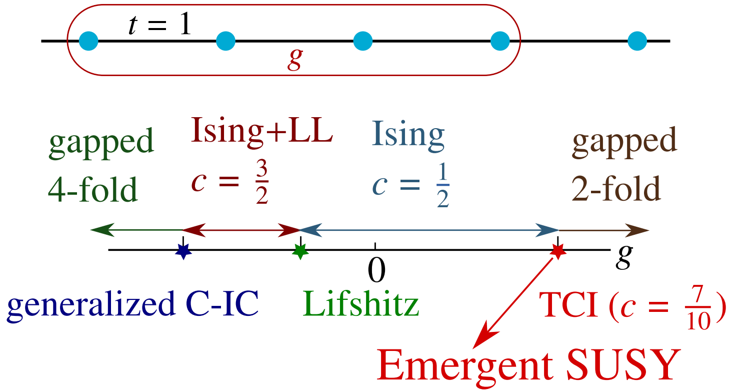
Interacting Majorana chain: SUSY and other novel phases

Armin Rahmani

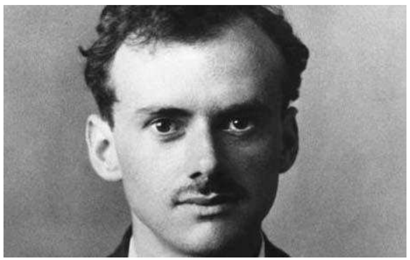
AR, X. Zhu, M. Franz, and I. Affleck, Phys. Rev. Lett. **115**, 166401 (2015).

AR, X. Zhu, M. Franz, and I. Affleck, arXiv:1505.03966.



Majorana fermions and SUSY

Most fermionic elementary particles are not their own antiparticle:



1928: complex equation for relativistic fermions

$e \neq e^+$

$c \neq c^\dagger$

1937: real equation for relativistic fermions

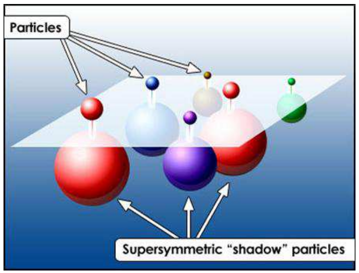
$=$

$\gamma = \gamma^\dagger$

Thus far elusive, status of neutrinos remain unclear

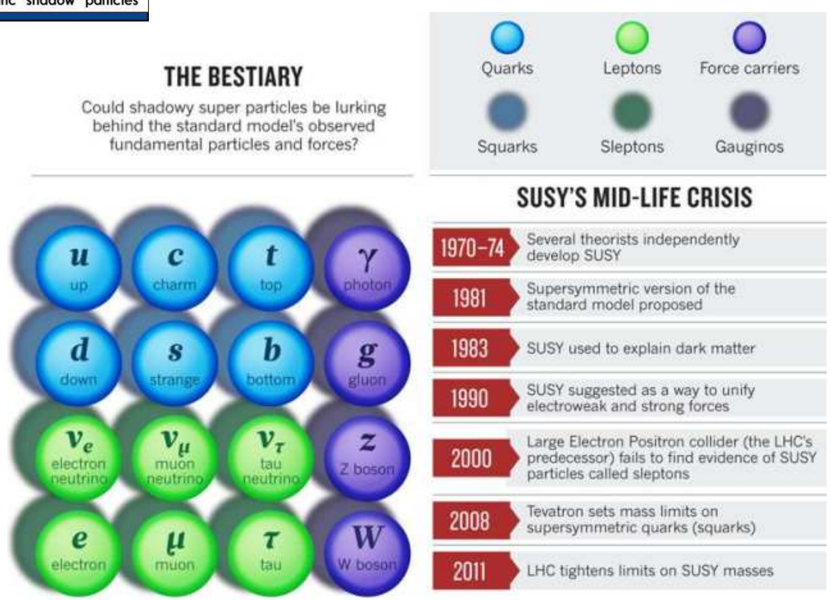
Majorana fermions and SUSY

Another pillar of theoretical physics: supersymmetry (SUSY)



Appealing theory

Still unconfirmed by experiments



Majorana fermions in condensed matter

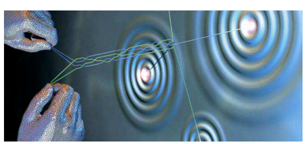
Majoranas can emerge as collective excitations when many electrons interacts.

The simplest realization relies on spinless superconductivity

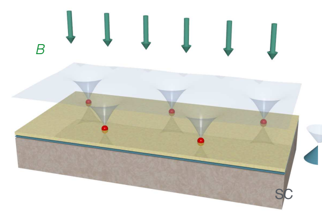
$$\gamma = c + c^\dagger$$

vortices in *p*-wave superconductors

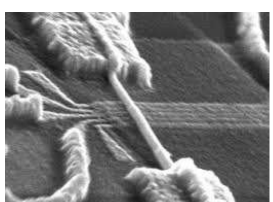
$$\langle c_\alpha(\mathbf{p}) c_\beta(-\mathbf{p}) \rangle \sim \delta_{\alpha\uparrow}\delta_{\beta\uparrow}\left(p_x + i p_y\right)$$

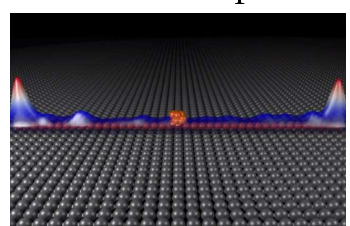


vortices in *s*-wave superconductors in proximity to TI surface states



also semiconducting wires with SOC, magnetic adatom on superconductor, ...





