_2^z|}{2|K_1^z| + 3|K_2^z|} + \frac{2|K_2^z|}{|K_1^z| + 4|K_2^z|} \right]$$

Quantum order-by-disorder: strong coupling expansion

- third type: driven by K_1 & K_2 perturbations

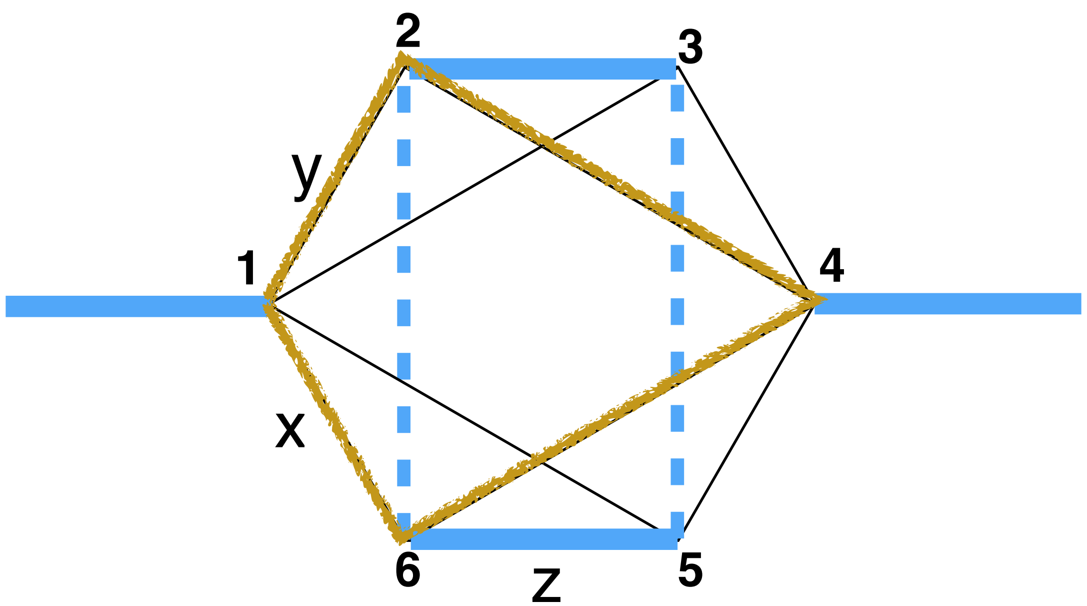


- *effective* Ising coupling J_2 between NNN ladders: $J_2 S_1^z S_4^z$

$$J_2 = \frac{K_1^x K_1^y K_2^x K_2^y}{4(|K_1^z| + 2|K_2^z|)^3} \left[\frac{|K_1^z| + |K_2^z|}{2|K_1^z| + 3|K_2^z|} + \frac{2|K_2^z|}{|K_1^z| + 4|K_2^z|} \right]$$

Quantum order-by-disorder: strong coupling expansion

- third type: driven by K_1 & K_2 perturbations



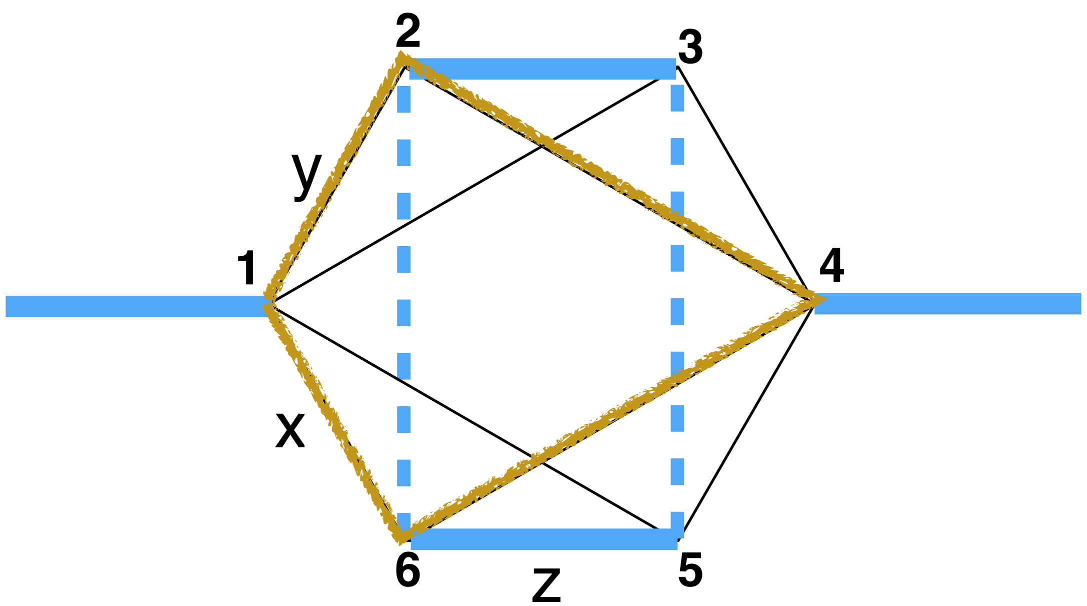
- *effective* Ising coupling J_2 between NNN ladders: $J_2 S_1^z S_4^z$