Majorana fermions in condensed matter

Models with emergent Majoranas as degrees of freedom

Let us focus on 1D:

Hubbard: $H = -t \sum_i \left(c^\dagger_{i\sigma} c_{i+1\sigma} + \text{H.c.} \right) + \sum_i U n_{i\uparrow} n_{i\downarrow}$

Spinless Hubbard: $H = -t \sum_i \left(c^\dagger_i c_{i+1} + \text{H.c.} \right) + \sum_i V n_i n_{i+1}$

Majorana fermions in condensed matter

Models with emergent Majoranas as degrees of freedom

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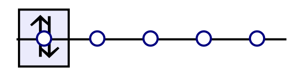
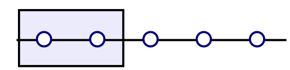
Hubbard: $H = -t \sum_i \left(c^\dagger_{i\sigma} c_{i+1\sigma} + \text{H.c.} \right) + \sum_i U n_{i\uparrow} n_{i\downarrow}$



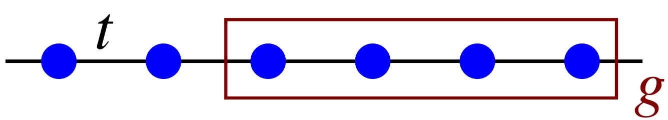
Spinless Hubbard: $H = -t \sum_i \left(c^\dagger_i c_{i+1} + \text{H.c.} \right) + \sum_i V n_i n_{i+1}$



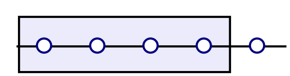
Nearest neighbor because $n_i n_i = n_i$

$$\gamma_j \gamma_j = 1$$

Minimal interaction involves four sites

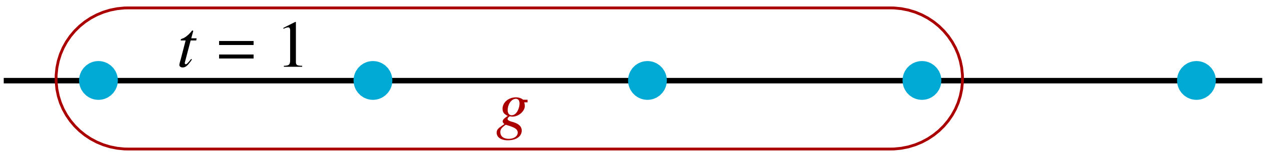


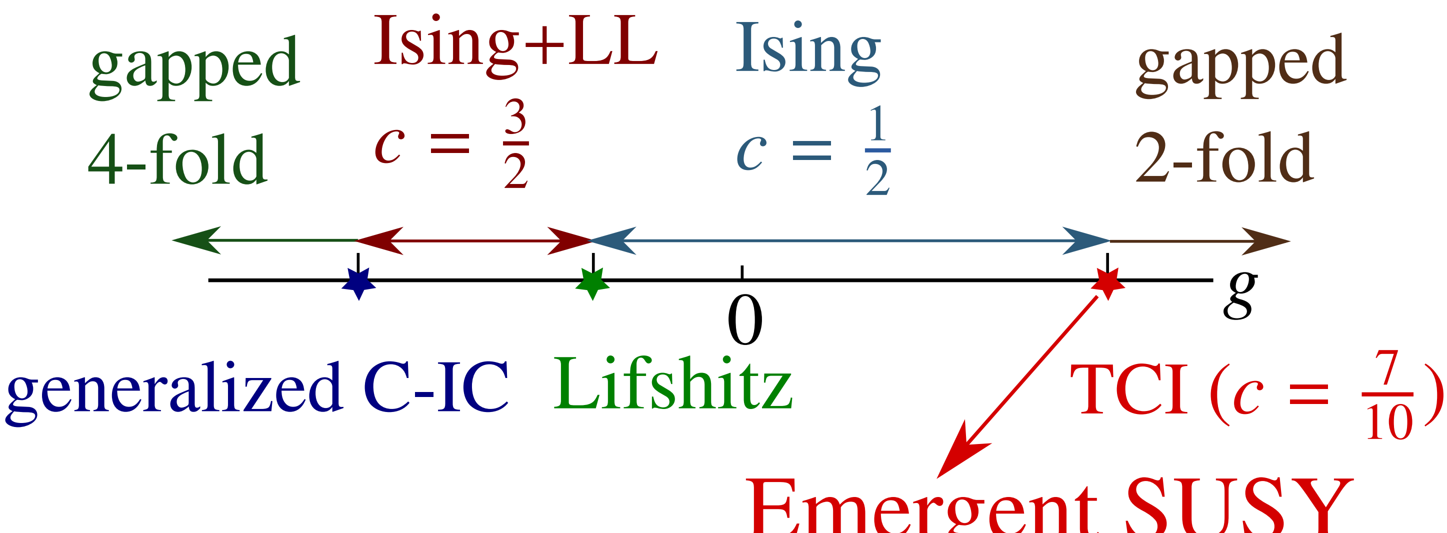
$$H = it \sum_j \gamma_j \gamma_{j+1} + g \sum_j \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3}$$



Interacting Majorana chain

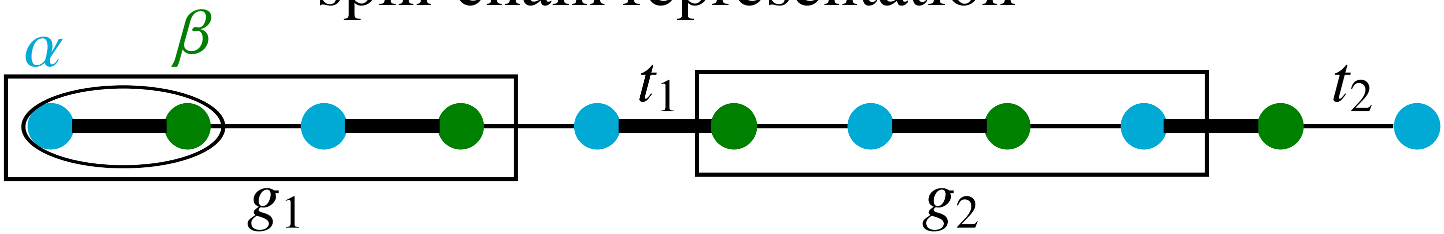
$$H = it \sum_j \gamma_j \gamma_{j+1} + g \sum_j \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3}$$



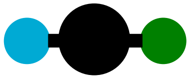


Interacting Majorana chain

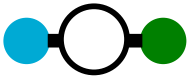
spin-chain representation



$$c_j = \alpha_j + i\beta_j,\ n_j = c_j^\dagger c_j,\ \sigma_j^z = 2n_j - 1$$



$n_j = 1,\ \sigma_j^z = 1$



$n_j = 0,\ \sigma_j^z = -1$

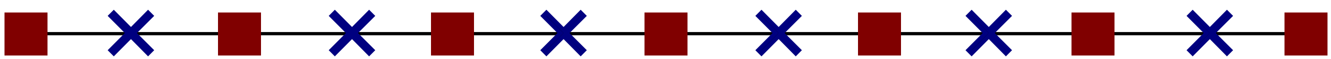
$$H = t_1 \sum_j \sigma_j^z - t_2 \sum_j \sigma_j^x \sigma_{j+1}^x - g_1 \sum_j \sigma_j^z \sigma_{j+1}^z - g_2 \sum_j \sigma_j^x \sigma_{j+2}^x$$

Interacting Majorana chain: strong coupling

$$H = t_1 \sum_j \sigma^z_j - t_2 \sum_j \sigma^x_j \sigma^x_{j+1} - g_1 \sum_j \sigma^z_j \sigma^z_{j+1} - g_2 \sum_j \sigma^x_j \sigma^x_{j+2}$$

$$t \rightarrow 0$$

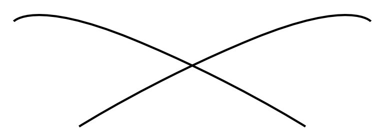
σ^z_j $\qquad\qquad$ $\tau^z_j = \sigma^z_j \sigma^z_{j+1}$



$t \rightarrow 0$ $\qquad H_g = -g_1 \sum_j \tau^z_j - g_2 \sum_j \tau^x_{j-1} \tau^x_j \tau^x_{j+1} \tau^x_{j+2}$

Similar to 8-state Potts model

We expect a <u>first-order</u> transition at self-dual $g_1 = g_2$



At $g_1 = g_2$, the system is gapped.

At $g_1 = g_2$, both states (dominated by g_1 and g_2) are present.

Interacting Majorana chain: strong coupling

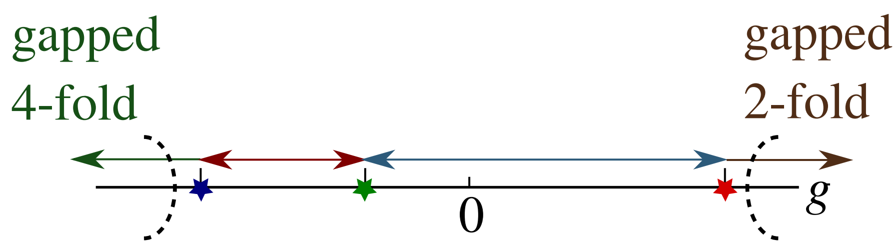
States dominated by g_1:

What does t_1 do to states dominated by g_1?

$$H = t_1 \sum_j \sigma_j^z - t_2 \sum_j \sigma_j^x \sigma_{j+1}^x - g_1 \sum_j \sigma_j^z \sigma_{j+1}^z - g_2 \sum_j \sigma_j^x \sigma_{j+2}^x$$

$g_1 < 0$: does not split degeneracy

$g_1 > 0$: splits degeneracy



Interacting Majorana chain: weak coupling

$t \to \infty$

$$H = t \sum_j \sigma^z_j - t \sum_j \sigma^x_j \sigma^x_{j+1} - g \sum_j \sigma^z_j \sigma^z_{j+1} - g \sum_j \sigma^x_j \sigma^x_{j+2}$$

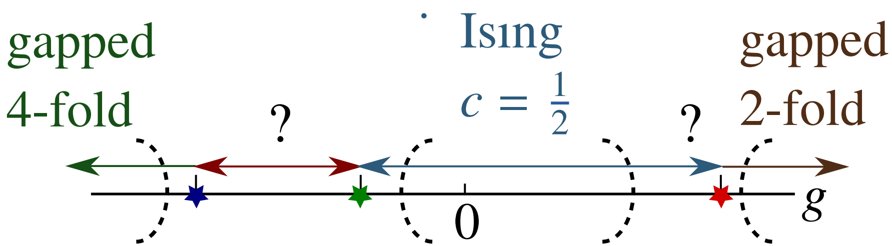
Ising critical point

$$H_t \approx iv \int dx \left(\gamma_R \partial_x \gamma_R - \gamma_L \partial_x \gamma_L\right)$$

$$H_g = -2g \int dx \gamma_L \left(\partial_x \gamma_L\right) \gamma_R \left(\partial_x \gamma_R\right) + \cdots$$

irrelevant in RG

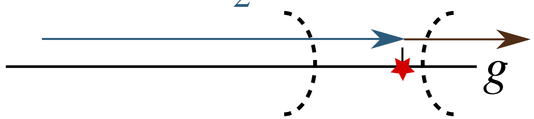
Low-energy Majoranas: $\gamma_j \approx 2\gamma_L(j) + 2(-1)^j \gamma_R(j)$



Interacting Majorana chain: TCI

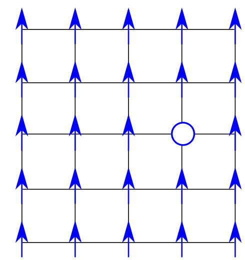
Let us assume a direct transition

2nd-order transition in $t_1 - t_2$ ← Ising $c = \frac{1}{2}$ gapped 2-fold → 1st-order transition in $g_1 - g_2$



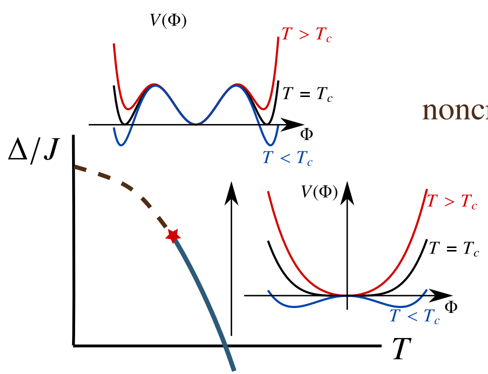
Tricritical Ising is the most natural choice.