$$J_2 = \frac{K_1^x K_1^y K_2^x K_2^y}{4(|K_1^z| + 2|K_2^z|)^3} \left[\frac{|K_1^z| + |K_2^z|}{2|K_1^z| + 3|K_2^z|} + \frac{2|K_2^z|}{|K_1^z| + 4|K_2^z|} \right]$$

always AFM

Quantum order-by-disorder: strong coupling expansion

- third type: driven by K_1 & K_2 perturbations



- *effective* Ising coupling J_2 between NNN ladders: $J_2 S_1^z S_4^z$

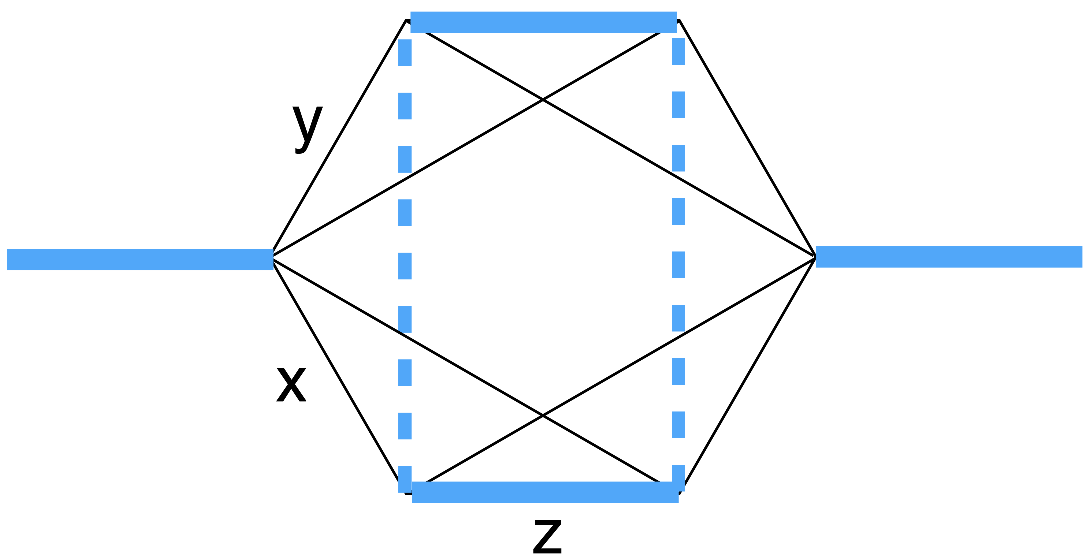
$$J_2 = \frac{K_1^x K_1^y K_2^x K_2^y}{4(|K_1^z| + 2|K_2^z|)^3} \left[\frac{|K_1^z| + |K_2^z|}{2|K_1^z| + 3|K_2^z|} + \frac{2|K_2^z|}{|K_1^z| + 4|K_2^z|} \right]$$