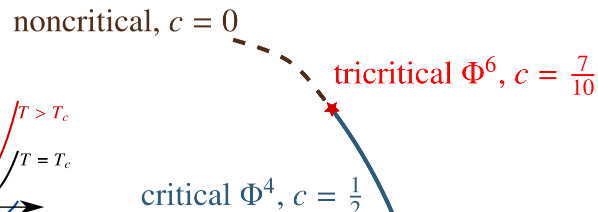
Classical Ising with vacancies: $H = -J \sum_{\langle ij \rangle} s_i s_j + \Delta \sum_i (s_i)^2$



$\Delta = -\infty$, Ising model, 2nd-order transition at T_c

$T = 0$, 1st-order transition at $\Delta = 4J$





Interacting Majorana chain: TCI

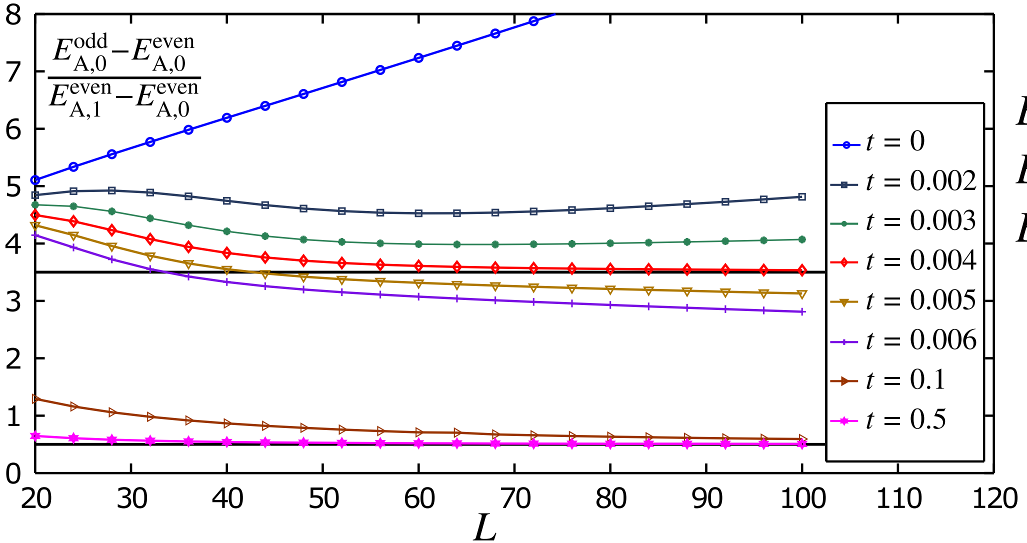
Ising

primary	$\mathbb{I}$	σ	ϵ
dimension	0	$\frac{1}{16}$	$\frac{1}{2}$

TCI

primary	$\mathbb{I}$	σ	σ'	ϵ	ϵ'	ϵ''
dimension	0	$\frac{3}{80}$	$\frac{7}{16}$	$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{2}$

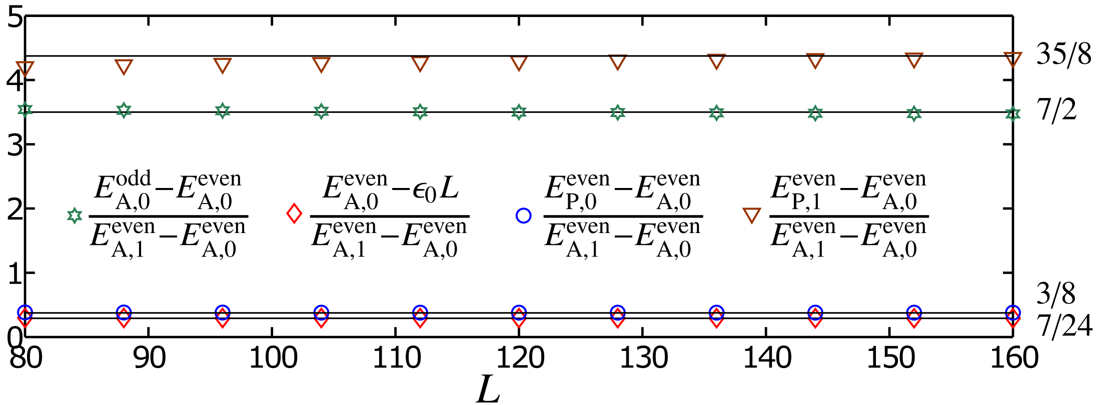
$E^{\rm even}_{A,0} \sim (0,0)$
$E^{\rm even}_{A,1} \sim (\frac{1}{2},\frac{1}{2})$
$E^{\rm odd}_{A,0} \sim (0,\frac{1}{2}),(\frac{1}{2},0)$



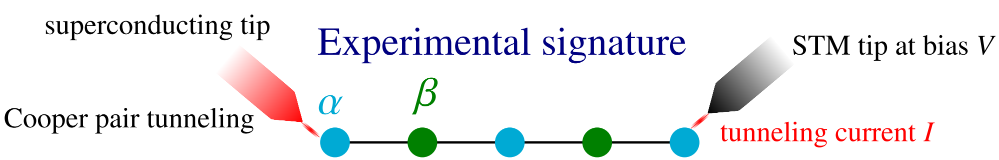
$E^{\rm even}_{A,0} \sim (0,0)$
$E^{\rm even}_{A,1} \sim (\frac{1}{10},\frac{1}{10})$
$E^{\rm odd}_{A,0} \sim (\frac{3}{5},\frac{1}{10})$

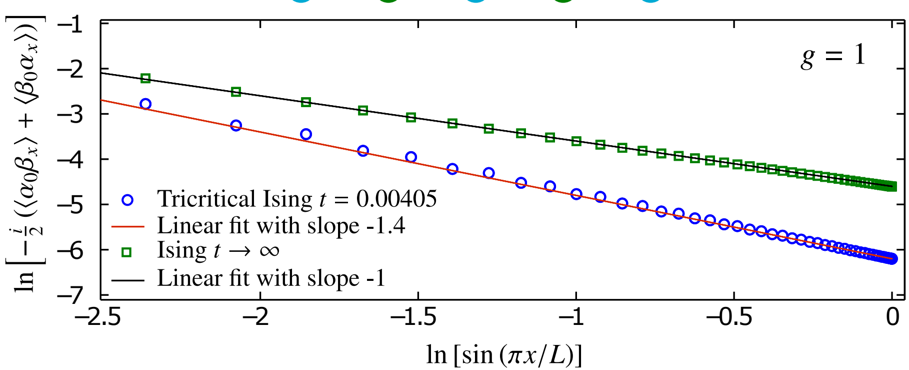
$h+\bar{h} = \frac{7}{10}$
$h-\bar{h} = \frac{1}{2}$

Interacting Majorana chain: TCI



Experimental signature





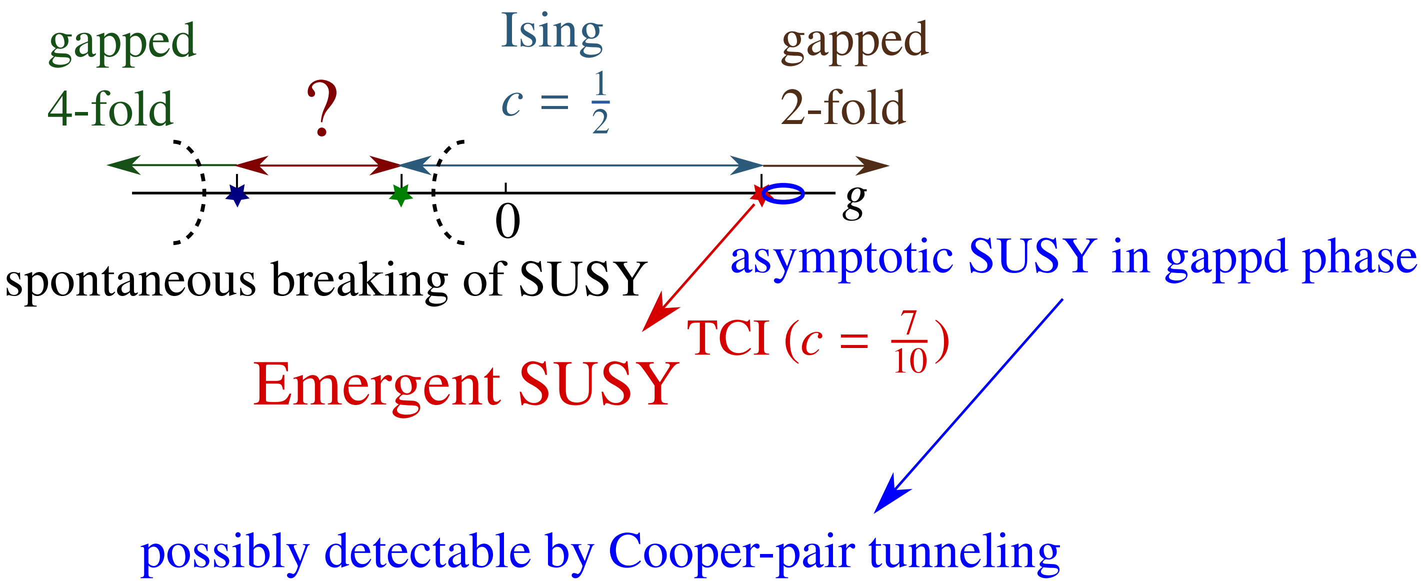
TCI

$I_{\rm TCI} \propto \text{sign}(V)|V|^{7/5}$

gapped

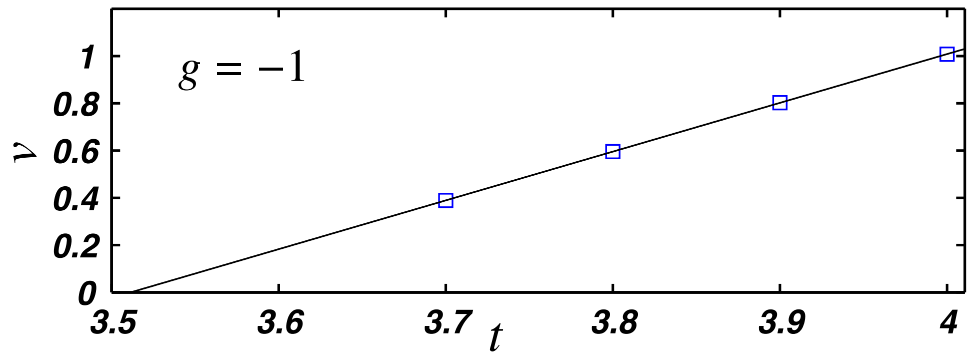
same "mass" for fermions and bosons

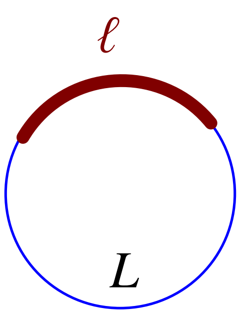
Interacting Majorana chain: TCI

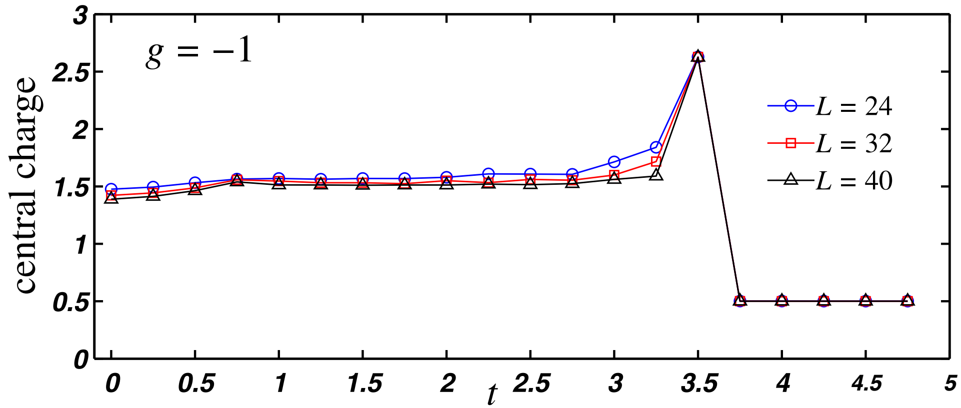


Interacting Majorana chain: $g < 0$

$$\Delta E = \frac{2\pi v}{L}\,(x + \bar{x})$$







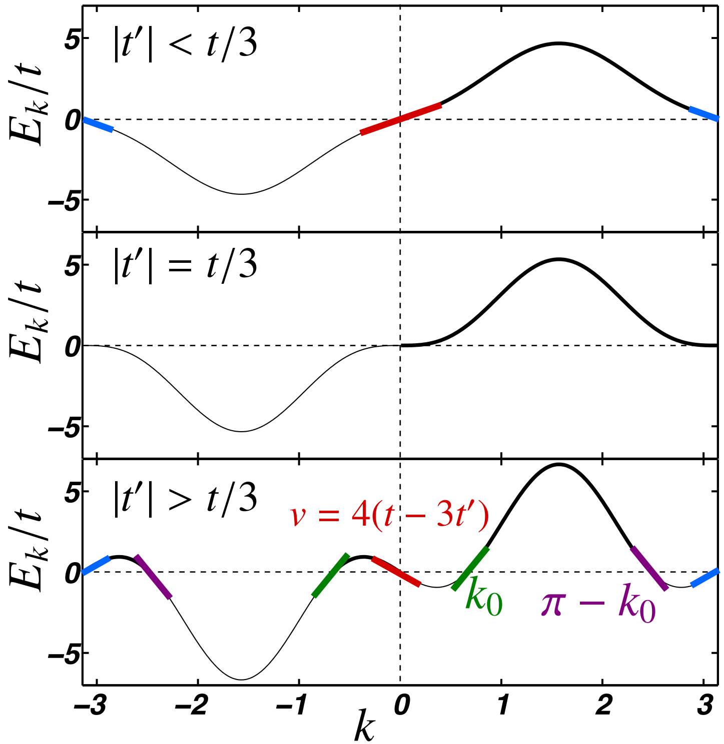
$$S = \frac{c}{3}\log\left[\frac{L}{\pi a}\sin\left(\frac{\pi\ell}{L}\right)\right] + \text{const.}$$