always AFM

large for ψ=π/4, 3π/4, 5π/4, 7π/4

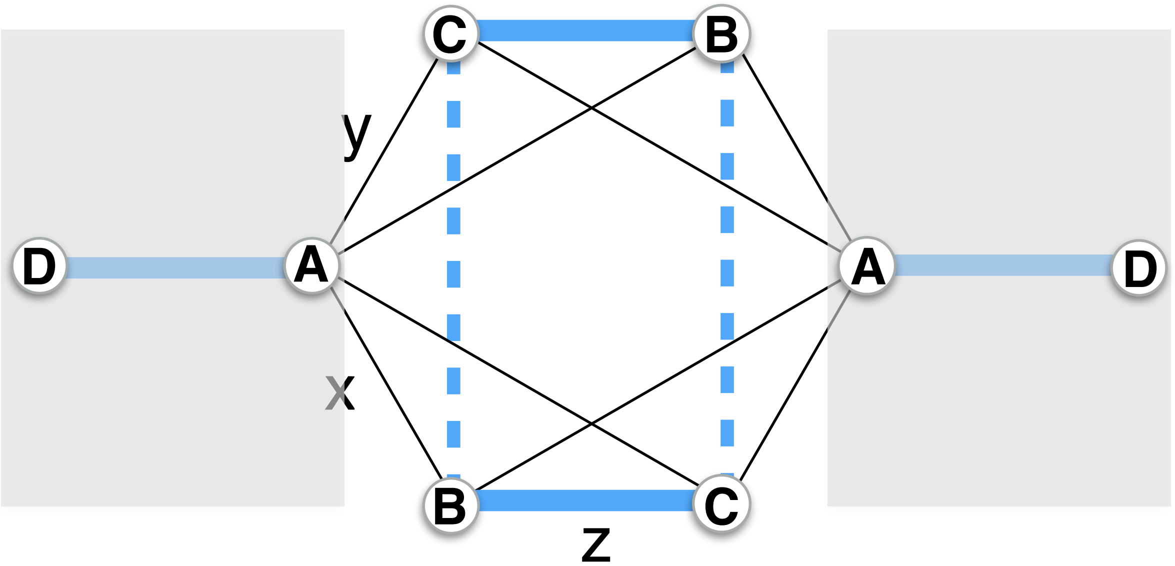
Quantum order-by-disorder: no coupling between NN ladders

- NN ladders do not talk to each other: true to all orders in pert. theory



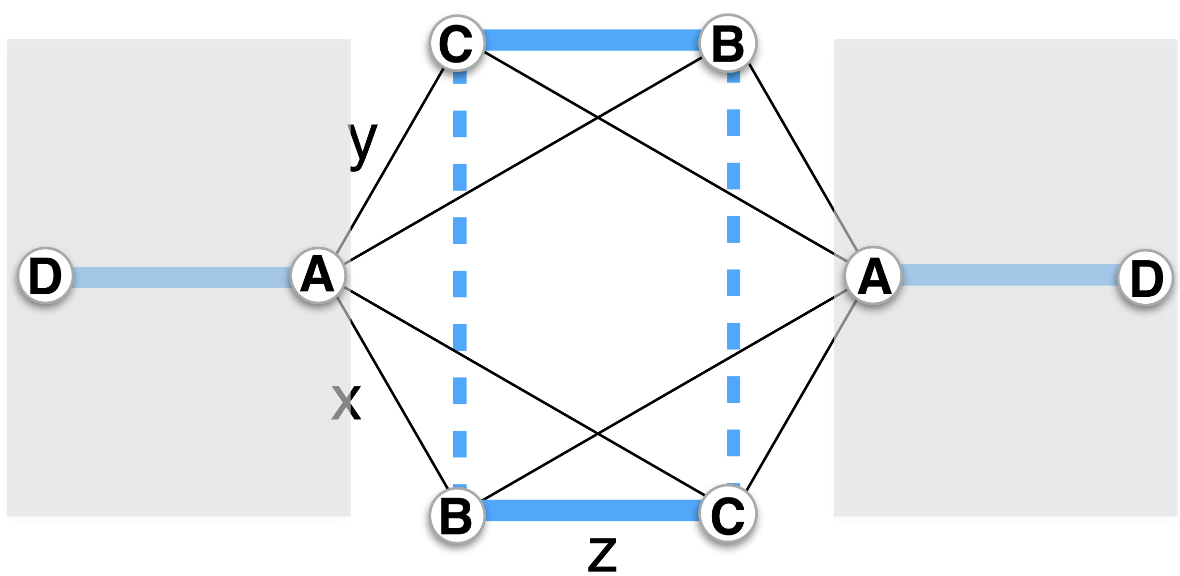
Quantum order-by-disorder: no coupling between NN ladders

- NN ladders do not talk to each other: true to all orders in pert. theory



Quantum order-by-disorder: no coupling between NN ladders

- NN ladders do not talk to each other: true to all orders in pert. theory

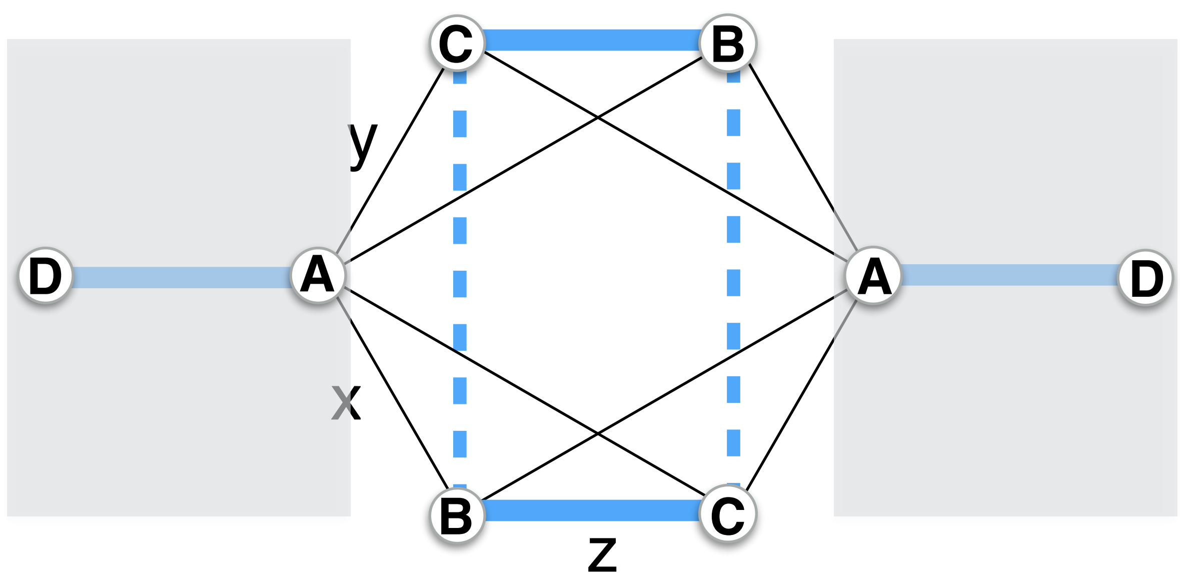


- gauge-like symmetry: $S_{xyz} = \begin{pmatrix} A & B & C & D \\ \mathbf{1} & C_{2x} & C_{2y} & C_{2z} \end{pmatrix}$

$(A_x, A_y, A_z) \rightarrow (A_x, A_y, A_z)$	$(B_x, B_y, B_z) \rightarrow (B_x, -B_y, -B_z)$
$(D_x, D_y, D_z) \rightarrow (-D_x, -D_y, D_z)$	$(C_x, C_y, C_z) \rightarrow (-C_x, C_y, -C_z)$

Quantum order-by-disorder: no coupling between NN ladders

- NN ladders do not talk to each other: true to all orders in pert. theory



- gauge-like symmetry: $S_{xyz} = \begin{pmatrix} A & B & C & D \\ \mathbf{1} & C_{2x} & C_{2y} & C_{2z} \end{pmatrix}$