Interacting Majorana chain: $g < 0$

A jump in c from $\frac{1}{2}$ to $\frac{3}{2}$ suggests a Lifshitz transition.

Similar transition can arise without interactions

Let's consider $H = i\sum_j \gamma_j[t\gamma_{j+1} - t'\gamma_{j+3}] = \frac{1}{2}\sum_k E_k\gamma(-k)\gamma(k)$

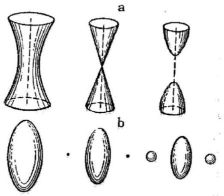


$$E_k = 4\,[t\sin k - t'\sin(3k)]$$

$$E_k \propto k^3, \text{ when } v \to 0$$

$$\boxed{z = 3}$$

$$\sin k_0 = \sqrt{\frac{3t'-t}{4t'}}$$



Lifshitz (JETP 1960)

Interacting Majorana chain: $g < 0$

Conjecture: interactions generate effective t' changing the dispersion.

$$\gamma_j \approx 2\gamma_L(j) + (-1)^j 2\gamma_R(j) + \left[e^{-ik_0 j}\psi_R(j) + e^{i(k_0-\pi)j}\psi_L(j) + \text{H.c.}\right]$$

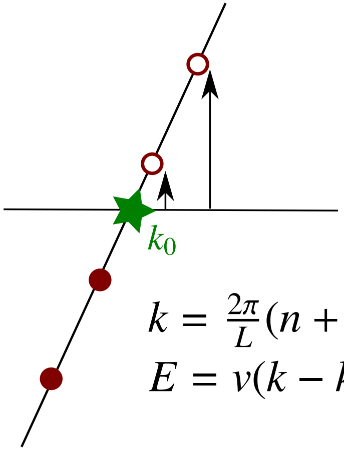
$$H_t = i\int dx[v_0(\gamma_R\partial_x\gamma_R - \gamma_L\partial_x\gamma_L) + v(\psi_R^\dagger\partial_x\psi_R - \psi_L^\dagger\partial_x\psi_L)]$$

$$H_g = \int dx[g_0 \underline{:\psi_L^\dagger\psi_L\psi_R^\dagger\psi_R:} + g_1 \underline{\gamma_R\gamma_L(\psi_L\psi_R + \psi_L^\dagger\psi_R^\dagger)}] + \ldots$$

emergent N and k_0

backscattering, marginal, K irrelevant for our range of K

Finite-size spectrum

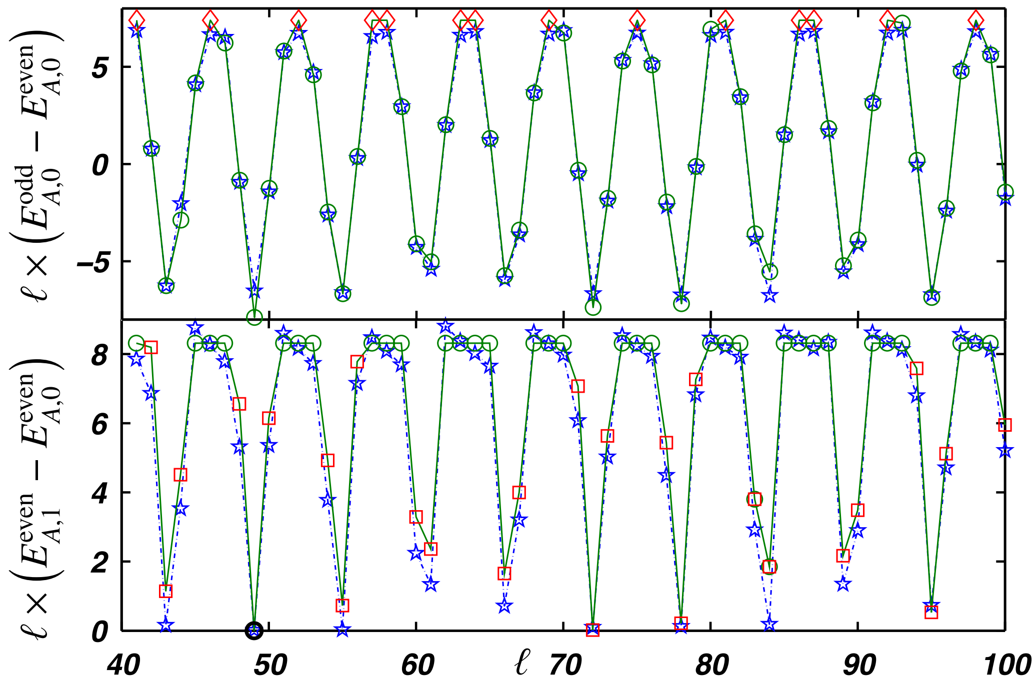


$$k = \frac{2\pi}{L}(n + 1/2)$$
$$E = v(k - k_0)$$

$$\Delta E = \frac{2\pi}{L}\left[\frac{v}{4K}(N - k_0 L/\pi)^2 + \frac{vK}{4}M^2 + \frac{v_0}{4}N_I^2 + \frac{v_0}{4}M_I^2\right]$$