$(A_x, A_y, A_z) \to (A_x, A_y, A_z)$

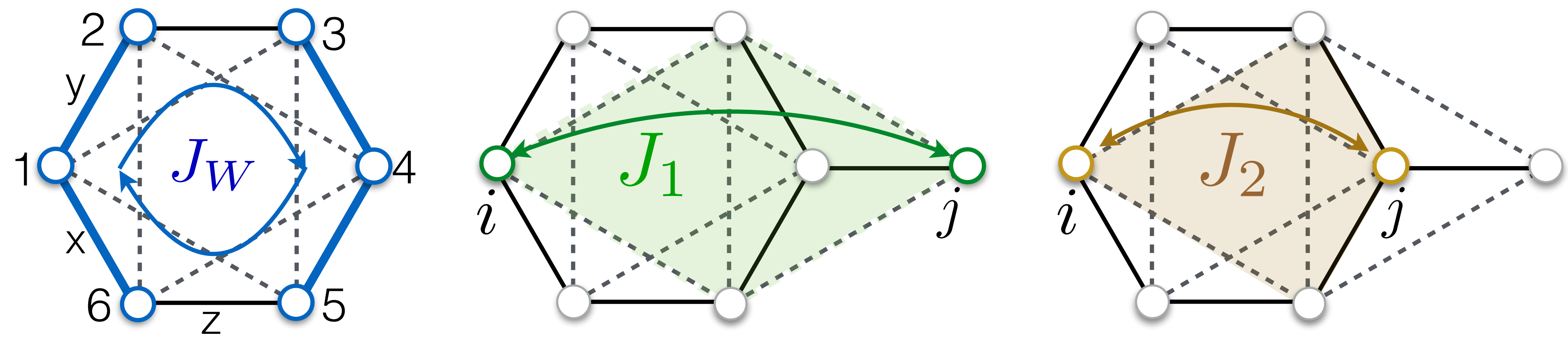
$(D_x, D_y, D_z) \to (-D_x, -D_y, D_z)$

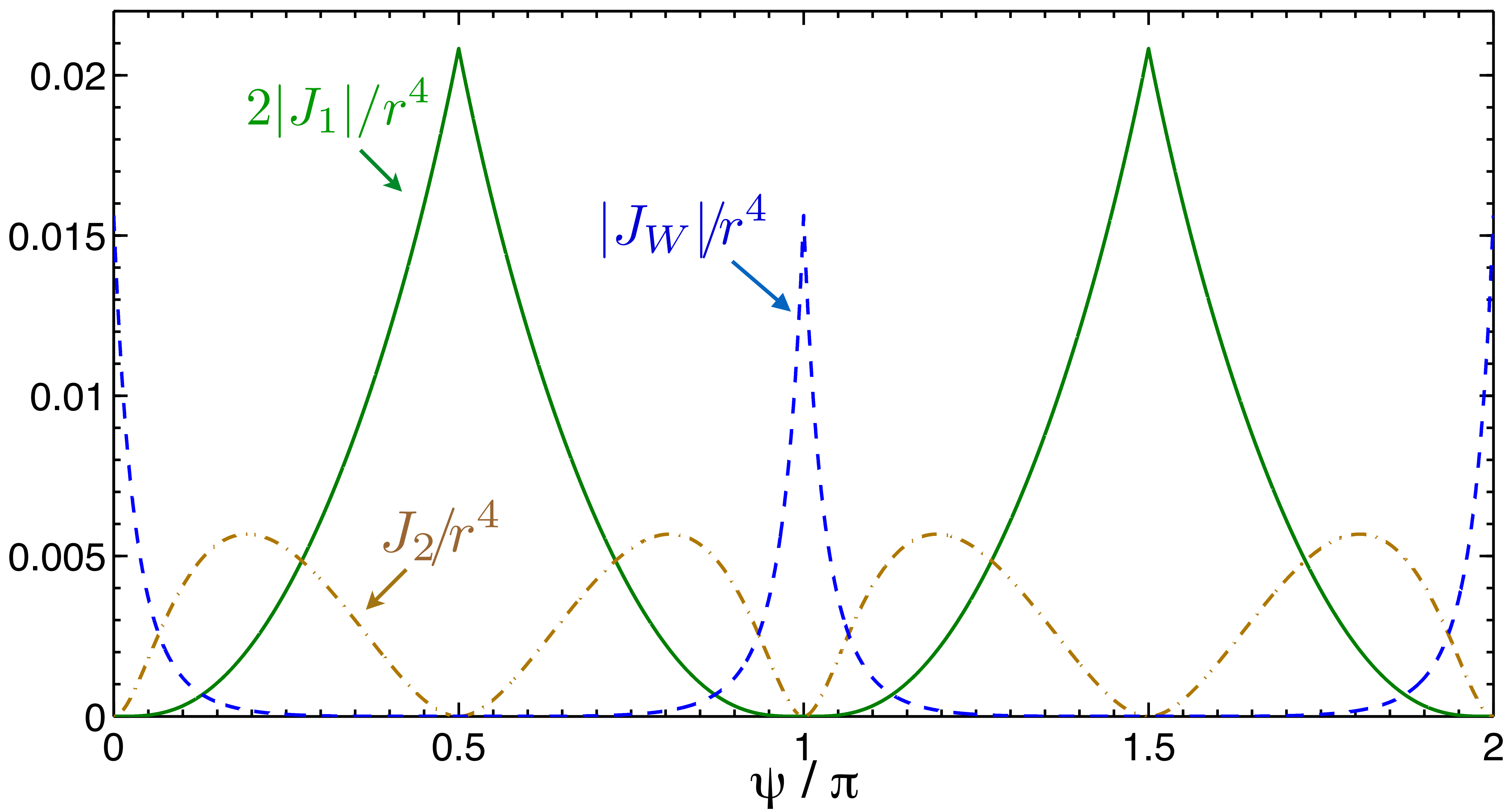
$(B_x, B_y, B_z) \to (B_x, -B_y, -B_z)$

$(C_x, C_y, C_z) \to (-C_x, C_y, -C_z)$

$J_{nn} A_z B_z \to -J_{nn} A_z B_z$, so J_{nn} must vanish identically !