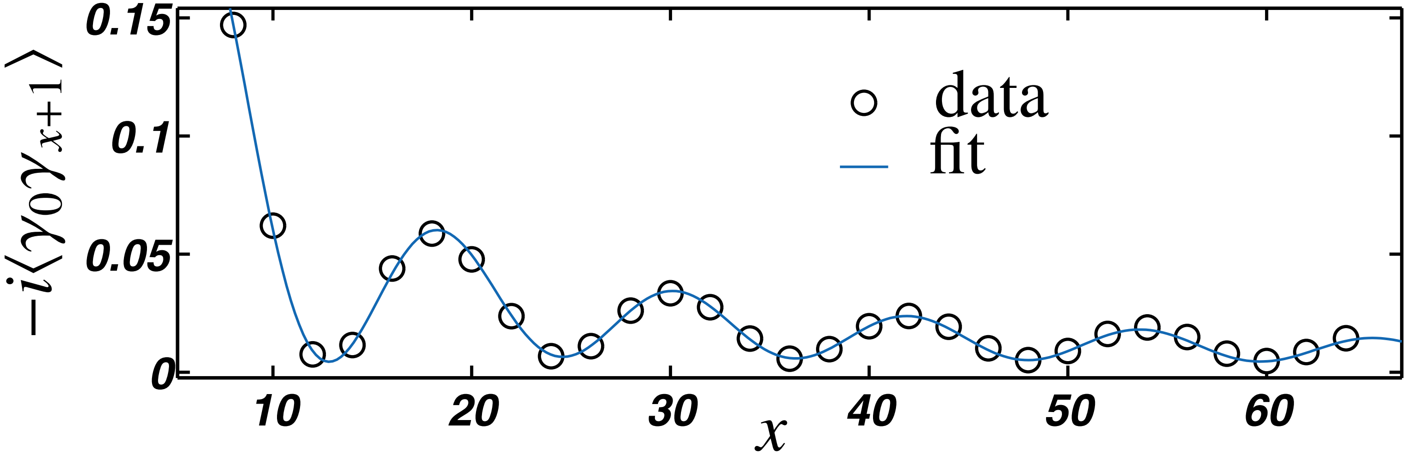
Interacting Majorana chain: $g < 0$

Finite-size spectrum



☆ data ○ fit, LL
□ fit, Ising ○ fit, LL+Ising

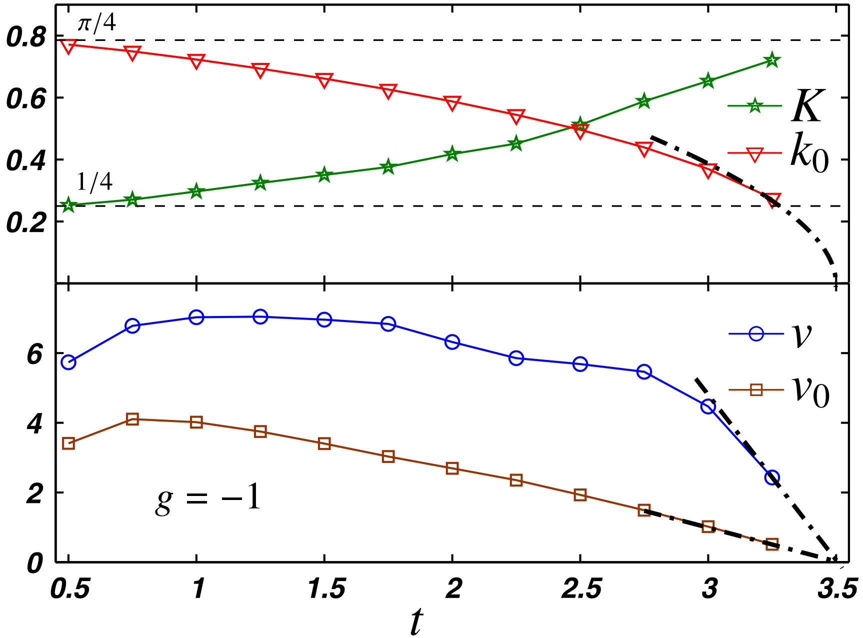
Interacting Majorana chain: $g < 0$

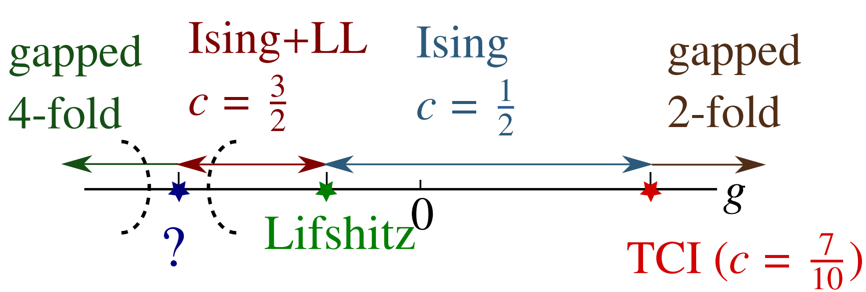


$$i\langle\gamma_0\gamma_{x+1}\rangle \sim a/x + b\sin(k_0 x + \phi)/x^{(K+1/K)/2} \quad \text{even } x$$

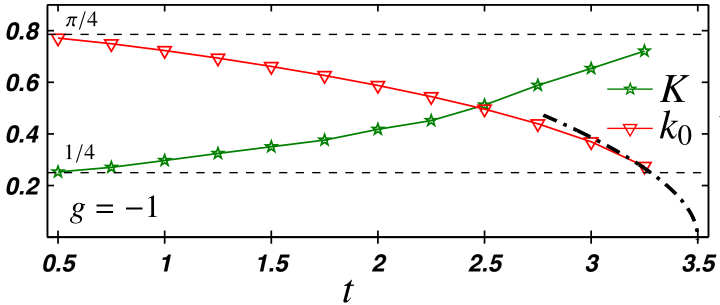
good agreement between fits to spectrum and correlation function