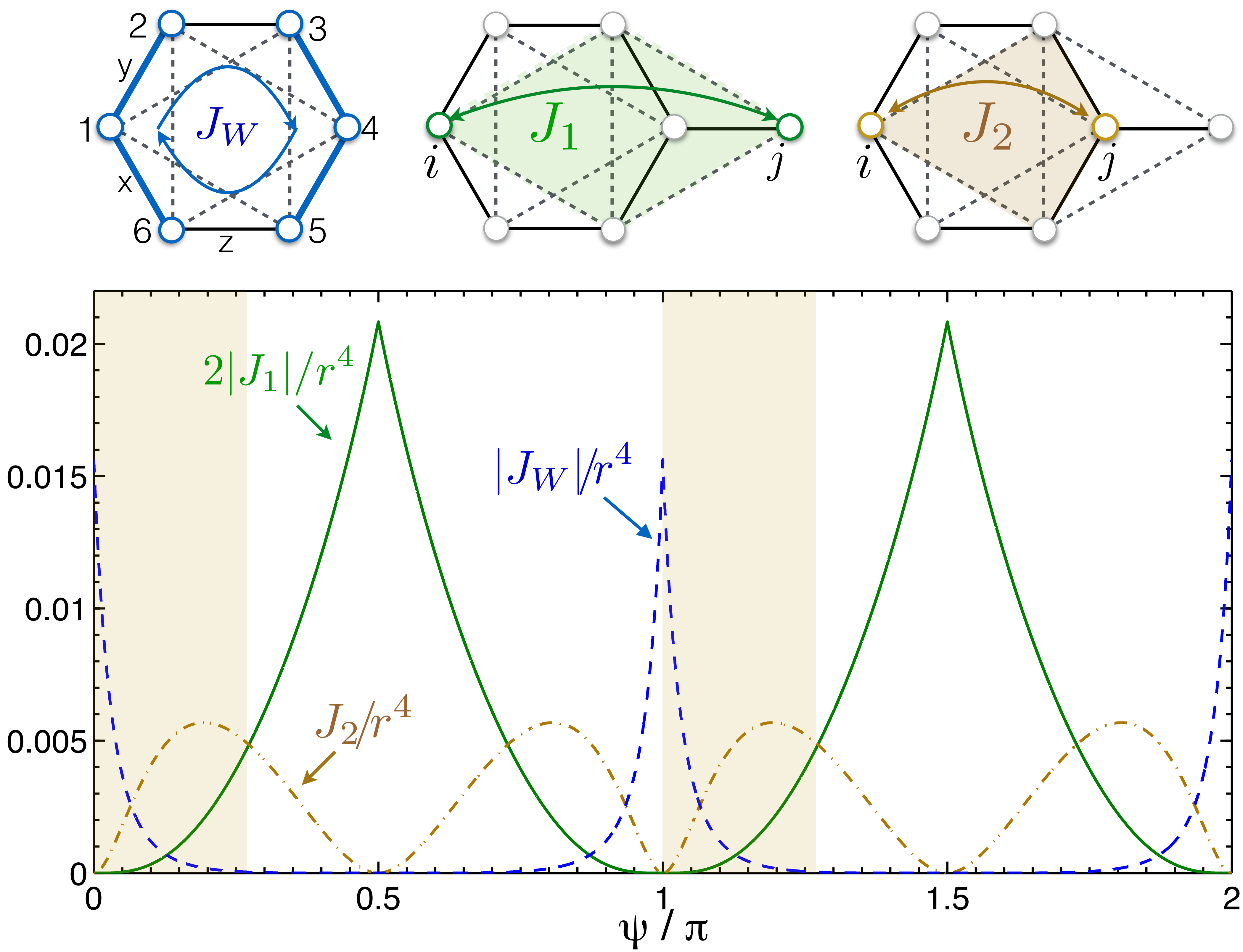
Quantum order-by-disorder: strong coupling expansion





→ all magnetic phases can be explained by J_1

Quantum order-by-disorder: strong coupling expansion

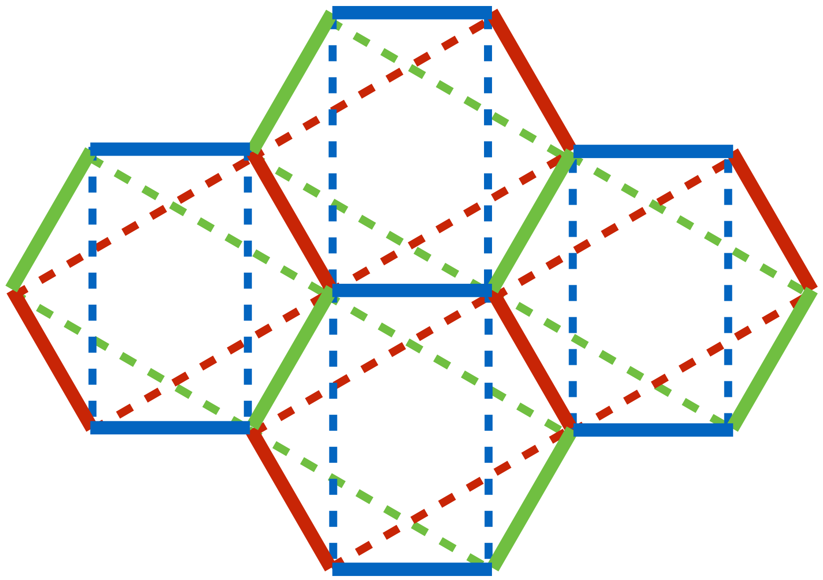


→ all magnetic phases can be explained by J_1

Take home messages

Take home messages

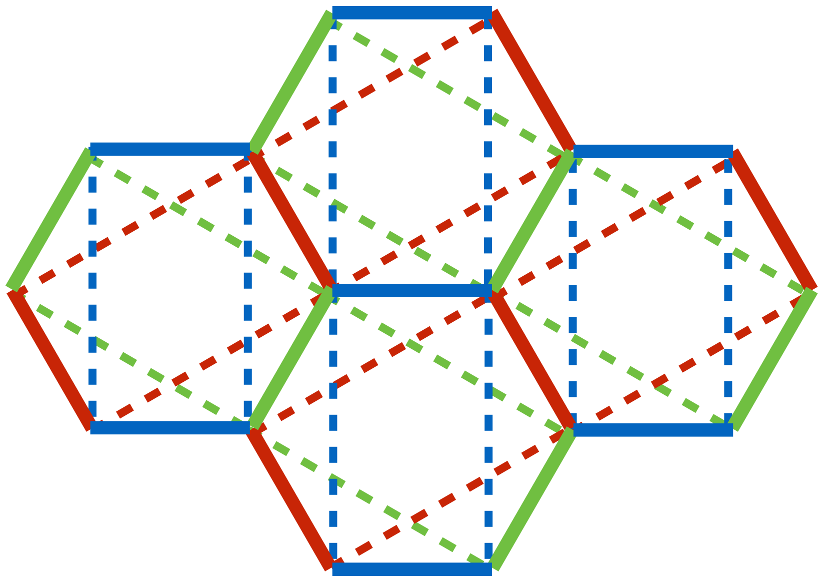
- on a conceptual level, K_1-K_2 model hosts some very striking & intriguing features:
  - several degenerate quantum GS's, some **qualitatively different** from others.
  - LRO states seem very classical. Yet, classical limit hosts very different physics.



Take home messages

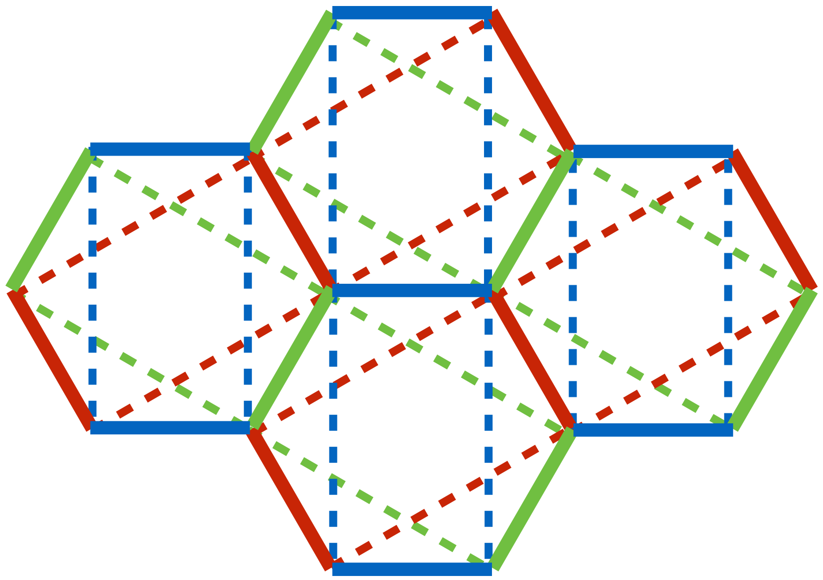
- on a conceptual level, K_1-K_2 model hosts some very striking & intriguing features:
  - several degenerate quantum GS's, some **qualitatively different** from others.
  - LRO states seem very classical. Yet, classical limit hosts very different physics.

- K_2 must be very relevant at least in Na213:
  - stabilizes zigzag state without introducing J_2 and J_3
  - accounts for the bond directional character of spin correlations
  - can account for a large piece of Curie-Weiss temperature



Take home messages

- on a conceptual level, K_1-K_2 model hosts some very striking & intriguing features:
  - several degenerate quantum GS's, some **qualitatively different** from others.
  - LRO states seem very classical. Yet, classical limit hosts very different physics.