Interacting Majorana chain: $g < 0$



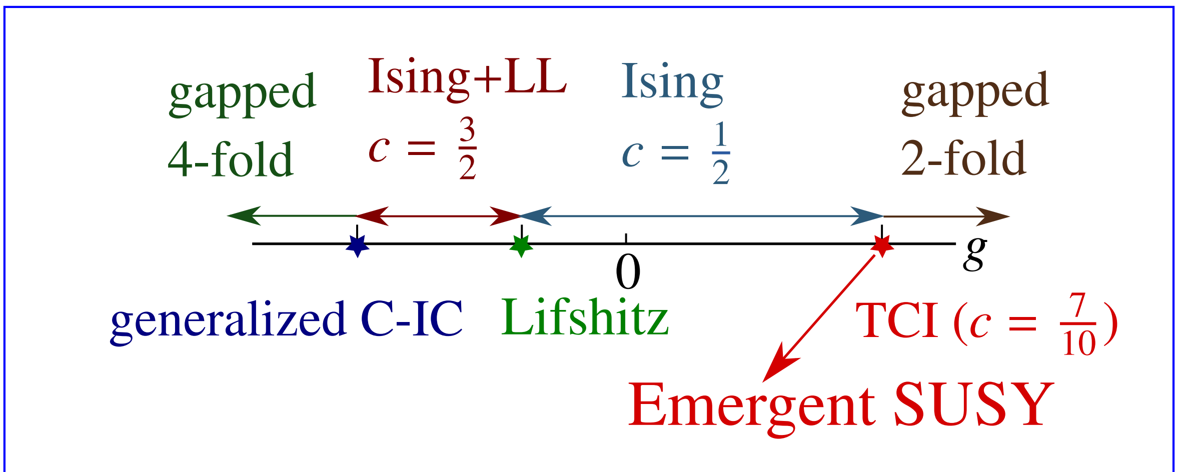


Interacting Majorana chain: $g < 0$



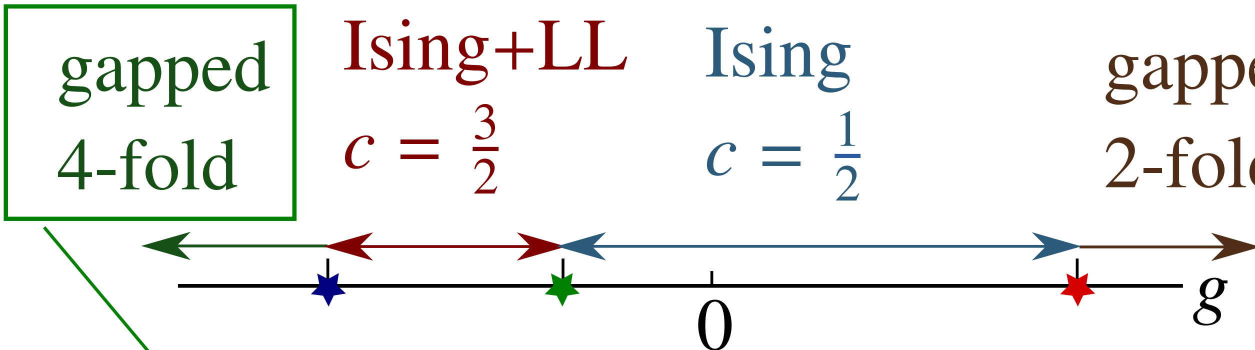
$$H_g = \int dx [g_0 \underbrace{:\psi_L^\dagger \psi_L \psi_R^\dagger \psi_R:}_{\text{marginal, } K} + g_1 \underbrace{\gamma_R \gamma_L (\psi_L \psi_R + \psi_L^\dagger \psi_R^\dagger)}_{\text{irrelevant for our range of } K}] + \ldots$$

$$e^{i(4k_0 - \pi)x} \gamma_R \gamma_L \psi_R^\dagger \partial_x \psi_R^\dagger \psi_L \partial_x \psi_L + \text{H.c.}$$

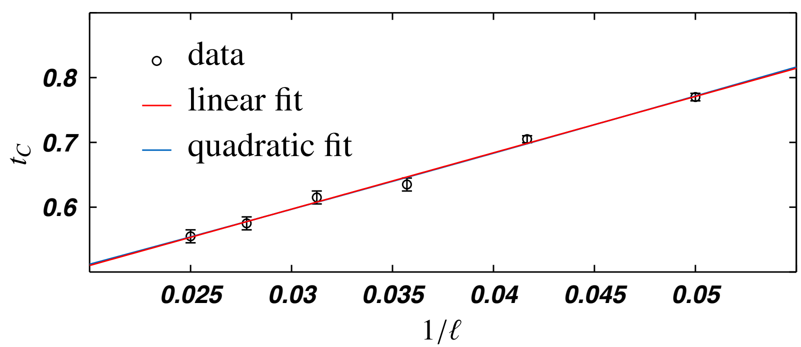


Interacting Majorana chain: $g < 0$

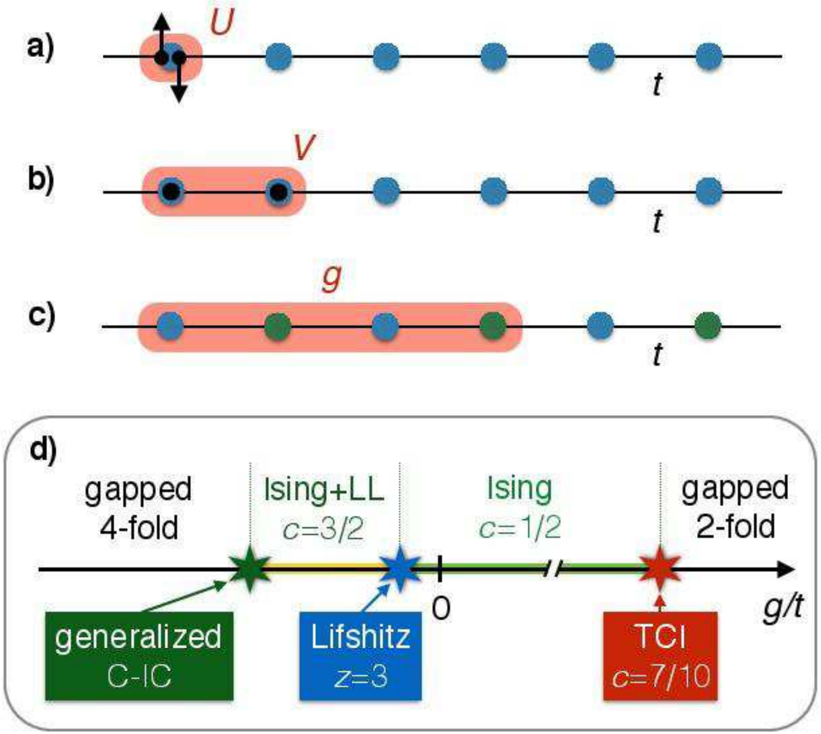
C-IC transition



exact degenercy with PBC and $4N$ Dirac fermions



Summary



a)
U
t
b)
V
t
c)
g
t
d)
gapped
4-fold
Ising+LL
c=3/2
Ising
c=1/2
gapped
2-fold
0
g/t
generalized
C-IC
Lifshitz
z=3
TCI
c=7/10