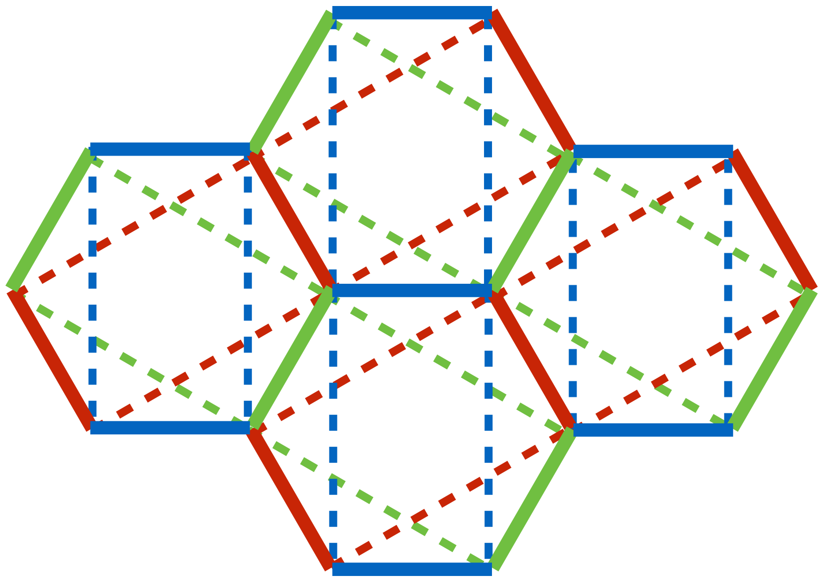
- K_2 must be very relevant at least in Na213:
  - stabilizes zigzag state without introducing J_2 and J_3
  - accounts for the bond directional character of spin correlations
  - can account for a large piece of Curie-Weiss temperature

- direction of the magnetic moments in Na213: Chun et al, Nat. Phys. ('15)

  can be accounted for by thermal fluctuations or **very small Γ** terms Perkins et al

Take home messages

- on a conceptual level, K_1-K_2 model hosts some very striking & intriguing features:
  - several degenerate quantum GS's, some **qualitatively different** from others.
  - LRO states seem very classical. Yet, classical limit hosts very different physics.



- K_2 must be very relevant at least in Na213:
  - stabilizes zigzag state without introducing J_2 and J_3
  - accounts for the bond directional character of spin correlations
  - can account for a large piece of Curie-Weiss temperature

- direction of the magnetic moments in Na213: Chun et al, Nat. Phys. ('15)

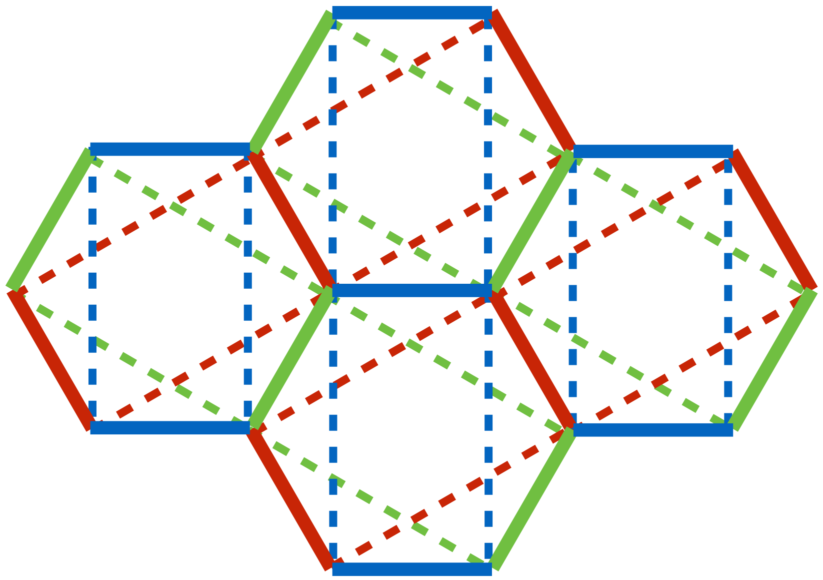
  can be accounted for by thermal fluctuations or **very small Γ** terms Perkins et al

- Kitaev spin liquid is unfortunately very fragile against K_2.

  K_2 can be controlled by chemical substitution of Na.

Take home messages

- on a conceptual level, K_1-K_2 model hosts some very striking & intriguing features:
  - several degenerate quantum GS's, some **qualitatively different** from others.
  - LRO states seem very classical. Yet, classical limit hosts very different physics.



- K_2 must be very relevant at least in Na213:
  - stabilizes zigzag state without introducing J_2 and J_3
  - accounts for the bond directional character of spin correlations
  - can account for a large piece of Curie-Weiss temperature

- direction of the magnetic moments in Na213: Chun et al, Nat. Phys. ('15)

  can be accounted for by thermal fluctuations or **very small Γ** terms Perkins et al

- Kitaev spin liquid is unfortunately very fragile against K_2.

  K_2 can be controlled by chemical substitution of Na.

Thank you very much for your attention !